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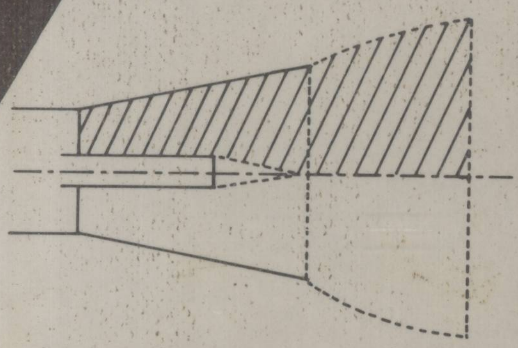
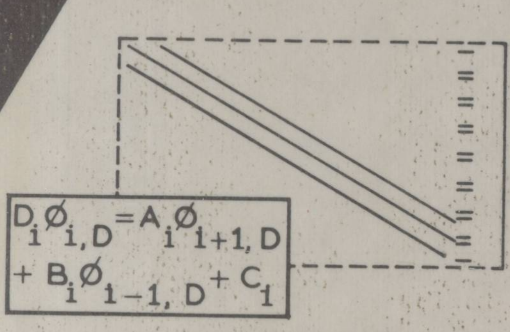
GENMIX — A General Computer Program for Two-Dimensional Parabolic Phenomena



$$\frac{\partial \phi}{\partial x} + (a+b\omega)\frac{\partial \phi}{\partial \omega} = \frac{\partial}{\partial \omega}\left(c\frac{\partial \phi}{\partial \omega}\right) + d$$



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GENMIX:
A General Computer Program for Two-dimensional Parabolic Phenomena

By

D. BRIAN SPALDING

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In order to make this volume available as economically and rapidly as possible the author's typescript has been reproduced in its original form. This method unfortunately has its typographical limitations but it is hoped that they in no way distract the reader.

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PREFACE

The computer program described in the present book is the outcome of many years of experience, gathered by the author, his students and associates, in the course of teaching, research, consulting work and design studies. The basic method remains that of Patankar and Spalding (P & S)(1967a). This method was first incorporated into a computer program by S.V. Patankar; and that program was published in a book (P & S, 1967b). The program was widely used and adapted; for example an ALGOL version was published recently in the USSR (Zhukauskas and Shlanchyauskas, 1973).

The second edition (P & S, 1970) of the 1967 book contained a program developed by the present author. It was called GENMIX; it is one of the two parents of the subject of the present work, which is also called GENMIX. The other parent is the program of CHAM Ltd., called PASS (parabolic axi-symmetric systems), which has been developed for use in engineering practice. The new GENMIX has also been adapted so as to be compatible with, and to demonstrate some of the major ideas of, the CHAM program suite: PHOENICS (parabolic, hyperbolic, or elliptic numerical-integration code series).

The new GENMIX is intended primarily for teaching purposes; and this book has been designed to assist the potential user to understand its physical and mathematical basis, and the ways in which it can be applied to practical problems, and extended in case of need.

The arrangement of material in the book, which intersperses mathematical, physical and computer-coding aspects of the matter, has been chosen so as to parallel a course of lectures, and associated computer-workshop sessions, in which the learner is enabled to make some elementary computations as soon as he has obtained a superficial knowledge of the method. However, the reader preferring to study the material in a different order should find it well enough sign-posted.

The book is not a guide to the literature of boundary-layer theory and practice. However, references to and remarks about the literature have been inserted at appropriate points in the text, to assist the reader to perceive the relationships between the present method and those used by other workers.

The listings supplied at the end, and the accompanying computer output, represent only a tiny fraction of the problems which have been solved with the aid of GENMIX and its forbears. Readers contemplating the use of GENMIX for a 2D parabolic problem which does not happen to be among the examples supplied, and wishing to know if any such application has already been made, are invited to make contact with the author.

Experience has shown that new users of even highly-automated computer codes encounter difficulties: sometimes they change the input data, and the computer produces error messages, unrealistic output, or nothing at all. Not knowing how to overcome the difficulties, they make a few random modifications without success, and then abandon the whole enterprise. Usually they attribute their failure, in large part, to the originator of the code; at the very least, they incline to think that he has misled them.

