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(editors)

The Application of Mathematics in Industry

THE APPLICATION OF MATHEMATICS IN INDUSTRY

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FOREWORD

This publication reports the proceedings of a one-day seminar on *The Application of Mathematics in Industry* held at the Australian National University on Wednesday, December 3, 1980. It was organized jointly by the Division of Mathematics and Statistics, CSIRO, and the Departments of Pure and Applied Mathematics, The Faculty of Science, Australian National University. A paper based on the talk "Some uses of statistically designed experiments in industrial problems" given by N.B. Carter at the Seminar was not received by the editors. Though R.M. Lewis of John Lysaght (Australia) Limited did not present a talk, the editors invited him to submit a paper. They only learnt about his work after the program for the seminar had been finalized and publicized. His paper appears as the last paper in these proceedings and is entitled "A simple model for coil interior temperature prediction during batch annealing".

The seminar was opened by Dr J.R. Philip, FAA, FRS, Director of the Physical Sciences Institute, CSIRO. He kindly agreed to supply an edited version of his comments for inclusion in the proceedings. They follow the Foreword as Opening Remarks.

An attempt was made to structure the program to the extent that (a) a major aim in organizing the seminar was to bring together the academic mathematician with little first hand experience with the application of mathematics to industrial problems, and non-academics with an ongoing industrial responsibility; (b) the seminar itself aimed to illustrate the different reasons why mathematics should be, is or must be used in the solution of industrial problems; and (c) all speakers were requested to organize their talks so that they first discussed an application, then showed why mathematics was necessary for the solution process, and only then discussed the mathematics itself.

The organizers would like to take this opportunity to record their appreciation and thanks for the contributions of the speakers, Dr John Philip for his Opening Remarks, the able assistance of the session chairmen (Dr Chris Heyde, Division of Mathematics and Statistics, CSIRO; Professor Archie Brown, Department of Applied Mathematics, The Faculty of Science, Australian National University; Professor Neil Trudinger, Department of Pure Mathematics, The Faculty of Science, Australian National University; and Dr John Philip, Director, Institute of Physical Sciences, CSIRO) and the active involvement of the participants. Thanks must also go to the Division of Mathematics and Statistics, CSIRO, and the Departments of Pure and Applied Mathematics, The Faculty of Science, Australian National University, for their support and assistance without which the Seminar could not have been the success that it was.

The excellent work of Barbara Harley and Dorothy Nash in assisting with organizational aspects associated with both the Seminar and Proceedings, of Clementine Krayshek who traced many of the figures and of Anna Zalucki who typed the proceedings is gratefully acknowledged with sincere thanks.

R.S. Anderssen and F.R. de Hoog
Canberra, ACT 2600
June, 1981

OPENING REMARKS

Let me begin by applauding the CSIRO Division of Mathematics and Statistics, and the mathematicians of the A.N.U., for their initiative in bringing into being this One-Day Seminar on Mathematics in Industry. So far as I can discover, the insight and the work which have led to this occasion have been provided largely by Bob Anderssen and Frank de Hoog. We are all most grateful to these two, and to everyone else involved in the preparations for the Seminar.

I have been commanded to provide some Opening Remarks. I am at something of a loss, since there is already in existence the Introduction which follows these remarks. That Introduction seems to say very well most things that might conceivably be said in Opening Remarks. So I shall do no more here than offer you one or two brief observations.

To begin with, I believe that one cannot repeat often enough just what is involved in applying mathematics to real world problems, whether in industry or in the wider context of natural science. Firstly, we must make a proper abstraction based on the real world problem. We must have sufficient insight into what is happening, and sufficient empirical knowledge of it, to be able to sort out the significant elements in the problem. We must not neglect important factors; and yet, on the other hand, we must not retain extraneous elements which make things complicated, obscure and difficult, but shed no light on the major issues.

Secondly, we must be able to set up the abstract problem so that it is amenable to the processes of mathematics.

