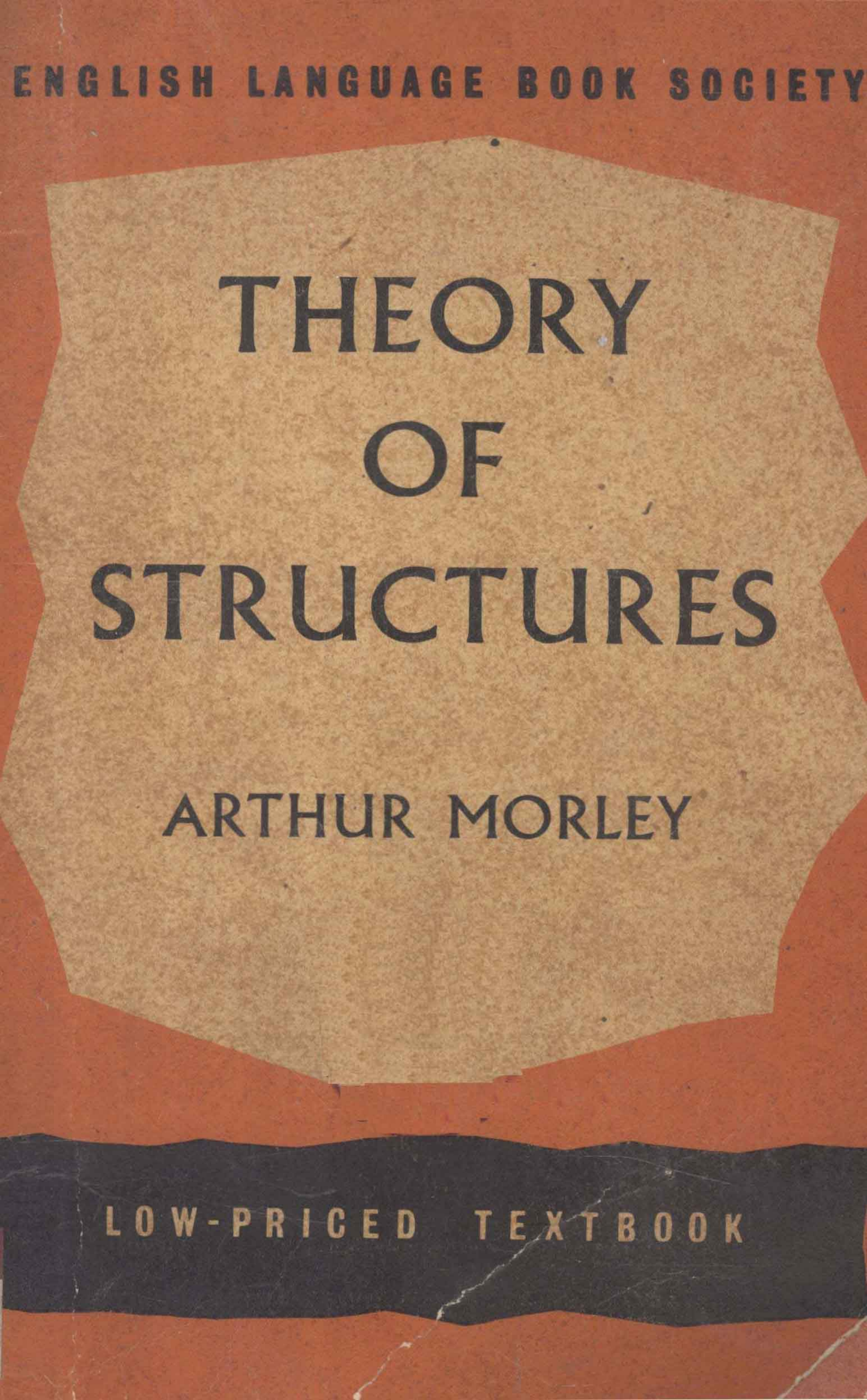

ENGLISH LANGUAGE BOOK SOCIETY

THEORY OF STRUCTURES

ARTHUR MORLEY

LOW-PRICED TEXTBOOK

THEORY OF STRUCTURES

by

ARTHUR MORLEY

O.B.E., D.Sc., Hon.M.I.Mech.E.