It would perhaps be possible, by the expenditure of many man-years of effort, to produce a "fool-proof" computer code, which, when provided with indigestible input data, would print out a message saying what was wrong and what the user should do about it. However, this would take so long, and be so costly, that no-one would be able to wait for it, or afford it when it was produced. For the time being therefore, would-be users of computer codes must either develop their own or make use of those like GENMIX, which are offered in good faith by their originators as <u>potentially</u> valuable, but which need to be handled with understanding.

It is regrettably impossible to mention individually the many people who have contributed to the development of GENMIX, whether by positive suggestion or by the provision of experience, favourable or adverse. The author is however able and glad to acknowledge the assistance:- of Peter Dale in continually testing, refining and reconstructing the program over many years, and of Colleen King, who, with Peter Dale, helped to prepare the diagrams, and of Christine MacKenzie who prepared the typescript.

LOCATIONS OF MAJOR ITEMS

Q. What differential equation does the GENMIX code solve?

A. $$\frac{\partial \phi}{\partial x} + (a + b\omega) \frac{\partial \phi}{\partial \omega} = \frac{\partial}{\partial \omega} (c \frac{\partial \phi}{\partial \omega}) + d \qquad . \ (2.1\text{-}1)$$

See page 14 et seq.

Q. What physical laws and processes are represented by this equation?

A. Conservation and transport, by convection and diffusion, of heat, mass and momentum, for two-dimensional boundary-layer flows.

See page 36 et seq.

Q. What is the basis of the finite-difference representations of the differential equations?

A. Integration over control volumes, coupled with interpolation assumptions, in such a way as to link downstream (unknown values) implicitly.

See page 66 et seq.

Q. How are the finite-difference equations solved?

A. By application of the tri-diagonal matrix algorithm (TDMA), sweeping once through the flow domain.

See page 79 et seq.

Q. What features of the GENMIX grid are mainly responsible for the economy of the method?

A. (1) Its width expands and contracts so as just to cover the region of interest.

See pages 12 and 115.

(2) The use of non-dimensional stream function ω as cross-stream variable permits lateral-convection terms to be computed accurately without iteration.

See page 65.

Q. How are turbulent flows handled in GENMIX?

A. The present version contains a form of the mixing-length model. However, versions containing more advanced turbulence models exist, and will be published.

See pages 106 et seq, and 125 et seq.

Q. What models of chemical-kinetic processes are built into GENMIX?

A. A single-step reaction is postulated. Its rate is controlled by an Arrhenius-type expression in laminar flow, and by an "eddy-break-up" expression in turbulent flow.

These are merely examples. Much more sophisticated models can be incorporated.

See pages 130 et seq.

Q. Does GENMIX solve the lateral (y-direction) momentum equation?

A. Not in the present version; but versions exist which do solve that equation both for supersonic (hyperbolic) and subsonic (partially-parabolic) problems.

See page 218 et seq.

Q. To what uses can GENMIX be put?

A. These are exemplified in Chapters 9 and 10.

See page 143, et seq.

Q. How can GENMIX be extended?

A. By the inclusion of more advanced turbulence models; by the introduction of further physical effects such as swirl, radiation, complex chemistry, suspended particle effects; and by allowance for lateral-momentum effects.

See Chapter 11.

ERRATA

Page 117:

In 3rd line from bottom:

replace "FACI which, being"
by "FACI which is being"

Page 118:

In 6th line from top:

replace "RM1"
by "RMI"

In 7th line from top:

replace "$0.4(\rho ur)_I$"
by "$0.4(\rho r)_I$.UDIF"

CONTENTS

Chapter 1 INTRODUCTION

1.1 The scope of two-dimensional boundary-layer theory

(a) Definitions

A boundary layer can be characterised as a region, in a moving fluid, in which there is a single predominant direction of flow; and in which transfers of momentum, heat and matter by molecular and turbulent intermingling occur only at right angles to the predominant direction.

