BERTRAND RUSSELL

Logic and Knowledge

ESSAYS 1901–1950

EDITED BY
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GEORGE ALLEN & UNWIN

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The Problem of China

Bertrand Russell's Best
Dear Bertrand Russell

PREFACE

THE ten essays in this volume represent work extending through fifty years in the life of one of the great philosophers of our times. All of these essays are representative, and several can be regarded as among the most important of his writings. None the less, only one of these papers has previously been available in a hard-cover edition, authorized by Lord Russell, and available through the normal channels of the book trade. Indeed, most of these papers have previously been available only in libraries with unusually full periodical collections, and this in itself would serve as ample justification for reprinting them in book form.

Two collections of essays with partly overlapping contents are all we have had, up to now, to preserve the shorter writings of Russell's most productive decades of work in logic, mathematics, and the theory of knowledge. Nothing included in *Philosophical Essays* (1910) or *Mysticism and Logic* (1918) appears here, and an examination of all three books is necessary for a comprehensive view of Russell's papers of the early years of the century. The period which marked the transition to the neutral monism of *An Analysis of Mind* (1921)—in other words, Russell's philosophical activity (apart from social philosophy) during and immediately after the 1914–18 war—has previously been difficult to study. The appearance here of three papers from those years, none of them previously available in an authorized edition, should serve to fill this troublesome gap in the chronology of Russell's available works.

It is the editor's belief that what is ultimately desired is a comprehensive edition of Russell's shorter writings arranged on a chronological basis by subject, eliminating only journalistic pieces of limited interest. Such a project is beyond the means of a commercial publisher, in all probability, but it deserves the attention of those interested in preserving in an appropriate form the writings which—for the most part—linked one of our most distinguished contemporaries with his audience.

Selection has been difficult, and I do not expect everyone to agree with my choices. I have reprinted the three papers of Russell which are starred in Church's *Bibliography of Symbolic Logic*. These are technical but important. To include them I was forced to omit a group of papers which appeared originally in French in the *Revue de Métaphysique et de Morale* which are still among the best general discussions of the problems they raise. I regret that I had to make such a choice, but I do not regret the choice that I made. The reader who is not prepared to cope with mathematical logic will, nonetheless, find in certain of the other essays writing as lucid and readable as any of Russell's more popular works.

I was introduced to Russell's philosophy by Professor Arthur H. Nethercot at Northwestern University in 1944. In 1951 I took a doctorate at Harvard University with a thesis on Russell's philosophy, and since that year I have had the pleasure, from time to time, of discussing philosophical questions with Lord Russell himself. In the production of this book, I alone am responsible for its contents and the views expressed in the introductory remarks which preface each of the essays. Lord Russell has been consulted on all matters relating to the text of the papers, and these, to the best of my knowledge, are here issued in the form which he wishes to be taken as final and definitive. For this aid, and for many other kindnesses, I am greatly in his debt.

The first paper has not been reset entirely: the greater part of the symbols are reproduced here by means of photo-engraving from the original edition. For that reason slight variations may be found in the typography of the English text and the symbolic material, since it has not been possible to duplicate exactly the type faces of Peano's Italian printer. There is no instance, however, in which this is likely to introduce an element of ambiguity. In resetting the second paper we have followed the style of the *Principia Mathematica* rather than the earlier typographical conventions Russell used in the initial publication of this essay. It was Lord Russell's desire that we take advantage of the resetting of the paper to introduce these minor revisions. The dates given for the papers are those of initial *publication*. In most instances the paper was written in the same year or that immediately preceding publication.

The scarcity of some of the papers now made generally available here can be seen from the fact that only one copy of the text of the lectures on logical atomism was known to exist in the whole of Cambridge, and when this disappeared from the University library during the preparation of the book it was necessary for me to borrow the missing material from the library of the University of

Bristol. I am greatly in debt to Bristol for the consideration shown me in making the original edition of these lectures freely available to me. Because of this kindness, future students of philosophy will be spared the inconvenience of inter-library loans or the necessity of resorting to theft.

This collection was planned shortly after I first went to Cambridge in 1953 and was eventually seen through the press during my second period there in 1954–56. I shall always associate Cambridge with a wonderful sense of having achieved freedom from departmentalism, so that I might function, not as a philosopher, or a musician, or an educator, but as a thinker with the right to do whatever he feels to be important.

