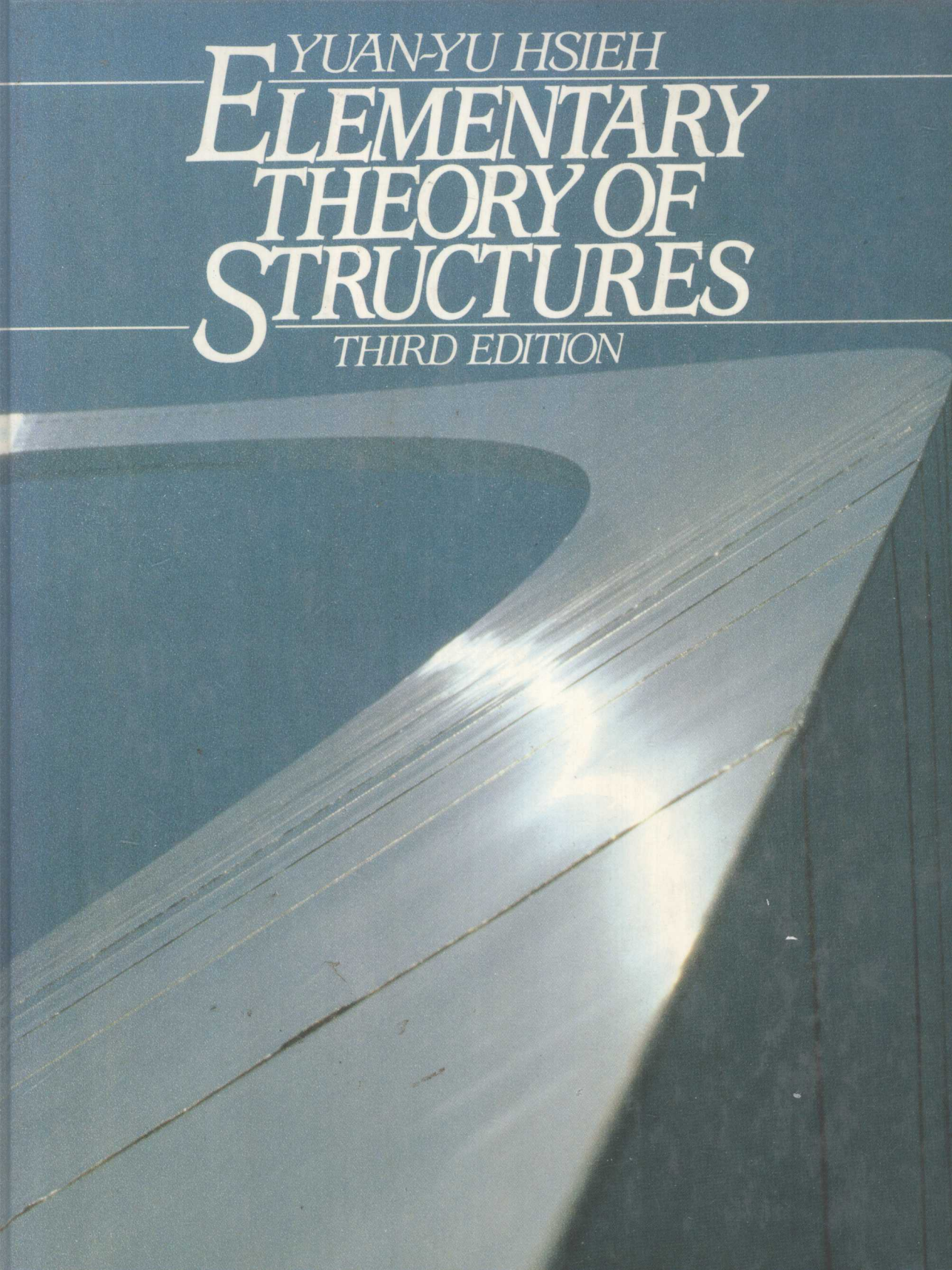


YUAN-YU HSIEH
**ELEMENTARY
THEORY OF
STRUCTURES**
THIRD EDITION



*Elementary Theory
of Structures*

Third Edition

YUAN-YU HSIEH



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of Structures*

Preface

The first two editions of this book were written within the scope of linear and static aspects of structure, and with equal weight on the classical and modern methods of solution. This new third edition is distinguished from the first two by including elements of nonlinear and dynamic behavior of structure and by shifting emphasis to the matrix analysis of structure. As a result, three new chapters have been added in this edition; namely, the direct stiffness method, elastic stability, and structural dynamics, all presented in matrix notation. The direct stiffness method is important in that it formalizes the structural analysis readily for computer programming. The matrix analysis of elastic stability demonstrates the direct stiffness approach to solving buckling problems. And the structural dynamics is arranged so that it leads to the course of earthquake engineering, which has become an increasingly popular topic, especially in California.