A two-dimensional boundary layer is a boundary layer in which each fluid property varies with only two of the three possible space coordinates. Both plane flows and axi-symmetric flows are two-dimensional (2D) according to this definition; the third dimension, in which the fluid properties are invariant, is the direction normal to the plane in the first case; and it is the angle of rotation about the symmetry in the second instance.

A steady flow is one in which all fluid properties are invariant with time. A flow in which this condition is not obeyed is called unsteady or transient.

(b) Examples of two-dimensional steady boundary-layer phenomena

Flow phenomena which satisfy the definition include the following:-

- Flow around an aerofoil of uniform section and large aspect ratio.
- Flow in plane or axi-symmetric jets, wakes, plumes, and diffusion flames. If buoyancy is influential, the gravitational field must be aligned with the symmetry axis.
- Flow in a wall jet, or in the region downstream of a film-cooling slot, where the slot is wide and of uniform width.
- Developed or developing turbulent pipe flow.
- Flows in circular-sectioned nozzles, diffusers and venturis.
- Flow over a blunt-nosed body of revolution, at zero angle of attack, rotating about its axis.

It should be noted that, in the last example, three velocity components are to be considered. However, the flow is 2D because no fluid property varies with circumferential position: the flow is axi-symmetrical.

(c) Examples of steady two-dimensional flow which are not boundary layers

It is important to distinguish boundary layers from flows which lack the "single predominant direction of flow", such as the following:-

- The stalled aerofoil of uniform section and large aspect ratio.
- Flow behind a bluff-body flame stabiliser.
- Flow downstream of a sudden enlargement in the diameter of a pipe.
- The flow which is brought about by the impingement of a jet perpendicularly on to a wall.
- The flow induced by the entry of an intensely-swirling fluid stream into an axi-symmetric chamber.
- The flow in a cavity, of rectangular cross-section, let into the wall of a duct through which fluid streams.

(d) Examples of flows which are boundary layers but not two-dimensional

Many boundary layers, in practice, are three-dimensional (3D). This means that there are variations of fluid properties, in both the directions normal to the predominant direction of flow, not just in one. Examples of such 3D boundary layers, which can not be analysed by the present method, include the following:-

- The aerofoil of non-uniform cross-section or small aspect ratio.
- Film cooling effected by the blowing of a coolant fluid along a surface from a row of orifices of circular cross-section.
- Flow through a duct of square cross-section.
- Flow through a duct of circular cross-section,

having a wall temperature which varies with circumferential position.

- Flow through a duct of circular cross-section, under the influence of buoyancy forces directed obliquely to the duct axis.

- Flow through a duct of arbitrary cross-section, which is in steady rotation about an axis at right angles to its length.

1.2 Some practical circumstances in which steady two-dimensional boundary layers often play important roles

2D steady boundary layers are too common in practice for it to be possible to provide a comprehensive list of their occurrences. The following short one is merely suggestive:

- The film cooling of gas-turbine combustion chambers.

- Heat transfer to the stator blades of gas turbines.

- The melting of the "batch" (i.e. the inflowing stream of sand, broken glass, ash and other materials) in a glass furnace.

- The burning of fuel gas in a turbulent diffusion flame confined in a duct.

- Mixing of two streams in an ejector.

- The spread of flame through a pre-mixed fuel-air mixture, well downstream of a bluff-body flame-stabilising baffle.

- Flow in an axi-symmetrical diffuser.

- The rocket exhaust plume, in which chemical reactions may occur as the excess fuel mixes with the oxygen of the air.

- The motion of air and water vapour in the lower atmosphere.

- Vaporisation of water from the surface of a lake.

- The heating of the cooling water in a steam condenser.

There exist also some practically-interesting phenomena which are mathematically similar, even though they are one-dimensional and unsteady: the "predominant direction" is that of time; and the transfers of momentum, heat and matter occur "at right angles to the time dimension", i.e., in space. Because their mathematical similarity implies that they can be analysed by the same method, and computed by the program described below, examples will now be given. They include:-

- Unsteady heat conduction into the earth, under the influence of daily and yearly changes.
- The growth and decay of the layer of turbulent fluid on the surface of a wide lake, stirred by a uniform wind stress.
- Unsteady propagation of a plane laminar flame through a pre-mixed reservoir of combustible gas.
- The growth of a spherical bubble of steam in a reservoir of superheated liquid.