Mr. Walter Beard of George Allen and Unwin undertook supervision of the production of the book. He has been obliged to deal with some vexing problems, and his contribution to the volume is not to be underestimated. I am grateful to him for the assistance he has given me, and for the effective but unobtrusive manner in which he attended to difficulties.

Trinity College,
Cambridge.

ROBERT CHARLES MARSH

I AM grateful to Mr. Robert Marsh for the industry, patience and accuracy which he has displayed in the following reprints of some of my less well-known writings. In regard to a considerable part of this volume he has undertaken the laborious work of collating versions which differed owing to difficulties arising from war-time censorship. Of many of the articles copies were not easily available and he has had much trouble in finding the material. Mr. Marsh has, in my opinion, shown good judgment in selecting what to reprint and also in his elucidatory introductions. It is not for me to judge whether it is worth while to perpetuate the record of what I thought at various times, but if any historian of bygone lucubrations should wish to study my development he will find this volume both helpful and reliable.

BERTRAND RUSSELL

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The Logic of Relations

In his autobiographical essay MY MENTAL DEVELOPMENT *Russell says: 'The most important year in my intellectual life was the year* 1900, *and the most important event in that year was my visit to the International Congress of Philosophy in Paris.'* He travelled to Paris with Whitehead, his former teacher and then colleague, and they were both struck by the skill shown in the discussion of mathematical and logical problems by Peano and his pupils. Impressed, Russell went home to master Peano's works and particularly his notation. Its influence on the later Russell-Whitehead notation of the* PRINCIPIA MATHEMATICA *is fairly easily traced.*

THE LOGIC OF RELATIONS *was written in* 1900 *and published the following year. It was composed in Peano's notation, although it represents work contemporary with the writing of much of the* PRINCIPLES OF MATHEMATICS *in which Russell uses an early form of the notation developed fully in the* PRINCIPIA MATHEMATICA. *Those unfamiliar with Peano's symbols will find a concise and admirable discussion of the system in Jørgen Jørgensen's standard work,* A TREATISE OF FORMAL LOGIC, *Copenhagen and London,* 1931, *Vol. I, p.* 176 *ff. The Peano notation is actually not difficult to read if one knows that of the* PRINCIPIA MATHEMATICA, *and the paper is therefore reproduced in its original form.*

Russell's first publication was in 1895, *the year following his first period of residence at Cambridge. It is of no particular interest, although his mathematical investigations during the following four years led to publications that repay examination. However it is with this paper that we see clearly the appearance in philosophy of a creative mind of the first order, and on its publication (in the year in which he was twenty-nine) Russell's eventual position as a 'thinker of reputation' seems to have been assured. Asked what idea in the paper he now feels to be the most important, Russell replied 'my definition of cardinal number'—which here appeared in print for the first time.*

It was largely on the basis of this paper and next but one in this collection that Russell was elected a Fellow of the Royal Society in 1908.

**The Philosophy of Bertrand Russell*, Evanston and Cambridge, 1944 *et seq.*, p. 12.

1901

THE LOGIC OF RELATIONS

With some Applications to the Theory of Series

This paper first appeared in French in Peano's *Rivista di Matematica* [*Revue de Mathématiques*] Vol. VII, pp. 115–48, Turin, 1900–01. The translation is by R.C.M. and has been revised and corrected by Lord Russell.

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THE LOGIC of relations, which we find in the works of Peirce and Schröder, is difficult and complicated to so great a degree that it is possible to doubt its utility. Since they lack the distinction between ϵ and $\supset$, these authors consider a class as a simple sum of individuals. For that reason, a relation for them is like a sum of pairs of individuals. It follows from this that the fundamental properties of relations are expressed by very long formulae of summation, the significance of which is not made very evident by the notation. It is, however, the logic of relations which must serve as a foundation for mathematics, since it is always types of relations which are considered in symbolic reasoning; that is, we are not required to consider such and such particular relation, with the exception of those which are fundamental to logic (like ϵ and ɔ), but rather relations of a certain type—for example, transitive and asymmetrical relations, or one-one relations. I point out in the present paper that it is possible to simplify the logic of relations enormously by making use of Peano's notation, knowledge of which is assumed in the following text. However it will appear that the logic of Peano is hardly complete without an explicit introduction to relations. Of the basic concepts, take for example the definition of *function* (1). The signs *xu* and *ux*, which appear on the right in that definition, are not made self-explanatory by the preceding text. The juxtaposition of two letters has not hitherto possessed any meaning other than logical multiplication, which is not involved here. The fact is that the definition of *function* is not possible except through knowing a new primitive idea, that of *relation*. We may observe, for example, the following consequence. From the definition cited, and from P§20 P9·4, §22 P2·4, §23 P1·02, P2·0, we derive