And thirdly, we must be able to translate our solutions of the abstract problem back to the real world accurately, and with a clear and correct recognition of the limitations to our solution.

The Irish mathematician, J.L. Synge, described this three-stage process vividly and succinctly as follows :

> "A dive from the world of reality into the world of mathematics; a swim in the world of mathematics; a climb from the world of mathematics into the world of reality, carrying a prediction in our teeth."

May I say that I find the enthusiasm for the application of mathematics in industry manifest in this seminar most commendable. I should like, however, to say that, in my opinion, we can do with considerably more enthusiasm directed towards the use of mathematics in the related but separate field of natural science in general.

It is my personal view that a great number of natural scientists abandon their mathematics far too early in their training, and live out their careers deprived of a central tool of their trade. I believe that one reason for this is the way in which mathematics is often taught.

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As the following Introduction hints, many good natural scientists find their motivation, not in the immediate pursuit of maximum generality, but in the process of working from the particular to the general. As many of you know, that viewpoint arouses only the disdain of many of our professional mathematicians. These, unfortunately, tend to be the people who teach mathematics to our potential natural scientists.

Let me close with a quotation from Professor Kaplan which, I am sure, has meaning for all of us; but I expect it holds special significance for the modellers amongst us.

> "Models are undeniably beautiful, and a man may justly be proud to be seen in their company. But they may have their hidden vices. The question is, after all, not only whether they are good to look at, but whether we can live happily with them"!

J.R. Philip

INTRODUCTION

"To explain the methods that have been devised to bring mathematics into some aspects of the everyday world is a more difficult task than to explain mathematics itself".

Norman Clark,
Secretary and Registrar, IMA,
in Foreword of
J. Lighthill, Newer Uses of Mathematics.

In examining "the application of mathematics in industry", it is not the range of mathematical techniques used which must be discussed, but the nature of how mathematics interacts with applications to the benefit of both. As the above quotation implies, that is easier said than done. The difficulty is the unstructured diversity of industrial applications that are amenable to mathematical analysis.

In order to give meaning to "The Application of Mathematics in Industry", it is necessary to choose from the huge range of possibilities a subset which reflects the utility of mathematics in industry.

In this seminar, we have chosen applications which illustrate the different reasons why mathematics should be, is or must be used. An intuitive understanding of the application of mathematics in industry is based on an appreciation of such reasons, and not solely on the mathematical tools applied. These include :

(i) For the optimal utilization of large and complex plant which cannot be modelled experimentally (such as a blast furnace), mathematical modelling is the only alternative.

(ii) In the examination of properties of new materials (such as composites), the use of mathematical models often avoids the necessity to perform long and/or expensive experiments.

(iii) The use of statistically designed experiments can often ensure that the control and monitoring of industrial processes is done with maximum efficiency.

(iv) Through the use of computational techniques, an increasing level of mathematical sophistication has been brought to bear on industrial processes.

(v) Industrial problems which can be simply explained do not necessarily have equally simple answers and can involve the use of highly sophisticated mathematics.

(vi) The efficiency and effectiveness of non-destructive testing and analysis.

(vii) The fact that, in many applications, the information required only comes from indirect measurement.

(viii) Even though the mathematics involved may be quite elementary, its need may be crucial in ensuring that an industrial process operates correctly.

Globally, applied mathematics and the application of mathematics in industry are similar in that they involve the three basic but interrelated steps of : (i) formulation; (ii) solution; and (iii) interpretations. However, as with the different branches of science, the inference patterns used in the application of mathematics to industrial problems often differ from those used in other branches of mathematics. These differences depend heavily on factors like :

(a) Compared with applied mathematics, which aims, through the use of mathematics, to seek knowledge and understanding of scientific facts and real world phenomena, the application of mathematics in industry aims to answer specific questions of immediate and direct concern.

(b) Compared with the academic situation where one has "a method without a problem", the situation in industry is very much one where one has "a problem without a method". In part, this is why computers have had such a big impact on applications as they allow, in no other way possible, for the problem to remain of central importance.