Formerly Professor of Mechanical
Engineering in University College,
Nottingham, and later H.M. Staff
Inspector of Technical Schools

WITH DIAGRAMS AND PLATES

THE ENGLISH LANGUAGE BOOK SOCIETY
and
LONGMANS GREEN AND CO LTD

Longmans, Green and Co Ltd
48 GROSVENOR STREET, LONDON W1
RAILWAY CRESCENT, CROYDON, VICTORIA, AUSTRALIA
AUCKLAND, KINGSTON (JAMAICA), LAHORE, NAIROBI
Longmans Southern Africa (pty) Ltd
THIBAULT HOUSE, THIBAULT SQUARE, CAPE TOWN
JOHANNESBURG, SALISBURY
Longmans of Nigeria Ltd
W. R. INDUSTRIAL ESTATE, IKEJA
Longmans of Ghana Ltd
INDUSTRIAL ESTATE, RING ROAD SOUTH, ACCRA
Longmans Green (Far East) Ltd
443 LOCKHART ROAD, HONG KONG
Longmans of Malaya Ltd
44 JALAN AMPANG, KUALA LUMPUR
Orient Longmans Ltd
CALCUTTA, BOMBAY, MADRAS
DELHI, HYDERABAD, DACCA
Longmans Canada Ltd
137 BOND STREET, TORONTO 2

First Edition . . . . *August* 1912
Second Edition . . . . . . *January* 1918
New Impressions . . . . *May* 1919, *February* 1920, *December* 1921, *October* 1923, *March* 1927
Third Edition . . . . *June* 1929
New Impression . . . . *June* 1931
Fourth Edition . . . . . . *November* 1934
New Impressions . . . . . . *January* 1937, *November* 1938, *August* 1940, *September* 1941, *March* 1942, *August* 1942, *April* 1943, *July* 1944, *May* 1946
Fifth Edition . . . . *November* 1948
New Impressions . . . . *April* 1950, *January* 1952
New Impression (with minor corrections) *July* 1955
New Impression . . . . *October* 1956
New Impression . . . . *September* 1959
New Impression . . . . *April* 1961
This edition first published 1960
New impression of this edition 1962

Made and printed in Great Britain by William Clowes and Sons, Limited, London and Beccles

Titles in the E.L.B.S. Low-Priced Textbook Series

Emeleus and Anderson, *Modern Aspects of Inorganic Chemistry.* 12s.
Finar, *Organic Chemistry,* Volumes I and II. 12s. each.
Kipping and Kipping, *Organic Chemistry,* Parts I and II. 9s. Part III. 9s.
Mee, *Physical Chemistry.* 12s.
Partington, *A Textbook of Inorganic Chemistry.* 12s.
Vogel, *A Textbook of Quantitative Inorganic Analysis.* 21s.

Besant and Ramsey, *A Treatise on Hydromechanics,* Parts I and II. 12s.
Ramsey, *Statics.* 9s.
Ramsey, *Dynamics,* Parts I and II. 9s. each.
Starling, *Electricity and Magnetism for Degree Students.* 12s.

Blakey, *University Mathematics.* 12s.
Copson, *An Introduction to the Theory of Functions of a Complex Variable.* 12s.
Hardy, *A Course of Pure Mathematics.* 9s.
Littlewood, *A University Algebra.* 12s.
Titchmarsh, *The Theory of Functions.* 12s.
Weatherburn, *A First Course in Mathematical Statistics.* 12s.