$$a,b \varepsilon \mathrm{N}_0 \ .\mathsf{ɔ}.\ a+b = ab = a \times b$$

This consequence shows that the notation adopted has need of modification. I shall give a more complicated notation from which we cannot derive an equivalent conclusion. I believe, moreover, that the introduction of relations gives occasion for a simplification and generalization of many mathematical theories; and it permits us to give *nominal* definitions whenever definitions are possible.

In the following text, I have adopted some of the sumbols of Schröder, for example, $\breve{\mathrm{R}}$, 0', 1'. I have not succeeded in making myself conform to the rule of formulation, to put all symbols in a line, and in the case of relations I have had to distinguish between RP and R⌢P. Otherwise I have adopted all that is given in the logic of Peano, and at the same time the notation Elm suggested by Padoa [*Rivista di Matematica*, Vol. VI, p. 117]; however I have distinguished ϱu, where *u* is a class contained in the range of a relation R, from $\varrho \frown u$. For that reason the logical product of a class *u* and a class represented by a Greek letter is always indicated by $\varrho \frown u$, or $\pi \frown u$ etc., and not by ϱu or $u \varrho$. [See §1, Prop. 1·33·34·35·36].

§1. GENERAL THEORY OF RELATIONS

$*$1·0 *Primitive idea*: Rel = Relation

·1 $R \varepsilon \mathrm{Rel} \,.\supset:\, xRy \,.=.\, x$ has the relation R with y.

·21 $R \varepsilon \mathrm{Rel} \,.\supset.\, \varrho = x\ni\{\exists y\ni(xRy)\}$ Df

·22 $R \varepsilon \mathrm{Rel} \,.\supset.\, \breve{\varrho} = x\ni\{\exists y\ni(yRx)\}$ Df

If R is a relation, ϱ can be called the *domain* of the relation R, that is to say, the class of terms which have that relation with a single term, or with several terms. I always use (except for the relations which are met with in the formulary) capital letters for relations and the corresponding small Greek letters for the domain of the relations. In definitions ·21 ·22 the letter R is assumed to be variable, that is to say, α shall be the domain of a relation A, β of a relation B, etc. I regard $\ni$ as a primitive idea of the sort that permits me to put this sign before propositions which are not reducible without its aid to the form $x\epsilon\alpha$.

·31 $R \varepsilon \mathrm{rel} \,.\, x\varepsilon\varrho \,.\supset.\, \breve{\varrho}x = y\ni(xRy)$ Df

·32 $x\varepsilon\breve{\varrho} \,.\supset.\, \varrho x = y\ni(yRx)$ Df

·33 $R \varepsilon \mathrm{rel}.\, u \varepsilon \mathrm{Cls} \,.\, u\supset\varrho \,.\supset.\, \breve{\varrho}u = y\ni\{\exists u \cap x\ni(xRy)\}$ Df

·34 $u\check{\varrho} = y\ni\{x\varepsilon u \,.\supset_x .\, xRy\}$ Df

·35 $u\supset\breve{\varrho} \,.\supset.\, \varrho u = y\ni\{\exists u\cap x\ni(yRx)\}$ Df

·36 $u\varrho = y\ni\{x\varepsilon u \,.\supset_x .\, yRx\}$ Df

·4 $R\varepsilon \mathrm{rel} \,.\supset.\, \exists\varrho \,.=.\, \exists\breve{\varrho}$

·5 $\exists R \,.=.\, \exists\varrho$ Df

·6 $R,R'\varepsilon \mathrm{rel} \,.\supset\therefore\, R\supset R' \,.=:\, xRy \,.\supset_{x,y}.\, xR'y$ Df

