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REPRESENTATIONS OF ALGEBRAS

Workshop Notes of the Third International Conference August 4-8, 1980

Edited by M. Auslander & E. Lluis

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Edited by A. Dold and B. Eckmann

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Representations of Algebras

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Edited by M. Auslander and E. Lluis

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FOREWORD

The Proceedings of the Third International Conference on Representations of Algebras held August 8-16, 1980 in Puebla, Mexico, appeared as Lecture Notes in Mathematics, Volume 903.

Here we present the notes of four series of lectures given at the Workshop held the week before the Third International Conference (August 4-8).

It is the editors' hope that they will provide a most interesting survey of some basic topics of the Theory, including the historical one.

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The history of algebras and their representations

William H. Gustafson

In this paper, I have attempted to outline the origins of the theory of finite dimensional algebras over fields, with particular attention to representation theory. Hence, the treatment is not exhaustive; for instance, we say nothing about the extensive work done on the structure of division algebras and little about group representations per se. Even with such provisos, much had to be omitted, due to the extensiveness of the literature (see for example the bibliography [276], which lists more than seven hundred related items for the years 1969-1979 alone). I have therefore chosen to consider what I myself feel are the main lines of development, realizing that many disagree with my choices. In order to maintain an upper bound on the length of this work, I have omitted familiar definitions, and provided no mathematical exposition at all. Hence, the beginner who stumbles upon this paper and wishes to learn more will have to go to the cited references. Good starting points for learning the classical theory are the text books of Albert [1], Artin, Nesbitt and Thrall [5], Curtis and Reiner [62], Deuring [67], Dickson [71,72,74] and Jacobson [143]. Unfortunately, no text is available at the time of this writing that covers the powerful methods developed in the last few years, but some guidance can be found in the expository papers of Gabriel [106,107,108]. Something of the approach of M. Auslander can be learned from the notes [6,14].

As I have said, the scope of this paper is strictly limited and the format is condensed -- it is more a chronology or an annotated bibliography than a scholarly history. I highly recommend that one also examine other sources, such as the excellent papers of Hawkins [132, 133,134], which are also summarized in Mackey [183], as well as Artin [4], Happel [129], Ringel [233] and Wussing [270]. I have used these sources freely in preparing this report, especially the works of Hawkins and Ringel.

It is not unreasonable to think that the theory of algebras begins in 1835, when William Rowan Hamilton represented complex numbers as ordered pairs of real numbers (his paper [125] on this subject appeared in 1837). This had been done before, in a sense, in the geometric representation of complex numbers. However, Hamilton was the first to see the algebraic significance of such a representation: it reflects the fact that the plus-sign in a+bi has a fundamentally different meaning than that which it has in the arithmetic of real numbers.

Hamilton examined the field operations in terms of ordered pairs, and undertook a search for corresponding operations on triples, in the hope that spatial physical phenomena might thereby be elucidated. His search was unsuccessful, as he required "uniqueness of division", i.e. he was looking for division algebras. So, he eventually turned his attention to quadruples, and finally discovered the quaternions on October 16, 1843. Only a month later, he presented a lengthy paper on the subject before the Royal Irish Academy; it appeared in print [127] in 1848, having been preceeded by a brief announcement [126] in the Philosophical Magazine. Hamilton devoted the rest of his life to the theory and applications of quaternions, and they were for quite some time a standard tool in geometry and physics.

At the same time, Grassmann had also been thinking about products of vectors, and his book [116] was in press at the time of Hamilton's discovery. However, Grassmann's work was so abstract and so veiled in mystical and philosophical speculation that it was ignored in his lifetime. Indeed, even the later, simplified and clarified version [118] was a failure initially - its importance was finally recognized through the efforts of Clifford [55]. In all, we must recognize that Grassmann was an original innovator in the theory of algebras, as he himself pointed out [117]. For more about Grassmann, see Crowe [60], Dieudonné [80] and Fearnley-Sander [95].

In contrast to Grassman, Hamilton was well known when he published his work on quaternions (he had done fundamental work on the calculus of variations, and he was the Royal Astronomer of Ireland). So, his ideas were widely studied, and aroused much interest. In particular, DeMorgan [66], inspired directly by Hamilton, gave the rudiments of the definition of an algebra, and Cayley, in the paper [50] where abstract finite groups were defined and shown to be representable as permutation groups, noted that the formal linear combinations of group elements form a system analogous to quaternions (i.e. he described the group algebra). He specifically demonstrated the multiplication law for elements of $\mathbb{C}S_3$.