Weatherburn, *Elementary Vector Analysis.* 9s.
Baker, *The Steel Skeleton,* Volumes I and II. 15s. each.
Bevan, *The Theory of Machines.* 12s.
Blyth, *A Geology for Engineers.* 12s.
Cattermole, *Transistor Circuits.* 15s.
Chapman, *Workshop Technology,* Parts I and II. 9s. each. Part III. 12s.
Clayton and Hancock, *The Performance and Design of D.C. Machines.* 15s.
Cotton, *Electrical Technology.* 9s.
Cotton, *The Transmission and Distribution of Electrical Energy.* 12s.
Coulson and Richardson, *Chemical Engineering,* Volume I. 15s. Volume II. 21s.
Gavin and Houldin, *Principles of Electronics.* 12s.
Golding, *Electrical Measurements and Measuring Instruments.* 15s.
Kearton, *Steam Turbine Theory and Practice.* 15s.
Lewitt, *Hydraulics and Fluid Mechanics.* 12s.
Morley, *Theory of Structures.* 12s.
Morley, *Strength of Materials.* 12s.
Rose, *Mathematics for Engineers,* Parts I and II. 9s. each.
Say, *The Performance and Design of A.C. Machines.* 12s.
Starr, *Generation, Transmission and Utilization of Electrical Power.* 12s.
Wrangham, *The Theory and Practice of Heat Engines.* 15s.

Benham, *Economics.* 9s.
Crowther, *An Outline of Money.* 9s.
Marshall, *Principles of Economics.* 12s.
Pigou, *The Economics of Welfare.* 12s.
Stonier and Hague, *A Textbook of Economic Theory.* 12s.

by MORLEY, A., O.B.E., D.Sc., Hon.M.I.Mech.E.

Applied Mechanics. With Diagrams.

Mechanics for Engineers. A Text-Book of Intermediate Standard. With 208 Diagrams and numerous Examples.

Strength of Materials. With 260 Diagrams and numerous Examples.

by MORLEY, A., O.B.E., D.Sc., Hon.M.I.Mech.E., and HUGHES, E., D.Sc., Ph.D., M.I.E.E.

Elementary Engineering Science. With Diagrams.

Mechanical Engineering Science. A Second Year Course. With Diagrams.

Electrical Engineering Science. A Second Year Course. With Diagrams.

Principles of Electricity. Based on the Rationalised M.K.S. System of Units.

by MORLEY, A., O.B.E., D.Sc., Hon.M.I.Mech.E., and INCHLEY, W., B.Sc., A.M.I.Mech.E.

Elementary Applied Mechanics. With 285 Diagrams and numerous Examples.

PREFACE TO FIFTH EDITION

FOR this edition the type has been re-set and new diagrams have been made. The opportunity has been used to re-write several sections of the book. Those on properties of materials and fundamental statics have been curtailed and others relating to later developments of stress analysis have been added. In particular, room has been found for an introduction to the methods of moment distribution, tension coefficients, the Williott-Mohr deflection diagram, wider use of elastic strain energy methods and a fuller treatment of reinforced concrete.

In the preparation of the diagrams I have had the assistance of Mr. G. F. Rodmell, M.I.Struct.E., to whom I am indebted for the design in Plate I. For this and for his valued co-operation in several sections I here express my thanks.

A. M.

BATH,
1948.

PREFACE TO FIRST EDITION

THE object of the following pages is mainly to set forth the theory of the simpler structures so far as it relates to strength, stiffness, and stability. The subject is largely based upon statics and the elastic properties of material, and has much in common with that called Strength of Materials. Consequently I have taken a considerable amount of matter in seven chapters out of the first nine, without great modification from my earlier book, " Strength of Materials," to which the present volume forms a companion.

Worked-out examples form an important feature of the text, and are generally essential to obtaining a sound knowledge of the subject. I have not hesitated to use examples which may be called academic, because they are simplified to illustrate particular points without unnecessary arithmetic complication ; this is particularly the case with statically indeterminate structures and secondary stresses on which little more than the principle is given as an introduction to the larger treatises. Students are apt to forget how many stress computations in structural design are necessarily of a conventional nature, and the attempt has been made to point out when this is specially the case. In some instances more exact estimates have been made to indicate the nature and degree of possible error involved by conventional assumptions.

Fairly free use has been made of influence lines, which form such clear and instructive means of understanding the stresses arising from moving loads.

The practical design of structures involves so much outside of what may reasonably be called theory that it can only be thoroughly learned in the drawing office, but a few examples have been included to illustrate the application of the theory to practice.