·61 $R=R' \,.=:\, R\supset R' \,.\, R'\supset R$ Df

·7 $R\varepsilon \mathrm{rel} \,.\supset.\, \exists\, \mathrm{rel} \cap R'\ni(xR'y \,.=.\, yRx)$ Pp

·71 $R\varepsilon \mathrm{rel} \,.\supset.\, \mathrm{rel} \cap R'\ni(xR'y \,.=.\, yRx) \;\varepsilon\; \mathrm{Elm}$

[$R_1,R_2 \,\varepsilon\, \mathrm{rel} \cap R'\ni(xR'y \,.=.\, yRx) \,.\supset\therefore\, xR_1y \,.=.\, yRx : xR_2y \,.=.\, yRx \therefore \supset: xR_1y \,.=.\, xR_2y : \supset.\, R_1 = R_2$]

·72 $R\varepsilon \mathrm{rel} \,.\supset.\, \breve{R} = ℩\, \mathrm{rel} \cap R'\ni(xR'y \,.=.\, yRx)$ Df

·8 $\exists\, \mathrm{rel}\cap R\ni(\varrho = \iota x \,.\, \breve{\varrho} = \iota y)$ Pp

This Pp is of great importance, chiefly in arithmetic. It affirms that between any two individuals there is a relation which does not hold for any other pair of individuals. It does not need a hypothesis, since x and y are not subject to any limitation. However one can limit it to the case where x and y are different, since the case where x and y are identical is derived from this by relative multiplication.

·9 $R \varepsilon \text{rel} . \supset . \breve{\breve{R}} = R$

[$x\breve{\breve{R}}y . = . y\breve{R}x . = . xRy$]

·91 $R, S \varepsilon \text{rel} . R = \breve{S} . \supset . \breve{\varrho} = \sigma . \varrho = \breve{\sigma}$

$$R = \breve{S} . = . \breve{R} = S$$

·93 $R_1, R_2 \varepsilon \text{rel} . \supset :. x(R_1 \cup R_2)y . =: xR_1y . \cup . xR_2y$ Df
·94 $K \varepsilon \text{Cls'rel} . \supset . \cup{}'K = R \ni \{xRy . = . \exists K \cap R' \ni (xR'y)\}$ Df
·95 $\cup{}'K \varepsilon \text{rel}$ Pp
·96 $R_1, R_2 \varepsilon \text{rel} . \supset :. x(R_1 \cap R_2)y . =: xR_1y . xR_2y$ Df
·97 $K \varepsilon \text{Cls'Rel} . \supset . \cap{}'K = R \ni \{xRy . =: R' \varepsilon K . \supset_{R'} . xR'y\}$ Df
·98 $\cap{}'K \varepsilon \text{rel}$ Pp

*2·1 $R_1, R_2 \varepsilon \text{rel} . \supset : x R_1R_2 z . = . \exists y \ni (xR_1y . yR_2z)$ Df
·11 $R_1R_2 \varepsilon \text{rel}$ Pp

It is necessary to distinguish $R_1 \cap R_2$, which signifies the logical product, from R_1R_2, which signifies the relative product. We have $R_1 \cap R_1 = R_1$, but not in general, $R_1R_1 = R_1$; we have $R_1 \cap R_2 = R_2 \cap R_1$, but not in general $R_1R_2 = R_2R_1$. For example, *grandfather* is the relative product of *father* and *father* or of *mother* and *father*, but not of *father* and *mother*.

·12 $R \varepsilon \text{rel} . \supset . R^2 = RR$ Df

·13 $R, S \varepsilon \text{rel} . \supset . \breve{(RS)} = \breve{S}\breve{R}$

[$x\breve{(RS)}y . = . yRSx . = . \exists z \varepsilon (yRz . zSx) . = . \exists z \varepsilon (x\breve{S}z \; z\breve{R}y) . = . x\breve{S}\breve{R}y$]

·2 Transitive = tr = $\text{rel} \cap R \ni (R^2 \supset R)$ Df
When one has $R^2 \supset R$, one has $xRy . yRz . \supset . xRz$.

