MATHEMATICS DICTIONARY

PREFACE

The guiding objective in the preparation of this dictionary has been to make it useful for students, engineers, and others using mathematics in their professions.

This dictionary contains all of the definitions which appear in the *James Mathematics Dictionary* (1942–1946). These comprise a well-nigh exhaustive coverage of terms in the range beginning with arithmetic and extending through the calculus. The basic terms in metric differential geometry, theory of functions of real and complex variables, advanced calculus, differential equations, theory of groups, theory of matrices, theory of summability, point-set topology, general analysis (referring in particular to integral equations and the calculus of variations), analytic mechanics and theory of potential have now been added. Many miscellaneous terms have been included because of their importance in applications and in the structure of most sequences of mathematical courses. An extensive coverage of statistical terms has been included.

The appendix contains the usual tables that one desires to have on his desk. Formulas of many sorts, in the fields covered, appear in the context.

Leading words are printed in bold capitals beginning at the left margin. Subheadings are printed in bold at the beginning of paragraphs and in alphabetic order on the basis of their leading words.

Different statements of essentially the same meaning of a term or phrase are separated by semicolons, whereas definitions of different meanings for the same term are numbered: (1), (2), (3), etc.

Citations give the leading word in capitals (unless the citation is under the leading word) followed by a dash and then by the subheading, if necessary, as: ANGLE—adjacent angle.

Although this is by no means a mere word dictionary, neither is it an encyclopedia. It is rather a correlated condensation of mathematical concepts, designed for time-saving reference work. Nevertheless the general reader can come to an understanding of concepts in which he has not been schooled by looking up the unfamiliar terms in the definition at hand and following this procedure down to familiar concepts.

Comments on definitions and discussions of any phase of this dictionary are invited.

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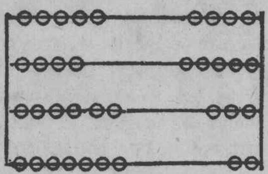
A

AB'A-CUS, *n.* [*pl.* **abaci**]. A counting frame to aid in arithmetic computation; an instructive plaything for children; used as an aid in teaching place value; a primitive predecessor of the modern computing machine. One form consists of a rectangular frame carrying as many parallel wires as there are digits in the largest number to be dealt with. Each wire contains nine beads free to slide on it. A bead on the lowest wire counts unity, on the next higher wire 10, on the next higher 100, etc. Two beads slid to the right on the lowest wire, three on the next higher, five on the next and four on the next denote 4532.



ABEL. Abel's identity. The identity

$$\sum_{i=1}^{n} a_i u_i = s_1(a_1 - a_2) + s_2(a_2 - a_3) + \cdots + s_{n-1}(a_{n-1} - a_n) + s_n a_n,$$

where

$$s_n = \sum_{i=1}^{n} u_i.$$

This is easily obtained from the evident identity:

$$\sum_{i=1}^{n} a_i u_i = a_1 s_1 + a_2(s_2 - s_1) + \cdots + a_n(s_n - s_{n-1}).$$

Abel's inequality. If $u_n \geq u_{n+1} > 0$ for all positive integers n, then $\left|\sum_{n=1}^{p} a_n u_n\right| \leq Lu_1$, where L is the largest of the quantities: $|a_1|$, $|a_1 + a_2|$, $|a_1 + a_2 + a_3|$, $\cdots$ $|a_1 + a_2 + \cdots + a_p|$. This inequality can be easily deduced from Abel's identity.

Abel's problem. Suppose a particle is constrained (without friction) to move along a certain path in a vertical plane under the force of gravity. **Abel's problem** is to find the path for which the time of descent is a given function $f(x)$ of x, where x is the horizontal axis and the particle starts from rest. This reduces to the problem of finding a solution $s(x)$ of the *Volterra integral equation of the first kind* $f(x) = \int_0^x \frac{s(t)}{\sqrt{2g(x-t)}}\,dt$, where $s(x)$ is the length of the path. If $f'(x)$ is continuous, a solution is $s(x) = \frac{\sqrt{2g}}{\pi}\frac{d}{dx}\int_0^x \frac{f(t)}{(x-t)^{1/2}}\,dt.$

Abel's tests for convergence. (1) If the series Σu_n converges and $\{a_n\}$ is a bounded monotonic sequence, then $\Sigma a_n u_n$ converges. (2) If $\sum_{n=1}^{k} u_n$ is equal to or less than a properly chosen constant for all k and $\{a_n\}$ is a positive, monotonic decreasing sequence which approaches zero as a limit, then $\Sigma a_n u_n$ converges. (3) If a series of complex numbers Σa_n is convergent, and the series $\Sigma(v_n - v_{n+1})$ is absolutely convergent, then $\Sigma a_n v_n$ is convergent.

Abel's test for uniform convergence. If the series $\Sigma a_n(x)$ is uniformly convergent in an interval (a, b), $v_n(x)$ is positive and monotonic decreasing for any value of x in the interval, and there is a number k such that $v_0(x) < k$ for all x in the interval, then $\Sigma a_n(x) v_n(x)$ is uniformly convergent.

Abel's theorem on power series. (1) If a power series, $a_0 + a_1 x + a_1 x^2 + \cdots + a_n x^n + \cdots$, converges for $x = c$, it converges absolutely for $|x| < |c|$. (2) If a power series converges to $f(x)$ for $|x| < 1$ and to s for $x = 1$, then $\lim_{x \to 1} f(x) = s$, $(0 \leqq x \leqq 1)$. The latter theorem is variously designated, most explicitly by "Abel's Theorem on Continuity up to the Circle of Convergence."

A-BRIDGED', *adj.* **abridged multiplication.** See MULTIPLICATION.

Plücker's abridged notation. A notation used for studying curves. Consists of the use of a single symbol to designate the expression (function) which, equated to zero, has a given curve for its locus; hence reduces the composition of curves to the study of polynomials of the first degree. *E.g.*, if $L_1 = 0$ denotes $2x + 3y - 5 = 0$ and $L_2 = 0$ denotes $x + y - 2 = 0$, then $k_1L_1 + k_2L_2 = 0$ denotes the family of lines passing through their common point (1, 1). See PENCIL—pencil of lines through a point.

AB-SCIS'SA, *n.* [*pl.* **abscissas** or **abscissae**]. The horizontal coordinate in a two-dimensional system of rectangular coordinates; usually denoted by x. Also used in a similar sense in systems of oblique coordinates. See CARTESIAN—Cartesian coordinates in the plane.

