

STRENGTH of MATERIALS

TO THE READER

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P. STEPIN

STRENGTH OF MATERIALS

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PUBLISHER'S NOTE

The book is intended as a textbook on Strength of Materials for students at technical institutes.

It deals with tension and compression, torsion, bending, complex resistance of materials. It presents strength theories, stability analysis, dynamic problems, fatigue analysis.

At the end of the book the author discusses briefly the limit-design method and the principal ideas of the statistical method of structural design.

The book includes numerous examples and their solutions which facilitate the understanding of the subject-matter and clarify the essence of mechanical phenomena.

The book is very popular among students and professors at the Soviet technical institutes. This is a revised edition in English.

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CHAPTER I

FUNDAMENTALS

1. Problems Dealt with in the Course

Various structures and machines with the design and construction of which the engineer is concerned in his practical work must necessarily possess, apart from other qualities, *strength*, i.e., the ability to resist failure when subjected to external forces (loads).

This demands that the elements (parts) of structures and machines should be made of appropriate material and have suitable dimensions.

The treatment of methods of structural design based on strength constitutes the first problem of the course.

In addition, in many cases it is necessary to determine the changes in shape and dimensions (deformations) which are produced by loads in structural elements.

The fact is that in reality there exist no absolutely rigid, non-deformable bodies which are studied in theoretical mechanics.

Of course, deformations produced by ordinary working loads are small and can be detected only with special instruments.

Small deformations do not appreciably affect the laws of equilibrium and motion of a body and are consequently neglected in theoretical mechanics. However, without a study of these deformations it is difficult to solve a problem which is very important to the engineer, that is, to discover under what conditions failure may occur and, on the other hand, what are the safe conditions.

Moreover, in many cases it is necessary to limit the magnitude of deformations in spite of their smallness as compared with the dimensions of the part itself because otherwise the normal service of the structure may become impossible. For example, when a part is machined

deformation of the part itself and of the machine elements may lead to a reduction in accuracy beyond what is permissible.

The ability of a structural element to resist deformation is called *stiffness*.

Hence the second problem of the course: the treatment of methods of structural design based on stiffness.

The next problem of strength of materials is concerned with the study of stability of equilibrium forms of real (i.e., deformable) bodies.

Stability is the ability of a body to retain under loading its *original* state of equilibrium.

Instability is a condition in which the load causes an abrupt qualitative change in the shape of a part and the character of its deformation.

A thin member compressed by a force acting along its axis is an example of instability. Up to a certain specific value of the compressive force depending on the material, dimensions and supporting conditions of the member, it will remain straight. With an increase in the load, however, the member will buckle and become curved.

Instability may occur at magnitudes of loads that are quite safe from the viewpoint of the strength or stiffness of a structural element.

Methods of structural design based on stability constitute the third problem of the course.

In carrying out the above types of design it is necessary to aim at the maximum *economy in materials*, at adequate but not excessive dimensions of machine and structural parts. Obviously this requires the most complete and profound study of the properties of the materials used and the nature of the loads acting upon the part in question. This can be achieved by thorough experimental research and a careful study of experience accumulated in the designing of structures and the service given by them.

On the other hand, in the derivation of basic design relations in strength of materials it is necessary to introduce various hypotheses and simplifying assumptions. The validity of these hypotheses and assumptions, and also the degree of inaccuracy they lead to in design formulas are verified by comparing the results calculated on the basis of these formulas with experimental data,

Most of the structures an engineer has to deal with are very complicated in form, but their individual elements can be reduced to the following simplest types.

1. A *rod* is a body two dimensions of which are small as compared with the third (Fig. 1.1a). In a particular case a rod may have a constant cross-sectional area and a straight-line axis.

The axis of a rod is a line passing through the centroids of its cross sections.

A rod with a straight-line axis is often called a *bar*.