Reinforced concrete structures are becoming so important as to demand a complete volume for their treatment, and no attempt has been made to deal with this subject except incidentally as an example of a beam of composite cross-section.

I take this opportunity of thanking numerous friends who have generously assisted me in reading proofs, preparation of designs or diagrams, and checking examples; particularly Messrs. S. W. Budd, R. T. McCallum, B.Sc., and W. N. Thomas, B.Sc. I also thank Sir Wm. Arrol & Co., Ltd., Messrs. Dorman Long & Co., Ltd., and Messrs. R. A. Skelton & Co., for the use of tables, diagrams, and technical information; and Mr. H. S. Prichard for much information regarding American practice relating to the treatment of live loads.

I should be grateful for intimation of any errors which readers may observe in the book.

ARTHUR MORLEY.

UNIVERSITY COLLEGE,
NOTTINGHAM.
April, 1912.

CONTENTS

CHAPTER I

STRESS AND STRAIN

CHAPTER II

WORKING STRESSES

CHAPTER III

STATICS

CHAPTER IV

BENDING MOMENTS AND SHEARING FORCES

CHAPTER V

STRESSES IN BEAMS

CHAPTER VI

MOVING LOADS

CHAPTER VII

DEFLECTION OF BEAMS

CHAPTER VIII

CONSTRAINED BEAMS

CHAPTER IX

FLEXURAL STRAIN ENERGY

CHAPTER X

DIRECT AND BENDING STRESSES

CHAPTER XI

FRAMED STRUCTURES

CHAPTER XII

STRESSES IN FRAMES

CHAPTER XIII

MOVING LOAD STRESSES IN FRAMES

CHAPTER XIV

SELECTED TYPICAL FRAMED STRUCTURES

CHAPTER XV

DEFLECTION AND INDETERMINATE FRAMES

CHAPTER XVI

SOME INDETERMINATE COMBINATIONS

CHAPTER XVII

FRAME MEMBERS AND STRUCTURAL CONNECTIONS

CHAPTER XVIII

PLATE GIRDERS AND BRIDGES

CHAPTER XIX

SUSPENSION BRIDGES AND METAL ARCHES

CHAPTER XX

EARTH PRESSURE, FOUNDATIONS, MASONRY STRUCTURES

PLATES

THEORY OF STRUCTURES

CHAPTER I

STRESS AND STRAIN

1. Introductory. The subject generally known as the *Theory of Structures* or *Mechanics of Structures* includes the study of the forces carried by structures and by the individual members of structures. It is largely an application of the subject of statics, but frequently the complexity of a structure or the uncertainty of the conditions of loading prevent anything like an exact mathematical analysis of the stresses, and assumptions have to be made which it is necessary to test by experiment and practical experience. It is important to realise the limits of much of our theory and the extent to which stress computations are frequently quite conventional rather than representing an actual physical state; *e.g.* the maximum intensity of stress in a flat bar axially pulled is not known within wide limits if the bar is perforated by a single hole.

The mechanics of structures is fundamental to structural design, but successful design involves commercial questions, such as cost and durability, which are not treated as theory, and which cannot well be taken into account except as the result of practical experience.

The " Theory of Structures " is closely related to the subject of the " Strength of Materials," and any boundary between the two is necessarily an arbitrary one. " Strength of Materials " has been treated in a separate volume, but to make this book serviceable to the reader who is concerned with structures only and not with machines, sufficient of the theory of stresses and strains in single pieces has been included to make it complete in itself.

2. Stress. The equal and opposite action and reaction which take place between two bodies, or two parts of the same body, transmitting forces constitute a stress. If we imagine a body which transmits a force to be divided into two parts by an ideal surface, and interaction takes place across this surface, the material there is said to be stressed or in a state of stress. The constituent forces, and therefore the stress itself, are distributed over the separating surface either uniformly or in some other manner. The *intensity of the stress* at a surface, generally referred to with less exactness as merely the stress, is estimated by the force transmitted per unit of area in the case of uniform distribution; if the distribution is not uniform, the stress intensity at a point in the surface must be looked upon as the limit of the ratio of units of force to units of area when each is decreased indefinitely. The intensity of stress is also sometimes called the unit stress.