AB'SO-LUTE, *adj.* **absolute constant.** See CONSTANT—absolute constant.

absolute continuity. A function $f(x)$ is absolutely continuous on a closed interval $[a, b]$ if for any positive number ϵ another positive number η can be determined so that if $(a_1, b_1), (a_2, b_2), \cdots (a_n, b_n)$ is any finite set of nonoverlapping intervals such that the sum of the lengths of the intervals is less than η, then $\sum_{i=1}^{n} |f(a_i) - f(b_i)| < \epsilon$. The definition remains equivalent to this if it is changed to allow a countable number of intervals. An absolutely continuous function is *continuous* and of *bounded variation*.

absolute convergence. See CONVERGENCE.

absolute inequality. See INEQUALITY—absolute inequality.

absolute maximum (minimum). See MAXIMUM (MINIMUM).

absolute number. A number represented by figures such as 2, 3, or $\sqrt{2}$, rather than by letters as in algebra.

absolute property of a surface. Same as INTRINSIC PROPERTY OF A SURFACE.

absolute symmetry of a function. See SYMMETRIC—symmetric function. Absolute symmetry requires that a function remain unchanged under all interchanges of variables, whereas cyclosymmetry requires only that a function remain unchanged under cyclic changes of the variables. The word absolute is usually omitted, symmetry and cyclosymmetry being sufficient. The function

$$abc + a^2 + b^2 + c^2$$

has absolute symmetry, whereas

$$(a - b)(b - c)(c - a)$$

has only cyclosymmetry.

absolute term in an expression. A term which does not contain a variable. *Syn.* Constant term. In the equation $ax^2 + bx + c = 0$, c is the only absolute term.

absolute value of a complex number, $a + bi$. The value $\sqrt{a^2 + b^2}$. See MODULUS—modulus of a complex number.

absolute value of a real number. Its value without regard for sign; its numerical value. The cardinal number 2 is the absolute value of both +2 and −2.

absolute value of a vector. See VALUE—absolute value of a vector.

AB'STRACT, *adj.* **abstract mathematics.** See MATHEMATICS—pure mathematics.

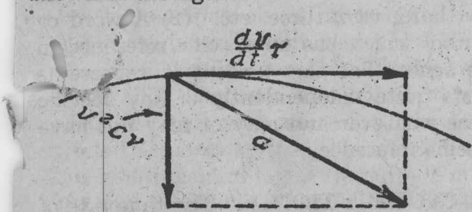
abstract number. Any number as such, simply as a number, without reference to any particular objects whatever except in so far as these objects possess the number property. Used to emphasize the distinction between a number, as such, and concrete numbers. See NUMBER, and CONCRETE.

abstract word or symbol. (1) A word or symbol that is not concrete; a word or symbol denoting a concept built up from consideration of many special cases; a word or symbol denoting a property common to many individuals or individual sets, as yellow, hard, two, three, etc. (2) A word or symbol which has no specific reference in the sense that the concept it represents exists quite independently of any specific cases whatever and may or may not have specific reference.

AC-CEL'ER-A'TION, *n.* The time rate of change of velocity. Since velocity is a directed quantity, the acceleration **a** is a vector equal to $\lim_{\Delta t \to 0} \frac{\Delta \mathbf{v}}{\Delta t} = \frac{d\mathbf{v}}{dt}$, where $\Delta \mathbf{v}$ is the increment in the velocity **v** which the moving object acquires in t units of time. Thus, if an airplane moving in a straight line with the speed of 2 miles per minute increases its speed until it is flying at the rate of 5 miles per minute at the end of the next minute, its *average acceleration* during that minute is 3 miles per minute per minate. If the increase in speed during this one minute interval of time is uniform, the average acceleration is equal to the actual acceleration. If the increase in speed in this example is not uniform, the instantaneous acceleration at the time t_1 is determined by evaluating the limit of the quotient $\frac{\Delta \mathbf{v}}{\Delta t}$ as the time interval $\Delta t = t - t_1$ is made to approach zero by making t approach t_1. For a particle moving along a curved path the velocity **v** is directed along the tangent to the path and the acceleration **a** can be shown to be given by the formula

$$\mathbf{a} = \frac{d\mathbf{v}}{dt}\boldsymbol{\tau} + \mathbf{v}^2 c \boldsymbol{\nu},$$

where $\frac{d\mathbf{v}}{dt}$ is the rate of speed v along the path, c is the curvature of the path at any point, and $\boldsymbol{\tau}$ and $\boldsymbol{\nu}$ are vectors of unit lengths directed along the tangent and normal to the path. The first of these terms, $\frac{d\mathbf{v}}{dt}$, is called the **tangential component,** and the second, $\mathbf{v}^2 c$, the **normal** (or **centripetal**) **component** of acceleration. If the path is a straight line, the curvature c is zero, and hence the acceleration vector will be directed along the path of motion. If the path is not rectilinear, the direction of the acceleration vector is determined by its tangential and normal components as shown in the figure.



acceleration of Coriolis. If S' is a reference frame rotating with the angular velocity $\boldsymbol{\omega}$ about a fixed point in another reference frame S, the acceleration **a** of a particle, as measured by the observer fixed in the reference frame S, is given by the sum of three terms: $\mathbf{a} = \mathbf{a}' + \mathbf{a}_t + \mathbf{a}_c$, where $\mathbf{a}'$ is the acceleration of the particle relative to S', $\mathbf{a}_t$ is the acceleration of the moving space, and $\mathbf{a}_c = 2\boldsymbol{\omega} \times \mathbf{v}'$ is the *acceleration of Coriolis*. The symbol $\boldsymbol{\omega} \times \mathbf{v}'$ denotes the vector product of the angular velocity $\boldsymbol{\omega}$, and the velocity $\mathbf{v}'$ relative to S', so that the acceleration of Coriolis is normal to the plane determined by the vectors $\boldsymbol{\omega}$ and $\mathbf{v}'$ and has the magnitude $2\mathbf{v}' \sin(\boldsymbol{\omega}, \mathbf{v}')$. The acceleration of Coriolis is also called the **complementary acceleration.**

acceleration of a falling body. See FALLING.

angular acceleration. The time rate of change of angular velocity. If the angular velocity is represented by a vector $\boldsymbol{\omega}$ directed along the axis of rotation, then the angular acceleration $\boldsymbol{\alpha}$, in the symbolism of calculus, is given by $\boldsymbol{\alpha} = \frac{d\boldsymbol{\omega}}{dt}$. See VELOCITY—angular velocity.

centripetal acceleration. See ACCELERATION.

normal acceleration. See ACCELERATION.

tangential acceleration. See ACCELERATION.

uniform acceleration. Acceleration in which there are equal changes in the velocity in equal intervals of time. *Syn.* Constant acceleration.

AC′CENT, *n.* A mark above and to the right of a quantity (or letter), as in a' or x'; the mark used in denoting that a letter is primed. See PRIME—prime as a symbol.

AC-CEPT′ANCE, *n.* **acceptance of a draft,** by the drawee (or payer). The writing of the word *accepted*, with the date and the signature of the drawee (or just the signature) across the front end or at the end of a commercial draft. This is *unqualified* acceptance or simply acceptance. If the payer stipulates that payment is to be made at a particular bank, the acceptance is said to be *special* acceptance; if he changes the amount or mode of payment it is said to be *qualified* acceptance. An *accepted* draft is negotiable.

trade acceptance. A bill of exchange drawn by the seller on the purchaser and accepted by the latter.

AC-CRUE′, *v.* To accumulate as the necessary result of some law or process. One speaks of accrued interest, profits, or dividends on stocks or bonds.

AC-CU′MU-LAT′ED, *adj.* **accumulated value.** Same as AMOUNT at simple or compound interest.

acculumated value of an annuity at a given date. The sum of the compound amounts of the annuity payments to that date. *Syn.* Accumulation of an annuity, amount of an annuity.