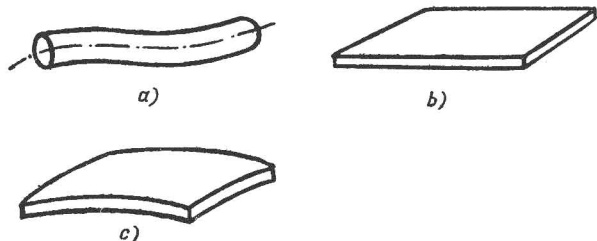


Fig. 1.1

2. A *plate* is a body bounded by two flat surfaces the distance between which is small as compared with the other dimensions (Fig. 1.1b).

3. A *shell* is a body bounded by two curvilinear surfaces the distance between which is small as compared with the other dimensions (Fig. 1.1c).

4. A *block* is a body all three dimensions of which are of the same order.

This course is concerned primarily with bodies having the form of rods of constant cross section and the simplest systems composed of such rods. It deals with rods having a considerable degree of stiffness, i. e., rods which do not become noticeably deformed under load.

In very slender bars (Fig. 1.2) such large deformations occur that they cannot be disregarded even in determining the reactions of supports. The determination of a new distance l_2 , however, which differs considerably from the original one l_1 , is a rather complicated problem.

Methods of analysis of slender bars, plates, shells and blocks are treated in the course on the theory of elasticity

free of the simplifying hypotheses which are introduced in the course on strength of materials. The methods of the theory of elasticity provide exact solutions of problems treated in the course on strength of materials as well as solutions of more complicated problems where it is not possible to state applicable simplifying hypotheses.

Methods of designing bar systems are studied in the course on the theory of structures (also known as structural mechanics). The development of the science of strength of materials as well as of the above related sciences is closely bound up with industrial progress.

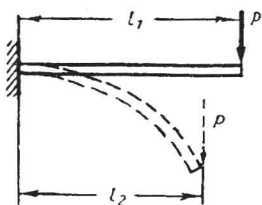


Fig. 1.2

The science of strength of materials originated in the 17th century as a result of Galileo's work. An important contribution to the development of strength of materials and theory of elasticity was made by a number of outstanding scientists—

Robert Hooke, Jacob Bernoulli, Barré de Saint-Venant, Augustin Cauchy, Gabriel Lamé and others—who formulated basic hypotheses and gave several design equations.

Of particular note is the remarkable research (18th cent.) done by the eminent scientist Leonhard Euler, a member of the St. Petersburg Academy of Sciences. Wide use is still made of his work on stability analysis of compressed members.

In the 19th century the work of the Russian scientists D. I. Zhuravsky, K. S. Golovin and others acquired world importance. D. I. Zhuravsky's formula for determining shearing stresses in bending is still in use today.

Important and interesting investigations on stability analysis of compressed members which have not lost their significance nowadays were carried out by F. S. Yasinsky at the end of the 19th century.

At the beginning of the 20th century Russian scientists began to make a still greater contribution to the theory of strength of materials. Valuable works were written by Professor I. G. Bubnov, Academician A. N. Krylov, Academician B. G. Galerkin and others further developing and refining methods in this field. Bubnov-Galerkin's methods

for solving complex problems in strength of materials are known throughout the world.

A very great contribution to the development of the theory of strength of materials was made by Professor S. P. Timoshenko, the author of several first-class textbooks and numerous publications dealing with structural design based on strength, stability and vibration.

An extensive network of research institutes dealing with structural design has been created in the U.S.S.R. In conjunction with our establishments of higher learning these institutes have solved many problems vital to technical progress, and developed new effective methods of designing parts of complicated shape under different loading conditions. It is in point to mention here the works of Academician N. N. Davidenkov on theories of strength, those of Academician S. V. Serensen concerned with the study of strength of parts under repeated loading, and those of Academician A. N. Dinnik on stability, among others.

Of particular note are the outstanding works of Professor V. Z. Vlasov on the analysis of thin-walled bars and shells, which are widely used in engineering today.