3. Simple Stresses. There are two specially simple states of stress which may exist within a body. More complex stresses may be split into component parts.

(1) *Tensile stress* between two parts of a body exists when each draws the other towards itself. The simplest example of material subject to tensile stress is that of a tie-bar sustaining a pull. If the pull on the tie-bar is say P lb., and we consider any imaginary plane of section X perpendicular to the axis of the bar, of area *a* sq. in., dividing the bar into two parts A and B (Fig. 1), the material at the section X is under a tensile stress. The portion B, say, exerts a pull on the portion A which just balances P, and is therefore equal and opposite to it. The average force exerted per square inch of section is

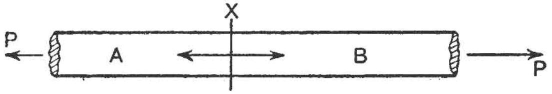

FIG. 1

$$p=\mathrm{P}/a$$

and this value p is the mean intensity of tensile stress at this section.

(2) *Compressive stress* between two parts of a body exists when each pushes the other from it.

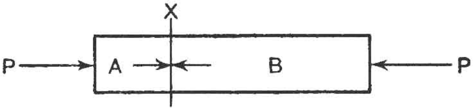

FIG. 2

If a bar (Fig. 2) sustains an axial thrust of P tons at each end, at a transverse section X of area *a* sq. in., dividing the bar into two parts A and B, the material is under compressive stress. The portion A, say, exerts a push on the portion B equal and opposite to that on the far end of B. The average force per square inch of section is

$$p=\mathrm{P}/a$$

and this value p is the mean intensity of compressive stress at the section X.

Shear stress exists between two parts of a body in contact when the two parts exert equal and opposite forces on each other laterally in a direction tangential to their surface of contact. As an example, there is a shear stress at the section XY of a pin or rivet (Fig. 3) when the two plates which it holds together sustain a pull P in the plane of the section XY. If the area of section XY is a sq. in., and the pull is P tons, the total shear at the section XY is P tons, and the average force per square inch is

$$q = \mathrm{P}/a$$

This value q is the mean intensity of shear stress at the section XY.

4. Strain. Strain is the alteration of shape or dimensions resulting from stress.

(1) Tensile strain is the stretch, and often results from a pull which causes a condition of tensile stress to be set up. It is in the direction of the tensile stress, and is measured by the fractional elongation. Thus, if a length l units is increased to $l+\delta l$, the strain is

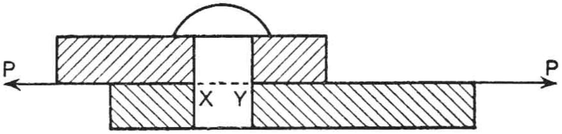

FIG. 3

$$\delta l/l$$

The strain is obviously equal numerically to the stretch per unit of length.

(2) Compressive strain is the contraction which is often due to compressive stress, and is measured by the ratio of the contraction to the original length. If a length l contracts to $l-\delta l$, the compressive strain is

$$\delta l/l$$

Tensile stress causes a contraction perpendicular to its own direction, and compressive stress causes an elongation perpendicular to its own direction.

(3) Distortional or shear strain is the angular displacement produced by shear stress. If a piece of material be subjected to a pure shear stress in a certain plane, the change in inclination (estimated in radians) between the plane and a line originally perpendicular to it, is the numerical measure of the resulting shear strain (see Art. 10).

5. Elastic Limits. The limits of stress for a given material within which the resulting strain completely disappears after the removal of the stress are called the elastic limits. If a stress beyond an elastic limit is applied, part of the resulting strain remains after the removal of the stress; such a residual strain is called a permanent set. The determination of an elastic limit will evidently depend upon the detection of the smallest possible permanent set, and gives a lower stress when instruments of great precision are employed than with cruder methods. In some materials the time allowed for strain to develop or to disappear will affect the result obtained.