AC-CU′MU-LA′TION, *n.* Same as ACCUMULATED VALUE.

accumulation of discount on a bond. Writing up the book value of a bond on each dividend date by an amount equal to the interest on the investment (interest on book value at yield rate) minus the dividend. See VALUE—book value of a bond.

accumulation factor. The name sometimes given to the binomial $(1 + r)$, or $(1 + i)$, where r, or i, is the rate of interest. The formula for compound interest is $A = P(1 + r)^n$, where A is the amount accumulated at the end of n periods from an original principal P at a rate r. See COMPOUND—compound amount, and TABLE III in the appendix.

accumulation point. Same as LIMIT POINT.

accumulation problem. The determination of the amount when the principal, or principals, interest rate, and time for which each principal is invested are given. See TABLES III and IV in the appendix.

accumulation schedule of bond discount. A table showing the accumulation of bond discounts on successive dates. Interest and book values are usually listed also.

AC′CU-RA-CY, *n.* Correctness, usually referring to numerical computations.

accuracy of a table. (1) The number of significant digits appearing in the numbers in the table (*e.g.*, in the mantissas of a logarithm table). (2) The number of correct places in computations made with the table. (This number of places varies with the form of computation, since errors may repeatedly combine so as to become of any size whatever.)

AC′CU-RATE, *adj.* Exact, precise, without error. One speaks of an accurate statement in the sense that it is correct or true and of an accurate computation in the sense that it contains no numerical error.

accurate to a certain decimal place. All digits preceding and including the given place being correct and the next place having been called zero if less than 5, 10 if greater than 5, and if 5, the most usual convention is to call it zero or 10 as is necessary to leave the last digit even. *E.g.*, 1.26 is accurate to two places if obtained from either 1.264 or 1.256 or 1.255. See ROUNDING—rounding off numbers.

AC′NODE, *n.* See POINT—**isolated** point.

A-COUS′TI-CAL, *adj.* **acoustical property of conics.** See ELLIPSE—focal property of ellipse; HYPERBOLA—focal property of hyperbola; and PARABOLA—focal property of parabola.

A′CRE, *n.* The unit commonly used in the United States in measuring land; contains 43,560 square feet, 4,840 square yards, or 160 square rods.

AC′TION, *n.* A concept in advanced dynamics defined by the line integral $A = \int_{P_1}^{P_2} m\mathbf{v}\cdot d\mathbf{r}$ called the **action integral,** where m is the mass of the particle, $\mathbf{v}$ is its velocity, and $d\mathbf{r}$ is the vector element of the arc of the trajectory joining the points P_1 and P_2. The dot in the integrand denotes the scalar product of the momentum vector $m\mathbf{v}$ and $d\mathbf{r}$. The *action A* plays an important part in the development of dynamics from variational principles. See **below,** principle of least action.

law of action and reaction. The basic law of mechanics asserting that two particles interact so that the forces exerted by one on another are equal in magnitude, act along the line joining the particles, and are opposite in direction. See NEWTON—Newton's laws of motion.

principle of least action. Of all curves passing through two fixed points in the neighborhood of the natural trajectory, and which are traversed by the particle at a rate such that for each (at every instant of time) the sum of the kinetic and potential energies is a constant, that one for which the action integral has an extremal value is the natural trajectory of the particle. See ACTION.

A-CUTE′, *adj.* **acute angle.** An angle numerically smaller than a right angle; usually refers to positive angles less than a right angle.

acute triangle. See TRIANGLE.

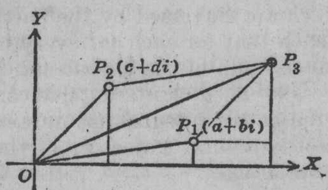
AD′DEND, *n.* One of a set of numbers to be added, as 2 or 3 in the sum $2 + 3$.

AD-DI′TION, *n.* **addition of algebraic quantities (algebraic addition).** Two positive numbers are added as in arithmetic; two negative numbers are added by adding their numerical values and making the result negative; a positive and a negative number are added by subtracting the lesser numerical value from the greater and giving the difference the sign of the number which has the greater numerical value. *E.g.*, (a). $(-2) + (-3) = -5$; (b). $(-2) + 3 = 1$; (c). $(-3) + 2 = -1$. The significance of this definition becomes apparent when we let positive numbers denote distances eastward and negative numbers distances westward and think of their sum as the place reached by travelling in succession the paths measured by the addends. *E.g.*, in illustration (c) one would travel 3 miles west, then 2 miles east, and finish 1 mile west of the starting point.

addition of angles. *Geometrically*, a rotation from the initial side through one angle, followed by a rotation, beginning with the terminal side of this angle, through the other angle, having regard for the signs of the angles; *algebraically*, the ordinary algebraic addition of the same kind of measures of the angles, degrees plus degrees or radians plus radians.

addition in arithmetic. See below, addition of integers.

addition of complex numbers. *Algebraically*, the addition of the real parts, and the imaginary parts, *e.g.*, $(2 - 3i) + (1 + 5i) = (3 + 2i)$; *geometrically*, same as the addition of the corresponding vectors in the plane. In the figure, $OP_1 + OP_2 = OP_3$, $(OP_2 = P_1P_3)$.



addition of decimals. Placing digits with like place value under one another, *i.e.*, placing decimal points under decimal points, and adding as with integers, putting the decimal point of the sum directly below those of the addends.

addition of directed segments of a line. Adding one to the other, keeping them both in the same straight line and giving consideration to their directions; connecting the segments in such a way that the initial point of one is on the terminal point of the other and they both have the same or exactly opposite directions. *E.g.*, 5 miles east plus 2 miles west is 3 miles east. See above, addition of algebraic quantities.

addition formulas of trigonometry. Formulas expressing the trigonometric functions of the algebraic sum of two angles in terms of the functions of the angles. The most important are

$$\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B,$$

$$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B,$$

and

$$\tan (A \pm B) = \frac{(\tan A \pm \tan B)}{(1 \mp \tan A \tan B)}$$

where the upper signs and the lower signs are taken together.

addition of fractions. The same process as the addition of integers, after a common denominator (common unit) has been established. *E.g.*, $\frac{1}{2}$ and $\frac{2}{3}$ are the same respectively as $\frac{3}{6}$ and $\frac{4}{6}$, and 3 sixths plus 4 sixths is 7 sixths.

addition of integers. Counting all the units of two sets into one; *e.g.*, the sum of 2 and 3 is obtained by counting 1, 2—3, 4, 5; the process of finding the number which represents all the units in the numbers to be added, taken collectively. *Tech.* The process of finding the number-class which is composed of (is the class of) the number-classes denoted by the addends.

addition of irrational numbers. Addition of the corresponding integral digits and decimals. *E.g.*,

$$2.3333\cdots + 5.1212\cdots = 7.4545\cdots.$$

The addition of irrationals is usually left in indicated form, after similar terms have been combined, until some specific application indicates the degree of accuracy desired. Such a sum as $(\sqrt{2} + \sqrt{3}) - (2\sqrt{2} - 5\sqrt{3})$ would thus be left in the form $6\sqrt{3} - \sqrt{2}$.

addition of matrices. See SUM—sum of matrices.