In order to accommodate these new materials while at the same time retaining the same length as previous editions, the entire second edition was restructured by deleting some of the original materials and by trimming and merging others without impairing the integrity and continuity of the whole book. With changes here and there, two goals remain intact: the treatment is kept simple but comprehensive, and the text is readable and teachable. The length of little more than four hundred pages is not intimidating to students and the contents are at an elementary level throughout.

As in the second edition, the author wishes to express his sincere appreciation to many university professors and students in various parts of the world, who have read, used, and supported this book. Special thanks are due to Dr. Y. C.

Fung, professor at the University of California at San Diego, and to Mr. D. Humphrey, senior engineering editor at Prentice Hall, for their enthusiastic encouragement and editorial guidance. The author is also grateful to Dr. B. Koo, professor at the University of Toledo, Dr. K. P. Chong, professor at the University of Wyoming, and Dr. Z. A. Lu of the University of California at Berkeley for their helpful suggestions and kind interest in the development of the revision, and to Mr. S. N. Yao of Washington State Highway Department for providing solutions to the problems in Chapters 11 and 13. Finally, the author is indebted to his wife, Nelly, for her careful typing of the manuscript.

Y. Y. Hsieh
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Introduction

1-1 ENGINEERING STRUCTURES

The word *structure* has various meanings. By an *engineering structure* we mean roughly something constructed or built. The principal structures of concern to civil engineers are bridges, buildings, walls, dams, towers, shells, and cable structures. Such structures are composed of one or more solid elements so arranged that the whole structures as well as their components are capable of holding themselves without appreciable geometric change during loading and unloading.

The design of a structure involves many considerations, among which are four major objectives that must be satisfied:

1. The structure must meet the performance requirement (utility).
2. The structure must carry loads safely (safety).
3. The structure should be economical in material, construction, and overall cost (economy).
4. The structure should have a good appearance (beauty).

Consider, for example, the roof truss resting on columns shown in Fig. 1-1. The purposes of the roof truss and of the columns are, on the one hand, to hold in equilibrium their own weights, the load of roof covering, and the wind and snow (if any) and, on the other hand, to provide rooms for housing a family, for a manufacturing plant, or for other uses. During its development the design is generally optimized to achieve minimum expenditure for materials and construction. Proper attention is also given to the truss formation so that it is both

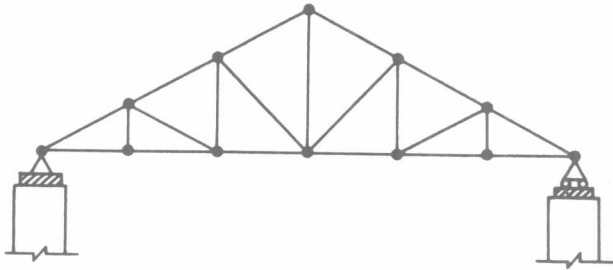


Fig. 1-1

practical and aesthetic. In this book, however, we are concerned only with the load-carrying function of structures.

1-2 THEORY OF STRUCTURES DEFINED

The complete design of a structure is outlined in the following stages:

1. *Developing a general layout.* The general layout of a structure is selected from many possible alternatives. The primary concern is the purpose for which the structure is to be built. This stage involves the choice of structural type, the selection of material, and a tentative estimation of cost based on a reasonable analysis of a preliminary structural design. It may also involve selecting the best location or adapting the structure to a site that has not been predetermined. There are many other considerations, including the legal, financial, sociological, cultural, aesthetic, and environmental aspects. It is clear that this stage of design calls for an engineer with a high order of experience, skill, general knowledge, and imagination.

2. *Investigating the loads.* Before a refined structural analysis can be carried out, it is necessary to determine the loads for which a given structure should be designed. General information about the loads imposed on a structure is usually given in the specifications and codes. However, it is part of the designer's responsibility to specify the load conditions and to take care of exceptional cases.

The weight of the structure itself together with the material permanently attached to it is called *dead load* and is regarded as fixed in magnitude and location. Since the dead load must be approximated before the structure is designed, the preliminary data are only tentative. Revision must be made if the initial estimation is not satisfactory.