(c) As long as the given problem is solved economically and effectively, the lack of mathematical sophistication or generality is of no major concern. The aim is to choose a framework in which one can rigorously answer the relevant questions.

(d) One of the major difficulties in solving industrial problems is ensuring right from the start that the question being answered is the question which should have been asked. The form of the mathematics used depends heavily on the question which must be answered.

Overriding all these points is the fact, which does not appear to be fully appreciated, that much useful mathematics is simple in nature. As a consequence, the average mathematician can contribute to the application of mathematics, as long as he has the motivation and interest. This point has been made indirectly by Jeffreys and Jeffreys in the Preface to their book on "Methods of Mathematical Physics" :

> "We think that many students ... have difficulty in following abstract arguments, not on account of incapacity, but because they need to "see the point" before their interest can be aroused."

SUMMARY OF THE TALKS

The Director, Physical Sciences Institute, CSIRO, Dr J.R. Philip, FAA, FRS, launched the Seminar with some opening remarks about the role of mathematics in industry. In the first talk, Dr John Lowke of the Division of Applied Physics, CSIRO, highlighted the fact that, in industrial problems where many physical processes occur simultaneously, it is difficult to identify which process, if any, dominates. Dr Lowke used two industrial examples to show how detailed mathematical modelling and experimentation can be used to identify the processes which dominate. In the first, the problem of determining the operating voltages of lasers was discussed; while, in the second, the prediction of the diameters of arcs which occur in circuit breakers was examined.

The next speaker was Dr Rys Jones of the Aeronautic Research Laboratories, Melbourne. His talk, which was about the mathematical analysis of *in situ* repairs of cracked aircraft components, illustrated clearly the need to use mathematical tools to avoid the need to perform long and/or expensive experiments. The next three talks by Mr David Jenkins, Dr Peter Swannell and Dr Val Pinczewski were all concerned in various ways with the use of computational tools in order to increase the level of mathematical sophistication that is brought to bear on industrial processes. Mr Jenkins spoke about the mathematical modelling of gas flow in blast furnaces; Dr Swannell discussed the dynamic behaviour of the Gateway Bridge just approved for construction across the Brisbane River; and Dr Pinczewski examined the simulation of lateral liquid flow in the hearth of a blast furnace. In different ways, they also illustrated points already made by the first two speakers as well as the fact that, for the optimal utilisation of large and complex plant, mathematical modelling is the only alternative.

The talk by Dr Ray Volker about a mathematical model for the storm water drainage system in Townsville, Queensland, aimed, among other things, at illustrating that practical problems which can be simply explained do not necessarily have equally simple answers, since the complexity of the underlying practical problem is not easily modelled mathematically.

Dr Greg Taylor then discussed why the survival of Non-Life Insurance Companies depends heavily on the sophistication and accuracy of their actuarial modelling. Dr Bob Johnston examined the mathematical methods which were now being used widely throughout industry to maximise the efficiency of the cutting of stock. Ms Nan Carter and Dr George Brown then described how statistically designed experiments can often ensure that the control and monitoring of industrial processing is done with maximum efficiency. Ms Carter explained the use of such work in the monitoring of ore separation machines, while Dr Brown discussed the use of acceptance sampling for the detection of bad batches with an example from the peanut industry.

In his talk about the grinding of contact lenses, Dr Bill Davis aimed at illustrating the important point that, though the mathematics involved may sometimes be elementary, the need for it to be correct is crucial for the success of the underlying industrial process. A further illustration of the point that, though industrial problems can often be simply explained, the underlying mathematics may be highly sophisticated was given by Dr Richard Cowan in his talk about sheet metal bending. The Seminar finished with Dr Bob Anderssen's talk about the fabrication of optical fibres. It was primarily concerned with the fact that, in many applications, the required information only comes from indirect measurement.