Important investigations have been carried out by Y. N. Rabotnov, A. A. Ilyushin, E. I. Grigolyuk, N. I. Bezukhov, V. V. Bolotin, S. D. Ponomarev, A. R. Rzhanitsyn, A. F. Smirnov, A. S. Grigoryev, I. A. Simvulidi, S. N. Sokolov and other Soviet scientists.

2. Assumptions in Strength of Materials

In view of the complexity of structural analysis certain simplifying assumptions are made in strength of materials concerning properties of materials, loads and the nature of the interaction of a part and loads.

Experimental verification of design relations obtained on the basis of the assumptions given below shows that the resultant error is so insignificant that for practical purposes it can be neglected.

Assumption 1. *The material of the body has a solid (continuous) structure.* This assumption is fully justified from the practical point of view as most structural materials have such fine-grained structure that they can be considered solid, continuous, without giving rise to appreciable

error. Calculations based on the assumption of solid structure give satisfactory results in practice even for such materials as timber, concrete and stone.

This is due to the fact that the dimensions of real parts are many times greater than interatomic distances.

The above assumption makes it possible to employ a method of analysing infinitesimal volumes for which the mathematical apparatus of continuous functions may be used and to apply the results obtained to real specimens.

Assumption 2. *The material of a part is homogeneous, i.e., it has identical properties at all points.* Metals possess a high degree of homogeneity, i. e., they have practically the same properties throughout a part. Timber, concrete, stone and reinforced plastics are less homogeneous. For example, concrete contains an aggregate—small stones, gravel and brick—the properties of which are different from those of cement. In timber there are knots whose properties also differ greatly from the properties of the remainder. In plastics the properties of a resin differ from those of a filler.

Nevertheless, as experience shows, calculations based on the assumption of homogeneity of the material of a part give satisfactory results for main structural materials.

Assumption 3. *The material of a part is isotropic, i.e., it has identical properties in all directions.* Investigations show that the crystals of which many materials consist have quite different properties in different directions. For example, copper crystals are more than three times as strong in one direction as in another.

However, in fine-grained materials the properties in different directions become uniform, they “level out” due to the disorderly arrangement of a great number of crystals, and these materials can be considered virtually isotropic.

For such materials as timber, reinforced concrete and plastics the above assumption is only approximate.

Such materials (whose properties are different in different directions) are called *anisotropic*.

In the solution of some problems, such as those concerned with the use of plastics, it is necessary to take into account the anisotropy of a material by the application of the methods of the theory of elasticity.

Assumption 4. *There are no internal (initial) forces in a body prior to loading.* The forces of interaction between the particles of a material, the distances between which vary, resist changes in the shape and dimensions of the body under load. Henceforth, speaking of internal forces (or forces of elasticity) we shall have in view just these forces, without taking into account the molecular forces existing in an unloaded body as well.

This assumption is not, strictly speaking, true of a single material. In steel parts there exist internal forces induced non-uniform cooling, and in timber by non-uniform drying; in concrete they arise during setting.

The magnitude of these forces is usually unknown to a designer. When there is reason to suppose that these forces are considerable an attempt is made to determine them (experimentally).

Assumption 5, or the principle of superposition. *The effect of a system of forces acting on a body is equal to the sum of the effects of these same forces applied to the body in succession and in any order.* The word "effect" implies deformations, internal forces produced in the body and displacements of individual points, depending on the particular case.

It should be borne in mind that the action of the separate forces of a system should be considered in conjunction with the corresponding reactions of constraints.

The principle of superposition, extensively used in theoretical mechanics for absolutely rigid bodies, can be applied to deformable bodies only under the following two conditions:

1. Displacements of the points of application of forces are small compared to the dimensions of a body.

2. Displacements resulting from the deformation of a body depend linearly on the acting forces.

In ordinary structures both these conditions are fulfilled, and therefore the principle of superposition is widely used in structural design.

Assumption 6, or Saint-Venant's principle. *At points of a body which are sufficiently distant from the places of application of loads the magnitude of internal forces depends to a very small extent on the particular manner in which these loads are applied.*