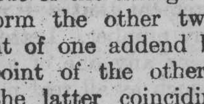
addition of mixed numbers. Adding the integral parts and fractions separately, then adding these results. *E.g.*, $2\frac{1}{2} + 3\frac{1}{4} = 2 + 3 + \frac{1}{2} + \frac{1}{4} = 5\frac{3}{4}$.

addition of series. See SERIES.

addition of similar terms in algebra. The process of adding the coefficients of terms which are alike as regards their other factors: $2x + 3x = 5x$, $3x^2y - 2x^2y = x^2y$ and $ax + bx = (a + b)x$. See DISSIMILAR TERMS.

addition of tensors. See TENSOR.

addition of two vectors in the plane. *Algebraically*, the addition of the corresponding components, *e.g.*, $2i + 3j$ plus $i - 2j$ equals $3i + j$; *geometrically*, finding the third side of the triangle of which the addends form the other two sides, the initial point of one addend being on the terminal point of the other, the initial point of the latter coinciding with the initial point of the sum. In the figure $OA + AB = OB$. See PARALLELOGRAM—parallelogram of forces; and VECTOR—vector components.



addition of two vectors in space. *Algebraically*, the addition of corresponding components, *e.g.*, $(2i + 3j + 5k) + (i - 2j + 3k) = 3i + j + 8k$; *geometrically*, the addition of the two space vectors in the plane which they determine when drawn so that they intersect, just as two vectors in the plane are added. See above, addition of two vectors in the plane.

higher-decade addition. The process of adding a one-place number to a two-place number. Thus, $24 + 3 = 27$, $34 + 9 = 43$ are examples of higher-decade addition.

proportion by addition. See PROPORTION.

proportion by addition and subtraction. See PROPORTION.

ADD'I-TIVE, *adj.* **additive function.** A function $f(x)$ which has the property that $f(x + y)$ is defined and equals $f(x) + f(y)$ whenever $f(x)$ and $f(y)$ are defined. A *continuous* additive function is necessarily *homogeneous*.

AD'I-A-BAT'IC, *adj.* **adiabatic curves.** Curves showing the relation between pressure and volume of substances which are assumed to have adiabatic expansion and contraction.

adiabatic expansion (or contraction). (*Thermodynamics*). A change in volume without loss or gain of heat.

AD IN'FI-NI'TUM. Continuing without end (according to some law); denoted by three dots, as $\cdots$; used, principally, in writing infinite series, infinite sequences, and infinite products.

AD-JA'CENT, *adj.* **adjacent angles.** See ANGLE—adjacent angles.

AD-JOINED', *adj.* **adjoined number.** See FIELD—number field.

AD'JOINT, *n.* **adjoint of a matrix.** The *transpose* of the matrix obtained by replacing each element by its *cofactor;* the matrix obtained by replacing each element a_{rs} (in row r and column s) by the cofactor of the element a_{sr} (in row s and column r). The adjoint is defined only for square matrices. The *Hermitian conjugate* matrix is fre-frequently called the adjoint matrix by writers on quantum mechanics.

AD VALOREM DUTY. A duty which is a certain fixed per cent of the value of the goods.

AF-FINE', *adj.* **affine transformation.** (1) A transformation of the form

$$x' = a_1x + b_1y + c_1, \quad y' = a_2x + b_2y + c_2,$$

$$\text{where} \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1b_2 - a_2b_1 \neq 0.$$

(2) A transformation of the form given in (1) except that the determinant of the coefficients may or may not be zero. The determinant of the coefficients is denoted by Δ. The following are important special cases of the affine transformation, $\Delta \neq 0$; (a) **a translation** ($x' = x + a$, $y' = y + b$); (b) **a rotation** ($x' = x \cos\theta + y \sin\theta$, $y' =$

$-x \sin\theta + y \cos\theta$); (c) **a stretching and shrinking** ($x' = kx$, $y' = ky$), called *transformations of similitude* or *homothetic transformations;* (d) **reflections** in the x-axis and y-axis, respectively, ($x' = x$, $y' = -y$ or $x' = -x$, $y' = y$); (e) **simple elongations and compressions** ($x' = x$, $y' = ky$ or $x' = kx$, $y' = y$), sometimes called *one-dimensional strains* and *one-dimensional elongations and compressions.* The affine transformation carries parallel lines into parallel lines, finite points into finite points and leaves the line at infinity fixed. An affine transformation can always be factored into the product of transformations belonging to the above special cases.

homogeneous affine transformation. An affine transformation in which the constant terms are zero; an affine transformation which does not contain a translation as a factor. (See FACTORIZATION—factorization of a transformation.) Its form is

$$x' = a_1x + b_1y, \quad y' = a_2x + b_2y,$$

$$\Delta = a_1b_2 - a_2b_1 \neq 0.$$

isogonal affine transformation. An affine transformation which does not change the size of angles. It has the form

$$x' = a_1x + b_1y + c_1,$$

$$y' = a_2x + b_2y + c_2,$$

where either $a_1 = b_2$ and $a_2 = -b_1$ or $-a_1 = b_2$ and $a_2 = b_1$.

nonsingular affine transformation. Same as the transformation given in (1) under AFFINE TRANSFORMATION. Usually called simply an **affine transformation.**

singular affine transformation. See above, affine transformation, (1). The form is that given except that $a_1b_2 - a_2b_1 = 0$.

AGE, *n.* **age at issue.** (*Life Insurance.*) The age of the insured at his birthday nearest the policy date.

age year. (*Life Insurance.*) A year in the lives of a group of people of a certain age. The age year l_x refers to the year from x to $x + 1$, the year during which the group is x years old.

AG'GRE-GA'TION, *n.* **Signs of aggregation: Parenthesis,** ( ); **bracket,** [ ]; **brace,** { } and **vinculum or bar,** ——. Each means that the terms enclosed are to be treated as a single term. *E.g* 3(2 − 1 + 4) means 3 times 5, or 15. See various headings under DISTRIBUTIVE.

AGNESI. witch of Agnesi. Same as WITCH.

AHMES (RHYND or RHIND) PAPYRUS. Probably the oldest mathematical book known, written about 1550 B.C.

A'LEPH, *n.* The first letter of the Hebrew alphabet, written ℵ.

aleph-null or **aleph-zero.** The *cardinal number* of countably infinite sets, written $\aleph_0$.

AL'GE-BRA, *n.* (1) A generalization of arithmetic. *E.g.*, the arithmetic facts that $2 + 2 + 2 = 3 \times 2$, $4 + 4 + 4 = 3 \times 4$, etc., are all special cases of the (general) algebraic statement that $x + x + x = 3x$, where x is any number. Letters denoting any number, or any one of a certain set of numbers, such as all real numbers, are related by laws that hold for any numbers in the set; *e.g.*, $x + x = 2x$ for all x (all numbers). On the other hand, conditions may be imposed upon a letter, representing any one of a set, so that it can take on but one value, as in the study of equations; *e.g.*, if $2x + 1 = 9$, then x is restricted to 4. Equations are met in arithmetic, although not so named. For instance, in percentage one has to find one of the unknowns in the equation, interest = principal × rate, or $I = p \times r$, when the other two are given. (2) A system of logic expressed in algebraic symbols.

fundamental theorem of algebra. See FUNDAMENTAL—fundamental theorem of algebra.