Elastic strain is that produced by stress within the limits of elasticity; but the same term is often applied to the portion of strain which disappears with the removal of stress even when the elastic limits have been exceeded.

Hooke's Law states that within the elastic limits the strain produced is proportional to the stress producing it. The law refers to all kinds of stress.

This law is not exactly true for all materials, but is approximately so for many.

6. Modulus of Elasticity. Assuming the truth of Hooke's Law, we may write

$$\text{intensity of stress} \propto \text{strain}$$

or

$$\text{stress intensity} = \text{strain} \times \text{constant}$$

The constant in this equation is called the modulus or coefficient of elasticity, and will vary with the kind of stress and strain contemplated, there being for each kind of stress a different kind of modulus. Since the strain is measured as a mere number, and has no dimensions of length, time, or force, the constant is a quantity of the same kind as a stress intensity, being measured in units of force per unit of area, such as pounds or tons per square inch. We might define the modulus of elasticity as the intensity of stress which would cause unit strain, if the material continued to follow the same law outside the elastic limits as within them, or as the intensity of stress per unit of strain.

7. Components of Oblique Stresses. When the stress across any given surface in a material is neither normal nor tangential to that surface, we may conveniently resolve it into rectangular components, normal to the surface and tangential to it. The normal stresses are tensile or compressive according to their directions, and the tangential components are shear stresses.

A simple example will illustrate the method of resolution of stress. If a parallel bar of cross-section a square inches be subjected to a pull of P tons, the intensity of tensile stress p is P/a in the direction of the length of the bar, or, in other words, normal to a surface, AB (Fig. 4), perpendicular to the line of pull.

Let p_n and p_t be the component stress intensities, normal and tangential respectively, to a surface, CD, which makes an angle θ with the surface AB. Resolving the whole force P normal to CD, the component is

$$P_n = P \cos \theta$$

and the area of the surface CD is *a* sec θ, hence

$$p_n = \frac{P \cos \theta}{a \sec \theta} = \frac{P}{a} \cos^2 \theta = p \cos^2 \theta$$

and resolving along CD, the tangential component of the whole force is

$$P_t = P \sin \theta$$

$$p_t = \frac{P \sin \theta}{a \sec \theta} = \frac{P}{a} \sin \theta \cos \theta = p \sin \theta \cos \theta, \text{ or } \frac{p}{2} \sin 2\theta$$

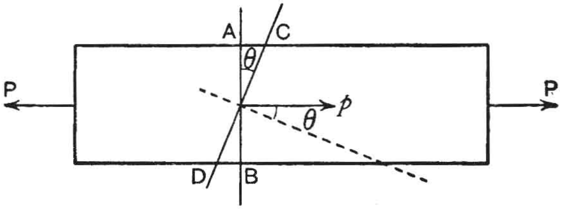

FIG. 4

Evidently p_t reaches a maximum value $\frac{1}{2}p$ when θ=45°, so that all surfaces, curved or plane, inclined 45° to AB (and therefore also to the axis of pull) are subjected to maximum shear stress. In testing materials in tension or compression, it often happens that fracture takes place by shearing at surfaces inclined at angles other than 90° to the axis of pull.

Example. The material of a tie-bar has a uniform tensile stress of 5 tons per sq. in. What is the intensity of shear stress on a plane the normal of which is inclined 40° to the axis of the bar? What is the intensity of normal stress on this plane, and what is the resultant intensity of stress?

Considering a portion of the bar, the section of which is 1 square inch normal to the axis, the pull is 5 tons. The area on which this load is spread on a plane inclined 40° to the perpendicular cross-section is

$$(1 \times \sec 40°) \text{ sq. in.}$$

and the amount of force resolved parallel to this oblique surface is

$$(5 \times \sin 40°) \text{ tons}$$

hence the intensity of shearing stress is

$$5 \sin 40° \div \sec 40° = 5 \sin 40° \cos 40° = 5 \times 0{\cdot}6428 \times 0{\cdot}7660$$
$$= 2{\cdot}462 \text{ tons per sq. in.}$$