The next basic advance in the theory of algebras was the distribution by Benjamin Pierce in 1870 of one hundred copies of a paper entitled Linear Associative Algebra. In this paper, Pierce defined nilpotent and idempotent elements of finite dimensional $\mathbb{C}$-algebras, introduced the Pierce decomposition and its properties, and catalogued many of the algebras of dimensions up to six. (Pierce specifically stated that enumeration of algebras by dimension lacked aesthetic appeal, but that it seemed a good way to start searching for generalities). In 1875, Pierce [218] pointed out that his algebras could be represented using the "logic of relatives" that had been developed by his son, the philosopher and logician C. S. Pierce. This system was essentially a representation in terms of matrix units, although it was not presented as such. Indeed, neither Pierce was aware at the time of the matrix caclulus that Cayley [51,52] had introduced some years before (and which was rediscovered by Laguerre [175], Frobenius [97] and Sylvester [252]). In 1881, Pierce's monograph was reprinted in the American Journal of Mathematics [219], with addenda by both father and son (B. Pierce had died the year before). In one of these, C. S. Pierce gave the proof that any algebra can be represented in "relative" (i.e. matrix) form. For more about this paper, see Nový [213] and Pycior [223].

In Europe, two separate threads led to a deeper structure theory of algebras. On one hand, Lie's theory of continuous groups was adapted to the problem of studying algebras (or hypercomplex number systems, as they were then called), through use of the regular representation. This approach was suggested by Poincaré [222]. On the other hand, an apparently unrelated series of papers on commutative ℂ-algebtras appeared, starting with Weierstrass [269] in 1884, where these algebras were defined and polynomial equations over them were discussed. Soon, sequels followed by Dedekind [65], Hölder [139], Petersen [217], Hilbert [138] and Frobenius [98]. In these papers, the basic structure theorem for semisimple commutative algebras was developed (Hilbert showed that it followed from the Nullstellensatz), and the parastrophic determinants were introduced. The latter led to the group determinant, which Frobenius later used as his main tool for developing the theory of group characters. The culmination of the Lie-theoretic line of thought was the discovery by Molien [190] and Cartan [49] of the structure theory of (noncommutative) semisimple ℂ-algebras. Later, Molien [191,192,193] applied his approach to the case of group algebras, and obtained many of the basic facts about complex representations of groups. It should be noted that Molien's work was independent of that of Frobenius, and that he was the first to view things from the vantage point of representations, which Frobenius discussed only after deriving the basic facts about characters from the factorization of the group determinant. Of course, the work of Molien and Cartan on the structure of algebras was superceded by that of Wedderburn [267], who showed that the theory could be developed by "rational" (i.e. purely algebraic) methods, and hence was applicable (in modified form) over arbitrary base fields (algebras had been defined in this generality by Dickson [69]). Later, Artin [3] extended the structure theory to semisimple rings with both chain conditions, and Hopkins [140] showed that DCC implies ACC in the presence of an identity element.

We now stand at the end of the first decade of this century, with the structure and representations of semisimple algebras understood, modulo the determination of all division algebras. There followed a lengthy period in which little happened directly from the viewpoint of structure theory of algebras. Wedderburn had discussed the radical and some basic properties of nonsemisimple algebras, and others pursued some of these ideas (c.f. Dickson [71]). However, much of the work of this time was concerned with classification and enumeration of algebras of fixed dimension with carefully specified properties (c.f. Hawkes [130,131] and Hazlett [135]; Happel [129] has investigated this period in detail). On the other hand, very useful information was being developed from the viewpoint of groups with operators. Of course, this theory includes the theory of modules as a special case, but that connection was not emphasized, probably because the concept of module was not widely known in sufficient generality at the time. Nonetheless, this era gave us many basic tools: the Jordan-Hölder theorem, the Krull-Schmidt-Remak theorem and the notion of Loewy series (see Krull [165], Loewy [178], Remak [226] and Schmidt [244).

The next major advance came in 1929, when Noether [212] showed the usefulness of viewing representations as modules, and pointed out the basic isomorphism theorems. Her ideas were rapidly disseminated in van der Waerden's influential text (see [264] for some recollections). This came at a fortuitous time, as interest in algebras was being revived because of the development of the theory of orders (see Gustafson [124] for a brief outline) and the related work on division algebras over algebraic number fields. At this point, one could also realize that some of the results of the previous century were really representation-theoretic. For instance, the theory of canonical forms for matrices could be interpreted as the representation theory of factor rings of polynomial rings in one variable. (These canonical forms originated from an idea of Weierstrass [268], the idea of elementary divisors. Jordan [150] gave the Jordan form in a special case; the general version is by Dickson [68]. The rational canonical form was introduced by Frobenius [97], although a precursor can be seen as early as 1856 in the work of Spottiswoode [251]; see also Muir [196, pp. 210-211]). Canonical forms for pairs of matrices under simultaneous equivalence seemed to escape notice as classifications of representations until recently. (Such forms, which classify representations of the quiver $\cdot \rightrightarrows \cdot$ and hence rings such as $K[X,Y]/(X,Y)^2$, were developed by Weierstrass [268], Kronecker [162] and Dickson [73,75], the latter by rational methods. The problem has been of continuing interest over the years; see Bautista Ramos [21,22], Brown [43], Dieudonné [79], Kantor [153,154], Levy [177], Sherwood [245] and Turnbull and Aitken [263]. Some of these works deal with special cases of the wild problem of three or more matrices under simultaneous equivalence. The last-named reference is a fine source for canonical forms and their history).