·3 $R \varepsilon \text{rel} . \supset :. R^2 = R . =: xRz . = . \exists y \ni (xRy . yRz)$

If R is a relation which yields a series (which requires that R be transitive and contained in diversity), $R^2 = R$ gives the condition for that series to be compact (*überall dicht*) that is to say it contains a term between any two of its terms. (See §5 below.)

·4 $R \varepsilon \mathrm{rel} .\supset: x - R y .=. -(xRy)$ Df

·5 $-R\, \varepsilon\, \mathrm{rel}$ Pp

·6 $(-\breve{R}) = -(\breve{R})$

I have not found the relative addition of Peirce and Schröder necessary. Here is its definition:

Let R and S be relations: their relative sum is a relation P such as

$$xPy .=\therefore x-Rz .\supset_z . zSy : z-Sy .\supset_z \; xRz \qquad \text{Df}$$

$$xPy .=. -\exists(-\breve{\varrho}x \cap -\sigma y) .=. -\{x(-R-S)y\}$$

yielding

✻ 3·1 $\epsilon\epsilon$ rel Pp

This Pp says that ϵ is a relation. In this case I have been obliged to abandon the rule of using capitals for relations.

·2 $e = x\ni\{\exists y\ni(x\varepsilon y)\}$ Df [e = individual]

·3 $\breve{e} = x\ni\{\exists y\ni(y\varepsilon x)\}$ Df [$e = \mathrm{Cls}-\iota\Lambda$]

·4 $\breve{e} \supset e$ [$y\varepsilon\breve{e} .\supset. y\varepsilon\mathrm{Cls} .\supset. y\varepsilon e$]

·5 $x\varepsilon\breve{\varepsilon}y .=. \exists z\ni(x\varepsilon z . y\varepsilon z)$

·51 $x\breve{\varepsilon}\varepsilon y .=. \exists z\ni(z\varepsilon x : z\varepsilon y) .=. x,y\varepsilon\mathrm{Cls} . \exists xy$

·6 $y\varepsilon \mathrm{Cls}'\mathrm{Cls} .\supset. \cup' y = x\ni(x\varepsilon^2 y)$

·7 $R\varepsilon \mathrm{rel} : x\varepsilon\varrho .\supset_x . y\ni(xRy) = x :\supset. R = \breve{\varepsilon}$

[$x\varepsilon\varrho .\supset_x : xRy .=. y\varepsilon x .\supset\therefore R = \breve{\varepsilon}$]

·8 $u,v\varepsilon \mathrm{Cls}-\iota\Lambda .\supset. \exists\, \mathrm{rel} \cap R\ni(xRy .=. x\varepsilon u . y\varepsilon v)$

[Prop 1·8 $.\supset. \exists \mathrm{rel} \cap P\ni(\iota u = \pi . \iota v = \breve{\pi})$

$P\varepsilon\,\mathrm{rel} . \iota u = \pi . \iota v = \breve{\pi} .\supset::$

$x\varepsilon u . y\varepsilon v .=_{x,y} . x(\varepsilon P\breve{\varepsilon})y .\therefore x-\varepsilon u .\cup. y-\varepsilon v .=_{x,y} . -\{x(\varepsilon P\breve{\varepsilon})y\} ::\supset.$ Prop]

This proposition proves that, if *u*, *v* are two non-null classes, there is a relation which holds between all terms of *u* and all terms of *v*, but which does not hold between any other pair of terms.

·81 $u\varepsilon \mathrm{Cls}-\iota\Lambda .\supset. \exists\, \mathrm{rel} \cap R\ni(\varrho = u : x\varepsilon u .=_x . xRu)$

[Prop 1·8 $.\supset. \exists\, \mathrm{rel} \cap P\ni(\pi = \iota u . \breve{\pi} = \iota u) :\supset\therefore$

$x\varepsilon u .\supset_x . x(\varepsilon P)u : x-\varepsilon u .\supset_x . -x(\varepsilon P)u .\supset\therefore$ Prop]

·82 $u\varepsilon \mathrm{Cls}-\iota\Lambda .\supset. \varepsilon u = \imath\, \mathrm{rel} \cap R\ni(\varrho = u : x\varepsilon u .=_x . xRu)$ Df