AL'GE-BRA'IC, *adj.* **algebraic addition.** See ADDITION—addition of algebraic quantities.

algebraic deviation. See DEVIATION.

algebraic expression, equation, operation, etc. An expression, etc., containing or using only algebraic symbols and operations, such as $2x + 3$, $x^2 + 2x + 4$, or $\sqrt{2} - x + y = 3$.

algebraic function. See FUNCTION—algebraic function.

algebraic multiplication. See MULTIPLICATION.

algebraic operations. Addition, subtraction, multiplication, division, evolution, and involution (extracting roots and raising to a power), no infinite processes being used.

algebraic proofs. Proofs which use algebraic symbols and no operations other than those which are algebraic. See above, algebraic operations.

algebraic subtraction. See SUBTRACTION—algebraic subtraction.

algebraic symbols. Letters representing numbers, and various operational symbols, including those of arithmetic. See MATHEMATICAL SYMBOLS in the appendix.

irrational algebraic surface. See IRRATIONAL.

real algebraic number. See REAL.

AL'GO-RITHM, *n.* Some special process of solving a certain type of problem.

Euclid's Algorithm. A method of finding the greatest common divisor (G.C.D.) of two numbers—one number is divided by the other, then the second by the remainder, the first remainder by the second remainder, the second by the third, etc. When exact division is finally reached, the last divisor is the greatest common divisor of the given numbers (integers). In algebra the same process can be applied to polynomials. *E.g.*, to find the highest common factor of 12 and 20, we have $20 \div 12$ is 1 with remainder 8; $12 \div 8$ is 1 with remainder 4; and $8 \div 4 = 2$, hence 4 is the G.C.D.

AL'IEN-A'TION, *n.* **coefficient of alienation.** See CORRELATION—normal correlation.

A-LIGN'MENT, *adj.* **alignment chart.** Same as NOMOGRAM.

AL'I-QUOT PART. Any exact divisor of a quantity; any factor of a quantity; used almost entirely when dealing with integers. *E.g.*, 2 and 3 are *aliquot parts* of 6.

AL'MOST, *adj.* **almost everywhere.** See MEASURE—measure zero.

AL'PHA, *n.* The first letter in the Greek alphabet, written α.

AL'PHA-BET, GREEK. See the APPENDIX.

AL'TER-NATE, *adj.* **alternate angles.** Angles on opposite sides of a transversal cutting two lines, each having one of the lines for one of its sides. See ANGLE—angles made by a transversal.

alternate exterior angles. *Alternate angles* neither of which lies between the two lines cut by a transversal. See ANGLE—angles made by a transversal.

alternate interior angles. Either of the two pairs of *alternate angles* lying between the two lines cut by a transversal. See ANGLE—angles made by a transversal.

AL-TER-NAT'ING, *adj.* **alternating group.** See GROUP—alternating group.

alternating series. A series whose terms are alternately positive and negative, as

$$1 - \tfrac{1}{2} + \tfrac{1}{3} - \tfrac{1}{4} + \cdots + (-1)^{n-1}/n + \cdots,$$

alternating series test for convergence. An alternating series converges if each term is numerically equal to or less than the preceding and if the nth term approaches zero as n increases without limit. This is a sufficient but not a necessary set of conditions—the term-by-term sum of any two convergent series converges and, if one series has all negative terms and the other all positive terms, their indicated sum may be a convergent alternating series and not have its terms monotonically decreasing. The series

$$1 - \tfrac{1}{2} + \tfrac{1}{3} - \tfrac{1}{4} + \tfrac{1}{9} - \tfrac{1}{8} + \tfrac{1}{27} - \tfrac{1}{16} + \cdots$$

is such a series. See NECESSARY—necessary condition for convergence.

AL'TER-NA'TION, *n.* **proportion by alternation.** See PROPORTION—proportion by alternation.

AL'TI-TUDE, *n.* **altitude of a celestial point.** Its angular distance above, or below, the observer's horizon, measured along a great celestial circle (vertical circle) passing through the point, the zenith, and the nadir. The altitude is taken positive when the object is above the horizon and negative when below. See figure under COUR—hour-angle and hour-circle.

altitude of a cone. See CONE.

altitude of a cylinder. The perpendicular distance between its bases.

altitude of a frustum of a cone. The perpendicular distance between its bases.

altitude of a parabolic segment. See PARABOLIC—parabolic segment.

altitude of a parallelogram. The perpendicular distance between two of its parallel sides. The side to which the altitude is drawn is called the base.

altitude of a prism. The perpendicular distance between its parallel bases.

altitude of a pyramid. See PYRAMID.

altitude of a rectangle. See above, altitude of a parallelogram.

altitude of a spherical segment. The perpendicular distance between the bases if the segment has two bases, otherwise the perpendicular distance between the base and the tangent plane to the segment, which is parallel to the base. See figures under SEGMENT—spherical segment.

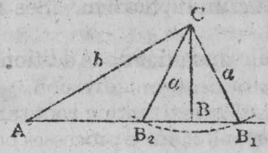
altitude of a trapezoid. The perpendicular distance between the parallel bases.

altitude of a triangle. The perpendicular distance, or the perpendicular, from a vertex to the opposite side. This side is then regarded as the **base.**

altitude of a zone. The perpendicular distance between the planes of the bases of the zone.

AM-BIG'U-OUS, *adj.* Not uniquely determinable.

ambiguous case in the solution of plane triangles. The case in which two sides and the angle opposite one of them is given. One of the other angles is then found by use of the law of sines; but there are always two angles less than 180° corresponding to any given value of the sine (unless the sine be unity, in which case the angle is 90° and the triangle is a right triangle). When the sine gives two distinct values of the angle, two triangles result if the side opposite is less than the side adjacent to the given angle (assuming the data are not such that there is no triangle possible, a situation that may arise in any case, ambiguous or nonambiguous). In the figure, angle A and sides a and b are given ($a < b$); triangles AB_1C and AB_2C are both solutions. If $a = b \sin A$, the right triangle ABC is the unique solution.



ambiguous case in spherical triangles. See SPHERICAL.

A-MER'I-CAN, *adj.* **American experience table of mortality.** (1) A table of mortality (mortality table) based upon the lives of Americans, constructed from insurance records about 1860. (2) A mortality table constructed from data obtained from American insurance companies and census records.

American men mortality table. A mortality table constructed from the records of the larger insurance companies in the United States covering the period from 1900 to 1915 inclusive.

AM'I-CA-BLE, *adj.* **Amicable numbers.** Two numbers, each of which is equal to the sum of all the exact divisors of the other except the number itself. *E.g.*, 220 and 284 are amicable numbers, for 220 has the exact divisors 1, 2, 4, 5, 10, 11, 20, 22, 44, 55 and 110, whose sum is 284; and 284 has the exact divisors 1, 2, 4, 71, and 142, whose sum is 220.