BACKGROUND READING

For people also interested in the application of mathematics in a wider context than covered by these proceedings, the following non-exhaustive list of references can be used as starting points for many of the possible directions that can be pursued.

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R.S. Anderssen and F.R. de Hoog
Canberra, ACT 2600
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PREDICTION OF OPERATION VOLTAGES OF CO_2 LASERS AND LIMITING CURRENTS OF CIRCUIT BREAKERS

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ABSTRACT

A major difficulty in doing calculations for industrial problems is that usually many physical processes occur simultaneously. However a detailed analysis frequently reveals that one process is dominant and that, as a consequence, approximate mathematical expressions can be derived that are adequate for the engineer. Two examples are given. Firstly, for CO_2-N_2-He gas lasers a two-term expansion of the electron distribution function in spherical harmonics yields electron attachment and ionization coefficients from the Boltzmann transport equation. Equating ionization and attachment at equilibrium enables predictions of the operating voltage of the laser to be made. Secondly, detailed solutions of the radiative transfer equation make it possible to predict the diameters of arcs that occur in circuit breakers in a simple way.

1. INTRODUCTION

There are a large number of differences between the university environment where a mathematician or physicist obtains his training, and the industrial environment where he must work and make a living. These differences I found to be rather daunting, even when moving to the relatively sophisticated environment of an American industrial research laboratory. For a mathematician working in Australian industry, the differences would be harsher.

It is of value to delineate some of the differences between university and industrial environments :

(i) In a university, courses in mathematics, physics and engineering tend to be dominated by theory. Ones colleagues at university understand, or at least can potentially understand, what one is doing. Furthermore, the output of the university is largely in the form of papers, where theory usually plays a dominant role.

On the other hand, industry is dominated by hardware, concrete and devices. The function of industry is not to produce theory or papers, but to sell devices or products and make a profit. In industry the mathematician, and physicist are in a minority. The mathematician's manager and most of his colleagues will have little appreciation for the sophisticated mathematics with which he is working. The mathematician or physicist is often regarded as being a parasite on the engineering establishment and the onus is on him to justify his existence.

(ii) In a university, the undergraduate or PhD student is given a reasonably defined problem. Furthermore the undergraduate knows that there are answers to these problems. A PhD student, provided his PhD supervisor is competent, knows that by using reasonably established methods, a given field can be extended. Even if a given approach is fruitless, a research paper can result and he is credited with a degree of success for professionally competent work.

In industry, one is presented not with a simple defined problem, but dozens of problems, all of which appear to be completely intractable and to have no rational solution. The engineers designing gas lasers want to get answers to the following types of questions : - what voltage power supply is needed to drive a laser of given dimensions, how are these requirements varied for different mixtures of gases, would the laser efficiency be increased by varying the mixture, what is the voltage required for initial breakdown, what is the effect of water vapour impurity? etc. The circuit breaker engineer would like to know what is the arc diameter as a function of current, would the current interrupting capability be increased by a longer or narrower nozzle, what is the effect of metal impurities from the vaporized electrodes? The problems go on and on, each problem appearing at least at first sight, to be quite unrelated to the skills of being able to invert matrices, manipulate Laplace transforms or understand group theory.

(iii) University type problems which, for example, involve the solution of differential equations, are usually linear and involve reasonably simple geometries. Fairly accurate solutions are possible.

Industrial problems are usually non-linear and geometries are usually complex. Usually material functions are required which are either unknown, or, at best, are known to 20%. Very accurate solutions are usually of no consequence - either the circuit breaker interrupts a given current or it doesn't.

In this paper, two topics are described in which mathematics was applied to solve a problem of direct industrial interest. The first is in the prediction of the power supply requirements to operate CO_2 gas lasers used for cutting and welding, and possibly for laser fusion and military defense. The second arises in the design of high-power circuit breakers used to protect generators in power stations. The experiences relate to a period when the author spent 11 years working at Westinghouse in the U.S. High technology research in secondary industry in Australia is notoriously weak, but as Australia's industry stengthens, it is expected that these U.S. experiences will become more relevant to Australian conditions.

