

Tracking and Data Association

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Preface

*"It is well known that through any three points in the plane,
one can pass a straight line.[†]"*

— Anonymous

This monograph presents the principles and techniques used to track targets in realistic situations. The background required consists of standard tools from linear algebra, probability theory, stochastic processes, and statistics (hypothesis testing and significance). This material is usually covered in advanced undergraduate courses or beginning graduate courses in Electrical and/or Systems Engineering.

The initial discussion deals with the principles of estimation for linear (and nonlinear) discrete-time systems, leading to the Kalman filter and the extended Kalman filter. This material is at the level of a graduate course dealing with linear estimation and covers material equivalent to about 2/3 of a semester. Other topics of estimation, specifically continuous-time systems and special numerical implementation techniques, are not included.

The book emphasizes a physical understanding of estimation in the presence of random disturbances—the concept of consistency for a dynamic estimator (filter) is stressed, for example. This is motivated by the authors' observation that knowing the mathematics of estimation does not guarantee that one's filter will perform adequately. The connection between mathematics, statistics, and engineering common sense is stressed in the discussion of the design of filters to track "unpredictable" (maneuvering) targets.

[†] Assuming the pencil is thick enough.

The data association problem is motivated by describing a realistic tracking environment, where false detections (clutter) are unavoidable, and then state-of-the-art algorithms are presented for data association and tracking of a single target in a cluttered environment. The issues of tracking performance evaluation in a cluttered environment and signal processor detection threshold optimization are also discussed.

Building on the single-target material, various algorithms for multi-target situations are developed, with realistic examples including multiple and diverse sensors. Finally, some techniques of track-to-track fusion in a multisensor/multitarget environment are discussed.

The presentation of various algorithms includes a discussion of their modeling assumptions and the resulting range of problems to which they are applicable, with emphasis on the connection between mathematical assumptions and physical reality.

This book is the outgrowth of many years of research in the field and of a series of continuously updated and expanding short courses on multitarget/multisensor tracking offered at the UCLA Extension, where the latest advances in algorithms and software are presented. Although it reflects the current state of the art, it is not definitive—the technology in this field continues to evolve.

A note on word processing:

The entire manuscript, except for the title page and figures, was prepared using a large-scale document production system, and the camera-ready copy was produced by a laser printer. The experience was somewhat less than satisfactory; details may be obtained from the second author.

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Chapter 1

Introduction

1.1 Background

The proliferation and increasing sophistication of surveillance systems, both civilian and military, has generated a great deal of interest in algorithms capable of tracking large numbers of objects using measurement data from many and possibly diverse sensors. The high bandwidth and sensitivity of modern sensors can lead to huge data loads, and the presence of countermeasures (or evasive maneuvers, decoys, etc.) can make tracking very difficult. Moreover, the tracking effort for n targets can be substantially more costly than n times the effort for a single target, because establishing the correspondence between targets and observations is not a trivial matter.

Major advances in hardware and algorithms have increased *signal processing* capabilities by one or more orders of magnitude in recent years. This has made the measurement data available for tracking even more numerous and complex, creating a demand for corresponding advances in *information processing* techniques to deal with them. Such techniques are the primary subject of this book.

It was recognized as far back as 1964 [211] that in tracking targets there can be an *uncertainty* associated with the measurements in addition to their *inaccuracy*, which is usually modeled by additive noise. This additional uncertainty is related to the origin of the measurements: a measurement that is to be used in the tracking algorithm may not have originated from the target of interest. This situation can occur in a surveillance system when a radar, sonar, or optical sensor is operating in

the presence of clutter, countermeasures, or false alarms. It also arises when a sensor can resolve (separate) the observed detections from two or more targets, but cannot associate them to tracks with any certainty. A similar situation occurs in the track formation problem when there are several targets but their number is not known and some of the measurements may be spurious.

The application of standard estimation algorithms, using the measurement nearest in some sense to the predicted measurement (a "nearest neighbor" filter), can lead to very poor results in an environment where spurious measurements occur frequently. This is because such an algorithm does not account for the fact that the measurement used in the filter might have originated from a source other than the target of interest.

1.2 Tracking and Data Association

Tracking is the processing of measurements obtained from a target in order to maintain an estimate of its current *state*, which typically consists of :

1. Kinematic components (position, velocity, acceleration, etc.).
2. Other components (radiated signal strength, spectral characteristics, "feature" information, etc.).
3. Constant or slowly-varying parameters (coupling coefficients, propagation velocity, etc.).

Measurements are noise-corrupted *observations* related to the state of a target, such as

1. Direct estimate of position.
2. Range and/or azimuth (bearing) from a sensor.
3. Time of arrival difference (obtained by cross-correlating data) between two sensors.
4. Frequency of narrow-band signal emitted by target.
5. Observed frequency difference (due to Doppler shift) between two sensors.
6. Signal strength.

The measurements of interest in multitarget applications are usually not raw data points, but rather the outputs of signal processing and detection subsystems, as shown in Figure 1-1.

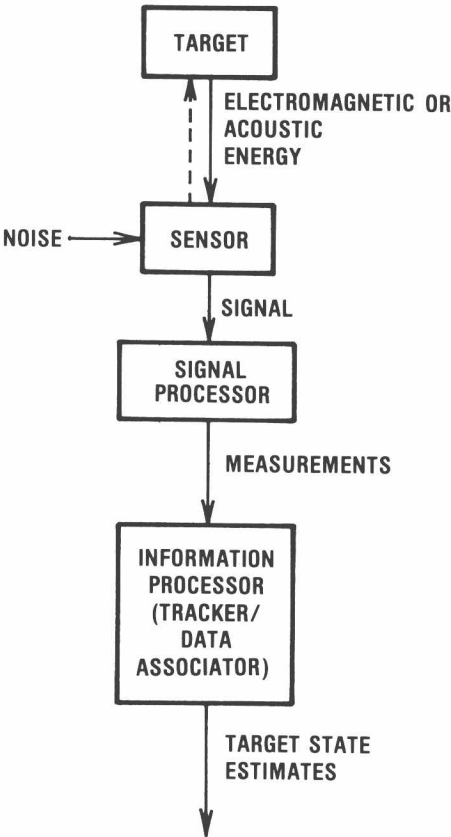


Figure 1-1: Components of a typical tracking system.