A-MOR'TI-ZA'TION, *n.* The process of amortizing.

amortization of a debt. See AMORTIZE.

amortization equation. An equation relating the amount of an obligation to be amortized, the interest rate, and the amount of the periodic payments. See AMORTIZE, and TABLE IV in the appendix.

amortization of a premium on a bond. Writing down (decreasing) the book value of the bond on each dividend date by an amount equal to the difference between the dividend and the interest on the investment (interest on the book value at the yield rate). See VALUE—book value of a bond.

amortization schedule. A table giving the annual payment, the amount applied to principal, amount applied to interest, and the balance of principal due. See AMORTIZE.

A-MOR'TIZE, *v.* To discharge a debt, including interest, by periodic payments, usually equal, which continue until the debt is paid without any renewal of the contract. The mathematical principles are the same as those used for annuities.

A-MOUNT', *n.* **amount of a sum of money at a given date.** The sum of the principal and interest (simple or compound) to the date; designated as *amount at simple interest* or *amount at compound interest* (or *compound amount*), according as interest is simple or compound. In practice the word *amount* without any qualification usually refers to amount at compound interest. See TABLE III in the appendix.

amount of an annuity. See ACCUMULATED—accumulated value of an annuity at a given date.

compound amount. See COMPOUND.

AM'PERE, *n.* A unit of measure of electric current; the current which flows through a conductor whose resistance is one *ohm* and for which the difference of potential between its two terminals is constantly one *volt*.

absolute ampere. One tenth of an electromagnetic unit (e.m.u.) of current, one e.m.u. of current being that current which will produce a force of 2π dynes on a unit magnetic pole placed at the center of a circular coil of wire of 1 turn and of 1 centimeter radius.

international ampere. The current which when passed through a standard solution of silver nitrate deposits silver at the rate of .001118 gram per sec.

AM'PLI-TUDE, *n.* **amplitude of a complex number.** The angle that the vector representing the complex number makes with the positive horizontal axis. *E.g.*, the *amplitude of* $2 + 2i$ is 45°. See COMPLEX—polar form of a complex number.

amplitude of a curve. The greatest numerical value of the ordinates of a periodic curve. The *amplitude* of $y = \sin x$ is 1; of $y = 2 \sin x$ is 2.

amplitude of a point. See POLAR—polar coordinates in the plane.

amplitude of simple harmonic motion. See HARMONIC—simple harmonic motion.

A-NAL'O-GY, *n.* A form of inference sometimes used in mathematics to set up new theorems. It is reasoned that, if two or more things agree in some respects, they will probably agree in others. Exact proofs must, of course, be made to determine the validity of any theorems set up by this method.

A-NAL'Y-SIS, *n.* [*pl.* **analyses**]. That part of mathematics which uses, for the most part, algebraic and calculus methods—as distinguished from such subjects as synthetic geometry, number theory, and group theory.

analysis of a problem. The exposition of the principles involved; a listing, in mathematical language, of the data given in the statement of the problem, other related data, the end sought, and the steps to be taken.

analysis of variance. See VARIANCE.

diophantine analysis. See DIOPHANTINE.

***n* way analysis.** (*Statistics.*) A general joint classification of values based on n joint factors gives an n way analysis.

one-way analysis. (*Statistics.*) One-way analysis is an analysis in which factors investigated as possible contributors to variances are classified under one general head, *e.g.*, male and female under sex.

proof by analysis. Proceeding from the thing to be proved to some known truth, as opposed to synthesis which proceeds from the true to that which is to be proved. The most common method of *proof by analysis* is, in fact, by *analysis* and *synthesis*, in that the steps in the analysis are required to be reversible.

trigonometric analysis. The study of the algebraic relations between the trigonometric functions, such as, for instance, the values of the functions of the sum (or difference) of two angles in terms of the functions of the angles.

two-way analysis. (*Statistics.*) Two-way analysis is an analysis in which two major factors jointly classify the observed values; *e.g.*, sex (male and female) and marital status (married, single, other).

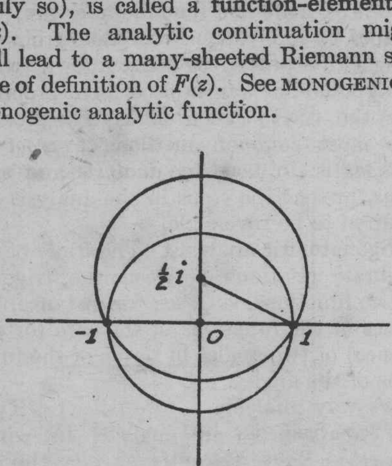
unitary analysis. A system of analysis that proceeds from a given number of units to a unit, then to the required number of units. Consider the problem of finding the cost of 7 tons of hay if $2\frac{1}{2}$ tons cost

\$25.00. Analysis: If $2\frac{1}{2}$ tons cost \$25.00, *1 ton costs* \$10.00. Hence 7 tons cost \$70.00.

AN-A-LYT'IC, *adj.* **analytic continuation of an analytic function of a complex variable.** If $w = f(z)$ is given to be a single-valued analytic function in a domain D, then possibly there is a function $F(z)$ analytic in a domain of which D is a proper subdomain, and such that $F(z) = f(z)$ in D. If so, the function $F(z)$ is necessarily unique. The process of obtaining $F(z)$ from $f(z)$ is called **analytic continuation.** *E.g.*, the function $f(z)$ defined by $f(z) = 1 + z + z^2 + z^3 + \cdots$ is thereby defined only for $|z| < 1$, the radius of convergence of the series being 1 and the circle of convergence having center at 0. The series represents the function $1/(1 - z)$, but if this function is given a new representation, say by

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(\frac{1}{2}i)}{n!}\left(z - \frac{1}{2}i\right)^n,$$

where the coefficients are determined from the original series, the new circle of convergence extends outside the old one (see the figure). The given function $f(z)$, usually given as a power series (but not necessarily so), is called a **function-element** of $F(z)$. The analytic continuation might well lead to a many-sheeted Riemann surface of definition of $F(z)$. See MONOGENIC—monogenic analytic function.



analytic curve. A curve in n-dimensional Euclidean space which, in the neighborhood of each of its points, admits a representation of the form $x_j = x_j(t)$, $j = 1, 2, \cdots n$, where the $x_j(t)$ are real analytic functions of the real variable t. If in addition we have $\sum_{j=1}^{n}(x_j')^2 \neq 0$, the curve is said to be a **regular analytic curve.**

analytic in a domain. A single-valued function $f(z)$ of the complex variable z is said to be **analytic in a domain D** if $f(z)$ has a derivative at each point of D.

analytic function of a complex variable. A single-valued function $w = f(z)$, or a multiple-valued function considered as a single-valued function on its Riemann surface, which has a derivative at each point of its *domain* (a non-null connected open set) of definition D **is said to be analytic in D.** It can be shown that an analytic function $f(z)$ of a complex variable has continuous derivatives of all orders and can be expanded as a Taylor series in a neighborhood of each point z_0 of D:

$$f(z) = \sum_{n=0}^{\infty} a_n(z - z_0)^n.$$

A function is sometimes said to be analytic in D if it is continuous in D and has a derivative at all except at most a finite number of points of D. If $f(z)$ has a derivative at all points of D, it is said to be a **regular function,** or a **regular analytic function,** or a **holomorphic function,** in D. See CAUCHY—Cauchy-Riemann partial differential equations.