1.3 The mathematical character of the problem of predicting 2D steady boundary-layer behaviour

(a) Marching integration

The most important characteristic of boundary-layer problems from the point of view of the practical mathematician is that they permit "marching integration".

"Integration" means establishment of the solution of the differential equations which describe the physical processes; so integration entails finding out what values of velocity, temperature, concentration, etc., prevail at each point in the domain of interest.

"Marching" integration is that kind of integration which starts by determining the values at one end of the domain, then determines the values over a front displaced just a little from that end, and so gradually moves the "integration front" towards the other end of the domain until the required values have been determined everywhere. Iteration is not required. (The metaphorical reference is to a line of soldiers sweeping shoulder-to-shoulder across the battlefield, and performing their task with such efficiency that they need not return to "mop up" isolated pockets of resistance; nor do they have to retreat and make renewed assaults until the enemy is finally subdued.)

The direction of the "march" is always that of the "predominant direction of flow" mentioned in the definition given in section 1.1 (a). Because convection cannot occur in the direction opposite to the direction of fluid flow, and because the transfers of momentum and heat by viscous and conductive action take place only in the direction at right angles, no influences from downstream locations can extend to upstream ones.* This is why, after having made an integration sweep in the downstream direction, there is no need to return; for, since the later-determined quantities cannot influence those determined earlier, no iterative correction is required. (N.B. In re-circulating flows, where there is no such predominant direction, convection can operate in all directions. Therefore, no matter what direction of sweep is chosen, it will always occur that earlier-determined values can be influenced by those determined later; so repeated integration sweeps are needed, and one must be content merely if these are few and if they result in a converged solution, i.e. one that in the end changes insignificantly from one sweep to the next.)

Why the ability to use marching integration is important in practice is that the confinement of the integration to a single sweep diminishes the necessary computer time; moreover, the freedom to visit each point in the field only once reduces the dimensionality of computer storage. Thus, only one-dimensional storage is needed for temperature (for example) in a two-dimensional boundary-layer calculation; for at any stage in the calculation, one is concerned only with the temperatures along a single line traversing the domain. (N.B. "Domain" and "field", "march" and "sweep", "integration" and "value-determining" are used as synonyms in this discussion, simply for variety. No significant distinctions are implied.)

Three mathematical terms can be usefully introduced at this point: parabolic, elliptic and partially-parabolic (Spalding, 1974, 1975a). The first is employed to describe mathematical problems which can be solved by a single marching integration; therefore all the problems discussed in the present book are parabolic. The second is employed for problems involving recirculation, or straight-through flow at

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*Footnote: Strictly speaking, non-uniformities of pressure arising from downstream disturbances can, if the flow velocity is subsonic, transmit themselves upstream. In the present book, attention is confined to circumstances in which such transmissions are negligible; the chief requirement for this to be true is that the radius of curvature of the streamlines should be much larger than the thickness of the boundary layer.

low Reynolds numbers, in which convective or diffusive (viscous, conductive) influences from downstream affect upstream locations; thus a stalled aerofoil presents an elliptic problem. The third term, partially-parabolic, describes flows of the kind indicated in the footnote on page 1.5, for which the downstream → upstream influence is exerted via pressure alone; the term "semi-elliptic" has also been used for such flows (Spalding, 1976a).

In the present work, attention is confined to 2D parabolic problems; however, the computer code GENMIX can be adapted to the solution of 2D partially-parabolic ones, as described in Chapter 11.