A *track* is a state trajectory estimated from a set of measurements that have been associated with the same target. The crux of the multitarget tracking problem is to carry out this *data association* (or *data correlation*) process for measurements whose origin is uncertain due to:

1. Random false alarms in the detection process.
2. Clutter due to spurious reflectors or radiators near the target of interest.
3. Interfering targets.
4. Decoys or other countermeasures.

Categories of Data Association

Data association problems may be categorized according to what is being associated with what:

1. Measurement-to-measurement (track initiation).
2. Measurement-to-track (track maintenance or updating).
3. Track-to-track (track fusion).

We shall be concerned mainly with categories 1 and 2, although the distinctions depend somewhat on one's point of view.

There is a major philosophical dichotomy between approaches that focus primarily on *targets*, selecting for further consideration those measurements that fall within "validation gates" generated by existing tracks, and those that focus on *measurements*, finding an existing track or creating a new one to go with each measurement.

The former approach is most appropriate in track maintenance, when the density of false measurements is high, or when an individual measurement (e.g., a passive sonar detection) does not contain sufficient information to localize the target geographically. The latter approach is more suitable for track initiation, or when most measurements correspond to real (trackable) targets and contain sufficient geographic information (e.g., range and azimuth from a sensor) to localize them.

Data Association Models

There are two fundamentally different models upon which data association algorithms can be constructed. The first model is a *deterministic model*, where the origin is certain or is so assumed for convenience, i.e.,

1. One takes the most likely of several "candidate" associations and ignores the fact that it is not necessarily correct, or
2. One carries out classical hypothesis testing, accepting the association hypothesis subject to a probability of error but then treating it as if it were certain.

The results of the deterministic association are then used in a standard state estimation algorithm—a linear or extended (linearized) Kalman filter.

The second model is a *probabilistic model*, utilizing a Bayesian framework in which the probabilities of individual events (e.g., measurement 3 belongs with target A) are computed and used in suitably modified state estimation algorithms.

Dynamic Models and Maneuvers

Virtually every tracking algorithm utilizes some model of a target's dynamic behavior. Although such models vary greatly in complexity, they generally share one assumption: that target motion is governed by well-known physical laws (straight line, great circle, ballistic, etc.) except when occasionally perturbed by an external force.

Unpredictable changes in target motion, commonly called maneuvers, represent a major challenge in tracking system design, particularly when they are combined with uncertainties in measurement origin. Various approaches to handling maneuvers, ranging from the inclusion of acceleration variables in the state vector to the detection of changes in the target model, will be discussed.

1.3 Applications

The applications of tracking are numerous, ranging from undersea surveillance and space-age weapon systems to bubble-chamber experiments and image processing.

Some of the earliest examples involved radar and sonar systems, where manual tracing of "blips" on video displays by human operators evolved into computer-controlled tracking algorithms. Radar and sonar applications continue to abound. Military uses include land, air, sea, and space surveillance involving a large variety of sensors, targeting and control of individual weapons and weapon systems, and overall battle management. Civilian uses include air traffic control, collision avoidance, and navigation.

New sensors based on infrared, laser, and other technologies and new applications such as machine vision require novel approaches to data association and tracking. The sophistication and complexity of the algorithms grows to keep pace with the ever-expanding frontiers of computer technology.

1.4 Scope and Outline of Book

The purpose of this book is to present the mathematical tools underlying various algorithms for measurement association and multi-target tracking. These tools rely on

- Techniques of system theory that utilize the state-space representation of linear and nonlinear dynamic systems, and
- Statistical techniques of estimation and hypothesis testing.

These techniques form the basis for the design of automated decision systems for association and tracking (also called correlator-trackers) in a multitarget/multisensor environment. The statistical models will account for false alarms, target detection probability, and finite resolution capability of sensors.

The material to be presented will deal mainly with information processing (the last block of Figure 1-1), consisting of data association and state estimation that uses the output from the signal processing, i.e.,

a macroscopic approach. A joint consideration of the signal processor and information processor (estimator) will also be undertaken.

Chapter 2 reviews the basic results of linear estimation, from simple parameter estimation through the Kalman filter; these are extended to nonlinear systems in Chapter 3. Chapter 4 introduces the problem of maneuvering targets and offers several alternative schemes for dealing with them.

The nature of the data association problem is outlined in Chapter 5, and the next four chapters attack various aspects of it: Chapter 6 covers the easiest case of single target in clutter, Chapter 7 treats the additional complication of maneuvers, Chapter 8 presents a means of optimizing estimation performance in various clutter densities by adjusting signal processing parameters, and Chapter 9 delves into the complexities of multiple targets in clutter.

The closely related problem of track fusion is discussed briefly in Chapter 10.

The basic linear algebra that forms the backbone of techniques useful in tracking and data association is reviewed in Appendix A. Similarly, a review of the tools from probability theory, random processes, and statistical hypothesis testing appears in Appendix B. Particular attention is paid to the concept of statistical significance, which is common in statistics and econometric research but has not received much attention in the engineering literature. A table of chi-square and Gaussian probability distributions is provided for reference in Appendix C.

Finally, Appendix D presents various proofs and derivations to which the main text refers.