In Section 2 we discuss the question of selecting a problem, in Section 3 the formulation of the problem in mathematical form, in Section 4 the mathematical solution, and Section 5 discusses the interpretation of the solution.

2. SELECTING A PROBLEM

The most pressing problems of any industry are usually insoluble mathematically. The laser industry may like to know the gases or combination of gases to make the most powerful laser possible. Predictions of laser performance while feasible, are dependent on a host of cross-sections for electronic excitation and atomic and molecular collisions, which just aren't known. The circuit breaker engineer would like to know whether a circuit breaker of given nozzle design in a particular circuit will interrupt a particular current. While scientific capability is tantalizingly close to being able to make such predictions, a host of uncertainties has so far denied such success.

But there are many problems in any given industry producing a particular product. The mathematician or physicist doesn't need to solve all of them, or even the most important problem, to justify his salary. In fact, because the industrial impact of success in any area is so great, he can be employed for many years just on the prospect that one day he might solve such a problem!

Usually the mathematician or physicist has some freedom to select a problem and, of course, he picks any problem that it is possible for him to solve. To be successful, he has to fulfill a double job. Firstly, he has to keep up with his subject from an academic point of view. He needs to be aware of recent developments in his subject, so that if he is a mathematician solving coupled differential equations he needs to be aware of new methods of inverting matrices, finite element methods, new computer routines that are available etc. If his problems involve electrical discharges, he needs to be aware of new insights obtained by physicists working in this field. He will need to attend specialist conferences and study current literature of the area.

Secondly, he will need to be aware of the practical problems of the engineers working in the industry. No academic can contribute to the practical problems of an industry unless he knows what these problems are. To obtain detailed knowledge of these problems is probably the most difficult task of an academic in a university or other research institution.

Furthermore, the obvious means of obtaining detailed knowledge of problems are frequently not productive. In a visit by an academic to an industry, the proud guide often does not reveal problem areas initially. Subsequent visits are likely to be regarded by the industry as a waste of their time.

Probably the best method of identifying problem areas is to go to specialist conferences. There are specialist laser meetings held annually; circuit breaker engineers meet at the "Current Zero Club" an international forum that is held biennially. For every professional group, be they welding engineers, illumination engineers or engineers interested in fuses, there is a regular society or forum where problems are discussed. All the above areas relate to ionized gases and gas discharges and so involve plasma physics. However even attending such conferences can be of limited value in that the papers presented merely outline solutions of problems that have been solved. It is only in the question period or discussions over coffee that one becomes acquainted with current problem areas.

3. MATHEMATICAL FORMULATION OF PROBLEM

1. INTRODUCTION

The problems discussed in this paper involve quantitative predictions i.e. predictions of the operating voltage of a gas laser and the diameter of an arc for a given current in a circuit breaker. Thus mathematics is an essential component of the solution process. We assume that the phenomena that we desire to predict obey the laws of physics, and that these laws are expressible in terms of mathematical relations - usually differential equations.

The laws of physics are reasonably well established. The behaviour of any industrial device may be the result of many complex phenomena, but it is unlikely that any new phenomena or physical processes will thwart our powers of prediction. The problem is simply that there are too many phenomena, all occurring together, in 3 dimensions and varying in time. Even with computers we need to make approximations and only treat the dominant physical processes.

We now discuss, in turn, the hardware and the dominant equations for these two problems.

2. HARDWARE

(1) CO_2 lasers -

The physical configuration of the electrodes of a CO_2 gas laser is shown in Figure 1. The laser is operated in the pulse mode, with a mixture of carbon dioxide, nitrogen and helium passed between the electrodes in a direction parallel to the electrode surface, in the plane of the diagram. The mirrors are mounted so that radiation intensity and lasing action build up in a direction perpendicular to the plane of the figure.