All loads other than dead load may be called *live loads*. Live loads are generally classified as movable loads and moving loads. *Movable loads* are loads that may be transported from one location to another on a structure without dynamic impact; for example, people, furniture, and goods on a building floor, or snow or ice on a roof. *Moving loads* are loads that move continuously over the structure, such as railway trains or tracks on a bridge, wind on a roof or wall, or hydrostatic pressure on an abutment. Moving loads may also be applied

suddenly to the structure—for example, the centrifugal and longitudinal forces induced by the acceleration of vehicles and the dynamic forces generated by earthquakes.

In an ordinary structural design all loads are treated as static loads in order to simplify the analysis. In this way the impact due to a moving live load is expressed as a percentage of the live load, and the earthquake force is commonly considered to be a horizontal force equal to a fraction of the weight of a structure.

Other load considerations may include thermal effects and resistance to bomb blasting.

3. *Stress analysis.* Once the basic form of the structure and the external loads are defined, a structural analysis can be made to determine the internal forces in various members of the structure and the displacements at some controlling points. When live loads are involved, it is important to determine the maximum possible stresses in each member being considered. The principles governing this phase of design are usually discussed in the theory of structures.

4. *Selection of elements.* The selection of suitable sizes and shapes of members and their connections depends on the results of the stress analysis together with the design provisions of the specifications or codes. A trial-and-error approach may be used in the search for a proportioning of elements that will be both economical and adequate. A sound knowledge of strength of material and process of fabrication is also essential.

5. *Drawing and detailing.* Once the makeup of each part of the structure has been determined, the last stage of design can begin. This final stage includes the preparation of contract drawing, detailing, job specification, and final cost; this information is necessary for construction to proceed.

These five stages are interrelated and may be subdivided and modified. In many cases they must be carried out more or less simultaneously. The subject matter of the theory of structures is stress analysis with occasional reference to loadings. The emphasis of structural theory is usually on the fundamentals rather than on the details of design.

1-3 THEORIES OF STRUCTURES CLASSIFIED

Structural theories may be classified from various points of view. For convenience of study, we shall characterize them by the following aspects:

1. *Statics versus dynamics.* Ordinary structures are usually designed under static loads. Dead load and snow load are static loads that cause no dynamic effect on structures. Some live loads, such as trucks and locomotives moving on bridges, are also assumed to be concentrated static load systems. They do cause impact on structures; however, the dynamic effects are treated as a fraction of the moving loads to simplify the design.

The specialized branch that deals with the dynamic effects on structures

of accelerated moving loads, earthquake loads, wind gusts, or bomb blasts is *structural dynamics*.

2. *Plane versus space*. No structure is really planar, that is, two-dimensional. However, structural analyses for beams, trussed bridges, or rigid frame buildings are usually treated as plane problems. On the other hand, in some structures, such as towers and framing for domes, the stresses between members not lying in a plane are interrelated in such a way that the analysis cannot be simplified in terms of component planar structures. Such structures must be considered as space frameworks under a noncoplanar force system.

3. *Linear versus nonlinear structures*. Linear structure means that a linear relationship is assumed to exist between the applied loads and the resulting displacements in a structure. This assumption is based on the following conditions:

a. The material of the structure is elastic and obeys Hooke's law at all points and throughout the range of loading considered.

b. The changes in the geometry of the structure are so small that they can be neglected when the stresses are calculated.

Note that if the principle of superposition is to apply, a linear relationship must exist, or be assumed to exist, between loads and displacements. The principle of superposition states that the total effect at some point in a structure due to a number of causes (forces or displacements) acting simultaneously is equal to the sum of the effects for the causes acting separately.

A nonlinear relationship between the applied actions and the resulting displacements exists under either of two conditions:

a. The material of the structure is inelastic.

b. The material is within the elastic range, but the geometry of the structure changes significantly during the application of loads.

The study of nonlinear behavior of structures includes *plastic analysis of structures* and *buckling of structures*.

4. *Statically determinate versus statically indeterminate structures*. The term *statically determinate structure* means that structural analysis can be carried out by statics alone. If this is not so, the structure is statically indeterminate.

A statically indeterminate structure is solved by the equations of statics together with the equations furnished by the geometry of the elastic curve of the structure in linear analysis. We note that the elastic deformations of the structure are affected not only by the applied loads on the structure but also by the material properties (e.g., the modulus of elasticity E) and by the geometric properties of the member section (e.g., the cross-sectional area A or the moment of inertia I). Thus, loads, material properties, and geometric properties are all involved in solving a statically indeterminate structure, while load factor alone dominates in a statically determinate problem.

5. *Force versus displacement*. Force and displacement are two categories of events that affect a structure. The objective of a structural analysis is to determine the forces and displacements pertaining to the structure and to analyze their