(b) Use of a grid

Although, in principle, the values of fluid variables at all points in the domain are of interest, in numerical computations it is necessary to confine attention to a limited number of points. This is done to save computer storage and time. If later the values of variables are required at places which do not coincide with the selected locations, they must be obtained from the considered-point values by interpolation. In the interests of easy organisation of the calculation, the considered points are arranged to lie at the nodes of a grid formed by two sets of lines intersecting at right angles (or nearly doing so). The lines of one set lie more or less along the "predominant direction of flow"; the lines of the other set are therefore more

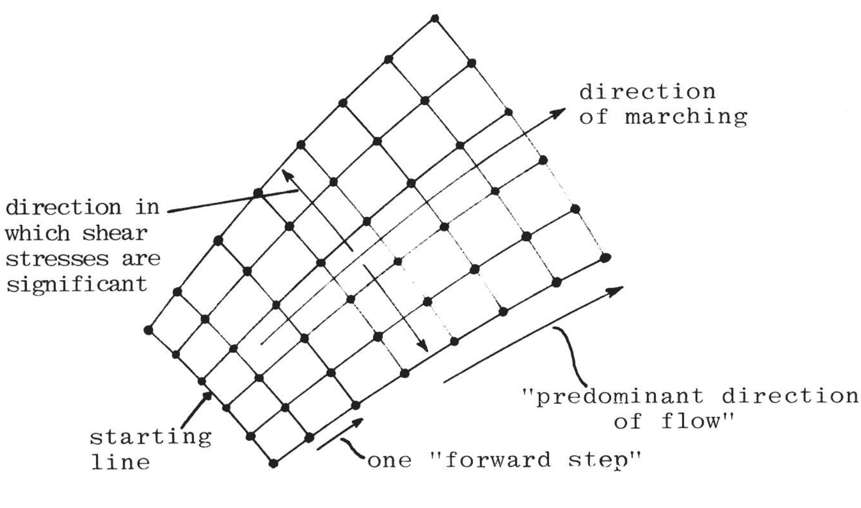

FIG. 1.3-1 ILLUSTRATION OF THE FINITE-DIFFERENCE GRID FOR A 2D PARABOLIC FLOW.

or less coincident with the directions in which "shear stresses, heat fluxes and diffusion fluxes are significant". (The reference is of course to the definition of 1.1 (a).) A marching integration therefore involves starting at the upstream edge of the grid, where the values of the fluid variables must be given, and proceeding line-by-line across the grid to the downstream edge, determining the fluid-variable values for the nodes for each successive line.

(c) Integration formulae

How are the values of the fluid variables at the nodes on the downstream line of a step to be obtained from those for the nodes at the upstream line of the step? Though more complex formulae are possible, most integration procedures use either four-node or six-node integration formulae answering this question.

Explicit formulae connect the values of fluid variables which are valid for groups of four nearby points, of which one is on the downstream line and the other three are on the upstream line. The latter values are always known quantities when an integration step is being performed; so there is only one unknown value, the downstream one, in the formula. This value can therefore be expressed explicitly in terms of known quantities.

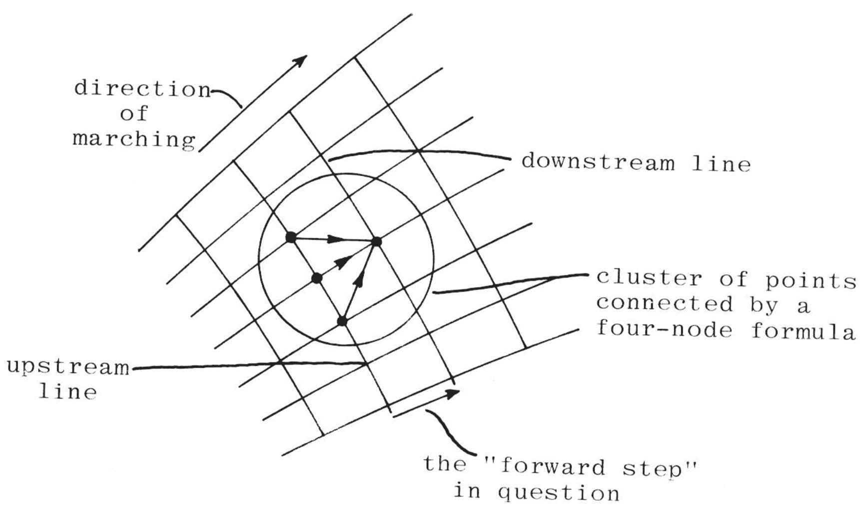

FIG. 1.3-2 ILLUSTRATION OF THE EXPLICIT FORMULAE FOR MARCHING INTEGRATION