The gas between the electrodes is initially preionized by placing a sharp pulse of high voltage on a row of spikes along the edge of the electrodes as shown in Figure 1. High voltage on the spikes produces corona and ultra violet radiation at the tip of the spikes. This ultra violet radiation photoionizes the gas.

Immediately following this voltage pulse on the spikes, a voltage is applied to the planar electrodes. Both voltage pulses are applied by transferring the voltage on a condensor bank by means of a spark gap.

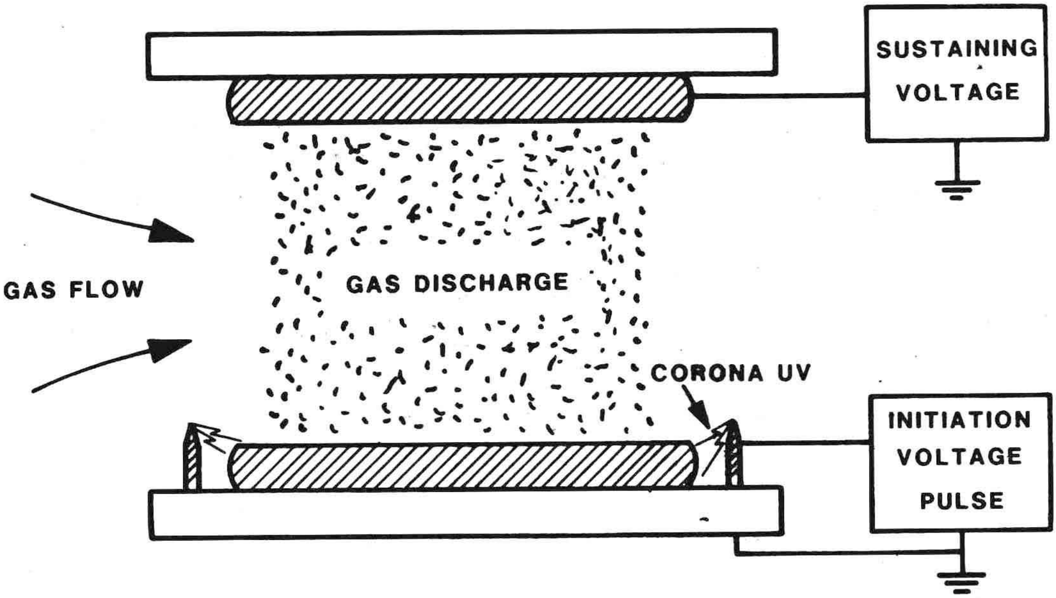

FIGURE 1 : Electrode configuration of a gas discharge laser.

The voltage across the planar electrodes, after an initial transient periods, settles down to a steady value. This voltage is independent of current and is a characteristic of the laser gas mixture rather than the power supply. Of course the voltage of the power supply must be sufficient to provide this voltage or the electrical discharge will not lase.

The problem to be solved is to determine, for a given gas mixture and electrode spacing, the magnitude of this steady voltage.

(2) Gas Blast Circuit Breakers -

To protect the generators of power stations, circuit breakers are required which can be the size of a large room. The heart of the breaker is shown diagramatically in Figure 2. Two contacts, shown as cylindrical metal nozzles, are initially in contact, and then are separated to the position shown in Figure 2. At the same time, as the contacts move apart, valves are opened to produce a high speed gas flow from a high pressure region outside of the nozzles, down the centre of the two nozzles.

On separation of the contacts an arc forms, initially, for example, at the two points A and B of Figure 2. The gas flow, however, forces the arc to the centre of the nozzles, as shown in the figure. The pressure driving the flow can be 20 atmospheres in the region outside of the nozzles and 1 atmosphere within the nozzles, so that the flow reaches supersonic velocities. The currents that are required to be interrupted can be up to 50 kA. The transmission voltages are 330 kV for New South Wales, but are being upgraded to 500 kV.

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