analytic function of a real variable. A function, $f(x)$, is analytic for $x = h$ if it can be represented by a Taylor's series in powers of $(x - h)$, which is equal to the function for any x in some neighborhood of h. The function is said to be analytic in the interval (a, b) if the above is true for every h in the interval (a, b). See TAYLOR—Taylor's series.

analytic geometry. See GEOMETRY—analytic geometry.

analytic at a point. A single-valued function $f(z)$ of the complex variable z is said to be analytic at the point z_0 if there is a neighborhood N of z_0 such that $f'(z)$ exists at every point of N. *I.e.*, $f(z)$ is analytic at z_0 if it is analytic in a neighborhood of z_0. *Syn.* Holomorphic, regular, or monogenic at a point. See above, analytic function of a complex variable.

analytic proof. A proof which depends upon that sort of procedure in mathematics called analysis; a proof which consists, essentially, of algebraic rather than geometric methods.

analytic solutions. Solutions which use, for the most part, algebraic methods. See above, analytic proof.

a-point of an analytic function. An a-point of the analytic function $f(z)$ of the complex variable z is a *zero point* of the analytic function $f(z) - a$. The order of an a-point is the order of the zero of $f(z) - a$ at the point. See ZERO—zero point of an analytic function of a complex variable.

normal family of analytic functions. See NORMAL.

singular point of an analytic function. See various headings under SINGULAR.

solid analytic geometry. See GEOMETRY—solid analytic geometry.

AN'A-LYT'I-CAL-LY, *adj.* Performed by analysis, by analytic methods, as opposed to synthetic methods.

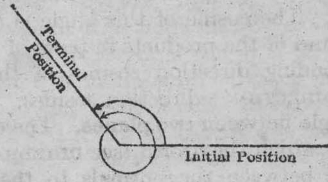
AN'A-LY-TIC'I-TY, *n.* **point of analyticity.** A point at which a function $f(z)$ of the complex variable z is analytic.

ANCHOR RING or TORUS. A surface the shape of a doughnut with a hole in it; a surface generated by the rotation, in space, of a circle about an axis in its plane but not cutting the circle. If r is the radius of the circle, k the distance from the center to the axis of revolution, in this case the z-axis, and the equation of the generating circle is $(y - k)^2 + z^2 = r^2$, then the equation of the anchor ring is

$$(\sqrt{x^2 + y^2} - k)^2 + z^2 = r^2.$$

Its volume is $2\pi^2kr^2$ and the area of its surface is $4\pi^2kr$.

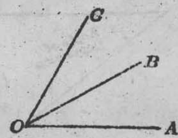
AN'GLE, *n.* In *geometry*, the inclination to each other (the divergence) of two straight lines; the figure formed by two straight lines drawn from the same point. In *trigonometry*, a figure which has been formed by one straight line (called the **terminal** line, or side) having been revolved about a fixed point on a stationary straight line (called the **initial line,** or side). If the motion is counterclockwise, the angle is said to be **positive;** if clockwise, it is said to be **negative.** "Angle" is used for "plane angle."



acute angle. See ACUTE.

addition of angles. See ADDITION—addition of angles.

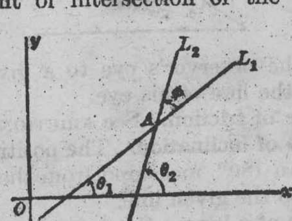
adjacent angles. Two angles having common side and common vertex and lying on opposite sides of their common side. In the figure AOB and BOC are adjacent angles.



angle between a line and a plane. The smaller (acute) angle which the line makes with its projection in the plane.

angle between two intersecting curves. The angle between the tangents to the curves at their point of intersection. See below, angle between two lines in a plane.

angle between two lines in a plane. (1) The angle from line L_1, say, to line L_2 is the smallest positive angle through which L_1 can be revolved counterclockwise about the point of intersection of the lines to



coincide with the line L_2, angle ϕ in the cut. (2) Otherwise defined as the least positive angle between the two lines and as the numerically least angle between them. The tangent of the angle from L_1 to L_2 is given by

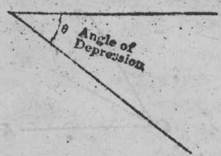
$$\tan\phi = \tan(\theta_2 - \theta_1) = \frac{m_2 - m_1}{1 + m_1m_2},$$

where $m_1 = \tan\theta_1$ and $m_2 = \tan\theta_2$.

angle between two lines in space. The angle between two intersecting lines which are parallel respectively to the two given lines. The cosine of this angle is equal to the sum of the products in pairs of the corresponding direction cosines of the lines. See DIRECTION—direction cosines.

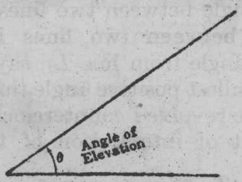
angle between two planes. The dihedral angle which they form (see DIHEDRAL); the angle between the normals to the planes (see above, angle between two lines in space). When the equations of the planes are in normal form the cosine of the angle between the planes is equal to the sum of the products of the corresponding coefficients (coefficients of the same variables) in their equations.

angle of depression. The angle between



the horizontal plane and the oblique line joining the observer's eye to some object lower than (beneath) the line of his eye.

angle of elevation. The angle between the horizontal plane and the oblique line



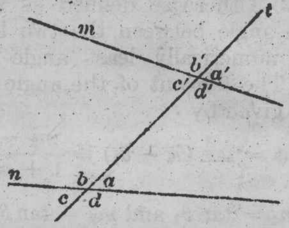
from the observer's eye to a given point above the line of his eye.

angle of friction. See FRICTION.

angle of inclination. The positive angle, less than 180°, measured from the positive x-axis to the given line.

angle of a lune. See LUNE.

angles made by a transversal. The



angles made by a line (the transversal) which cuts two or more other lines. In the figure, the transversal t cuts the lines m and n. The angles a, b, c', d' are **interior angles**; a', b', c, d are **exterior angles**; a and c', and b and d' are the pairs of **alternate-interior angles**; b' and d, a' and c are the pairs of **alternate-exterior angles**; a' and a, b' and b, c' and c, d' and d are the **exterior-interior** or **corresponding angles.**

angles of a polygon. The angles made by adjacent sides of the polygon and lying on the interior of the polygon. This definition suffices for any polygon, even if concave, provided no side (not extended) cuts more than two sides. If this condition does not hold, the sides must be directed in some order when defining the polygon in order to uniquely define the angles between them. See DIRECTED—directed line.

angle of reflection. See REFLECTION.

angle of refraction. See REFRACTION.

base angles of a triangle. The angles in the triangle having the base of the triangle for their common side.

central angle. See CENTRAL.

complementary angles. See COMPLEMENTARY.

conjugate angles. Two angles whose sum is a perigon; two angles whose sum is 360°. Such angles are also said to be *explements* of each other.

coterminal angles. See COTERMINAL.

dihedral angle. See DIHEDRAL.

direction angles. See DIRECTION—direction angles.

eccentric angle. See ELLIPSE—parametric equations of an ellipse.

equal angles. Angles which can be made to coincide, vertex upon vertex and sides upon corresponding sides.

Euler's angles. See EULER—Euler's angles.

explementary angles. See above, conjugate angles.

exterior angles. See EXTERIOR.

face angles. See below, polyhedral angle.

flat angle. A straight angle.

general formula for all angles having the same sine, cosine, or tangent. See GENERAL—general formula.

hour angle of a celestial point. The angle between the plane of the meridian of the observer and the plane of the hour circle of the star—measured westward from the plane of the meridian. See HOUR—hour angle and hour circle.

interior angle. See INTERIOR.

measure of an angle. See DEGREE, MIL, and RADIAN.

negative angle. An angle generated by revolving a line in the clockwise direction from the initial line. See ANGLE.

negatively oriented angle. An oriented angle for which the rotation is clockwise. Same as negative angle.

obtuse angle. An angle numerically greater than a right angle and less than a straight angle; sometimes used for all angles numerically greater than a right angle.

opposite angle. See OPPOSITE—opposite vertices (angles) of a polygon.

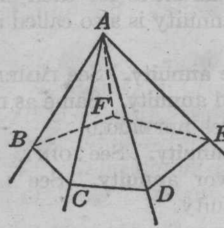
oriented angle. An angle with which the idea of directed rotation is associated.

perigon (angle). An angle containing 360°.

plane angle. See above, ANGLE.

polar angle. See POLAR—polar coordinates in the plane.

polyhedral angle. The configuration formed by the lateral faces of a polyhedron which have a common vertex (*A-BCDEF* in the figure); the positional relation of a set of planes determined by a point and the sides of some polygon whose plane does not contain the point. The planes (*ABC* etc.) are called **faces** of the angle; the lines of



intersection of the planes are called **edges** of the *polyhedral angle.* Their point of intersection (*A*) is called the **vertex.** The angles (*BAC*, *CAD*, etc.) between two successive edges are called **face angles.**

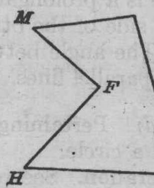
positive angle. An angle generated by revolving a line in the counterclockwise direction from the initial line. See ANGLE.

positively oriented angle. An oriented angle for which the rotation is counterclockwise. Same as positive angle.

quadrant angles. See QUADRANT.

quadrantal angles. See QUADRANTAL.

reentrant angle. An angle which is an interior angle of a polygon and greater than 180° (angle *HFM* in figure).



reference angle. Same as RELATED ANGLE. See RELATED.

reflex angle. An angle greater than a straight angle and less than two straight angles; an angle between 180° and 360°.

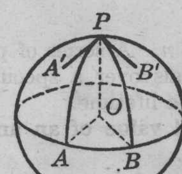
related angle. See RELATED.

right angle. Half of a straight angle; an angle containing 90° or $\frac{1}{2}\pi$ radians.

sides of an angle. The straight lines forming the angle.

solid angle. See SOLID.

spherical angle. The figure formed at the intersection of two great circles on a sphere; the difference in direction of the arcs of two great circles at their point of intersection. In the figure the spherical angle is *APB*. It is equal to the plane angles *A′PB′* and *AOB*. See DIRECTION—direction of a curve.



straight angle. An angle whose sides lie on the same straight line, but extend in opposite directions from the vertex; an angle of 180° or π radians. *Syn.* Flat angle.

supplementary angles. See SUPPLEMENTARY.

tetrahedral angle. A polyhedral angle having four faces.

trihedral angle. A polyhedral angle having three faces.

trisection of an angle. See TRISECTION.

vertex angle. The angle opposite the base of a triangle.

vertex of an angle. The point of intersection of the sides.

vertical angle. The angle at the vertex of a triangle. Usually called *vertex angle.*

vertical angles. Two angles such that each side of one is a prolongation, through the vertex, of a side of the other.

zero angle. The angle between two coincident or two parallel lines.

AN'GU-LAR, *adj.* Pertaining to an angle; circular; around a circle.

angular acceleration. See ACCELERATION—angular acceleration.

angular distance. See DISTANCE—angular distance between two points.

angular momentum. See MOMENTUM—moment of momentum.

angular speed. See SPEED.

angular velocity. See VELOCITY.

AN'HAR-MON'IC RATIO. See RATIO—anharmonic ratio.

AN'NU-AL, *adj.* Yearly.

annual premium (net annual premium). See PREMIUM—net annual premiums.

annual rent. Rent, when the payment period is a year. See RENT.

AN-NU'I-TANT, *n.* The life (person) upon whose existence each payment of a life annuity is contingent, *i.e.*, the beneficiary of an annuity.

AN-NU'I-TY, *n.* A series of payments at regular intervals over a specified term of years, or over a lifetime.

accumulated value of an annuity. See ACCUMULATED.

accumulation of an annuity. See ACCUMULATED—accumulated value of an annuity at a given date.

amount of an annuity. See ACCUMULATED—accumulated value of an annuity.

annuity bond. See BOND.

annuity certain. An annuity that provides for a definite number of payments, as contrasted to a life annuity.

annuity contract. The written agreement setting forth the amount of the annuity, its cost, and the conditions under which it is to be paid.

annuity due. An annuity in which the payments are made at the beginning of each period.

annuity policy. A contract to pay a certain annuity for life beginning at a certain age. Sometimes used instead of *contract* in the case of *temporary* annuity contracts.

apportionate annuity. Same as COMPLETE ANNUITY. See COMPLETE.

cash equivalent of an annuity. Same as PRESENT VALUE. See VALUE—present value.

complete annuity. See COMPLETE—complete annuity.

complete joint annuity. See COMPLETE—complete joint annuity.

consolidated annuities (consols). See CONSOLIDATED—consolidated annuities.

contingent reversionary annuity. See CONTINGENT.

continuous annuity. See CONTINUOUS.

curtate annuity. See CURTATE.

deferred annuity. See DEFERRED.

deferred reversionary annuity. See DEFERRED.

deferred temporary annuity. See DEFERRED.

discounted value of an annuity. Same as PRESENT VALUE. See VALUE—present value.

forborne annuity. A life annuity whose term began sometime in the past, *i.e.*, the payments have been allowed to accumulate with the insurance company for a stated period. In case a group contributes to a fund over a stated period and at the end of the period the accumulated fund is converted into annuities for each of the survivors, the annuity is also called a *forborne annuity.*

immediate annuity. See IMMEDIATE.

intercepted annuity. Same as DEFERRED ANNUITY. See DEFERRED.

joint life annuity. See JOINT.

last survivor annuity. See LAST—last survivor annuity.

life annuity. See LIFE.

ordinary annuity. An annuity whose payments are made at the end of the periods.

reversionary annuity. See REVERSIONARY.

single life annuity. See LIFE—life annuity.

temporary annuity. See TEMPORARY.

temporary annuity due. See TEMPORARY.

temporary reversionary annuity. See TEMPORARY.