In 1935, there appeared two papers that introduced motifs that have carried on ever since. Köthe [161] discussed modules over artinian rings that are products of primary rings and which have the property that each principal indecomposable module (on either side) has a unique composition series; he called these rings einreihig (uniserial). Köthe showed that any module over such a ring is a direct sum of cyclic modules. Conversely, he showed that a commutative artinian ring for which all modules are direct sums of cyclic modules must be uniserial (and hence, a principal ideal ring).

Three threads have followed from this work. First, the theory of uniserial rings has been developed extensively. A few years after Köthe, Nakayama removed the hypothesis that the ring be a direct product of primary rings, and thereby introduced the class of generalized uniserial rings (now called serial rings or Nakayama rings). Nakayama [203] showed that even in this case, modules are direct sums of cyclic ones, and he used these rings in his study of Frobenius algebras [204] (this class of algebras had been introduced by Frobenius [99], as a generalization of group algebras). Ever since, serial rings have been an important source of examples, as their structure and module theory can be determined very precisely; see Amdal and Ringdal [2], Eisenbud and Griffith [93,94], Fuller [100], Ivanov [142], Janusz [147], Kupisch [166,170,171], Murase [200,201] and Riedtmann [227].

Another offshoot of Köthe's paper involves finding rings whose modules decompose in predictable ways. One aspect of this problem is the search for commutative rings for which all finitely generated modules are direct sums of cyclic ones; we will not comment further on this. The corresponding problem for noncommutative algebras was solved by Kawada [156], who gave a list of nineteen necessary and sufficient conditions, with a correspondingly lengthy proof. This series of papers, which I have not yet seen, is said to contain many interesting results that have been subsequently rediscovered. Another aspect of this problem, of interest to representation theorists, is that of trying to determine the rings for which every module is a direct sum of finitely generated ones. Every ring of finite representation type has this property, for modules on either side; see Auslander [7] and Tachikawa [256]. Conversely, a ring having this property on both sides is of finite representation type; see [7] or Fuller [101]. However, it is not known whether the assumption of this property on one side only ensures finite representation type in general (it is known for artin algebras; see Auslander [10]). For further discussion, see Hullinger [141] and Simson [246].

The third idea introduced by Köthe was the method of interlacing. In order to show that a commutative artin ring with all indecomposable modules cyclic was necessarily uniserial, Köthe argued by contradiction. Hence, if R was not uniserial, some $M = Re/(\text{rad } R)^k e$ (with e a primitive idempotent) would have to have two distinct isomorphic simple submodules. One could then form a new module by taking the direct sum of two copies of M and identifying simple submodules. Köthe was able to show that the resulting module was indecomposable, but not cyclic. This is as far as Köthe took the idea himself. In 1939, Brummund [44] who I believe was Köthe's student, took the method much farther. Using Köthe's idea, he was able to show that a noncyclic p-group has indecomposable p-modular representations of arbitrarily high degree. Combined with Köthe's result on uniserial commutative rings (or the theory of canonical forms), this completely characterized p-groups of finite representation type (Brummund also characterized uniserial group algebras in the case of a normal Sylow p-subgroup). In [203], Nakayama reported on Brummund's work, and posed the problem of finding algebras of unbounded representation type. Yoshii [273] noted that repeated composition factors in the socle of $Re/(\text{rad } R)^k e$ are not in general sufficient for infinite representation type; his example was the tensor algebra of a species of type B_2. Hence, interlacing does not supply an easy answer to Nakayama's question, which is, of course, still open. In an interesting example of the cyclic theory of history, the interlacing method was rediscovered by Snapper [250], in 1949. He made essentially the same application that Köthe had. Again, the method was extended to exhibit algebras of infinite representation type, by Dickson [76]. Later, he and Kelly gave a general description of the method [78]. In the meantime, Tachikawa [254] used Brummund's formulation to extend the results of Jans [144] to the case of nonsplit algebras. For more about interlacing, its consequences and generalizations, see Brenner [37], Dickson and Fuller [77], Gordan and Green [115], Janusz [145] and Müller [199].

The second influential paper of 1935 was that of Brauer [32], in which results on modular representations of groups were first presented from an algebra-theoretic point of view (Dickson [70] obtained some information much earlier by use of the group determinant). Brauer's work, which has had many group-theoretic sequels, also helped to initiate renewed interest in the structure of nonsemisimple algebras. In particular, it led to the block theory and to the effective use of the principal indecomposable (= indecomposable projective) modules (see also Köthe [160] for some earlier work). Brauer's student Nesbitt wrote down the general assertions corresponding to what Brauer had announced about group algebras [210]. Nesbitt and Scott [211] continued this work by studying symmetric algebras and basic algebras. It is important to note that they essentially described the Morita equivalence between an algebra and its basic algebra. Some of these ideas were made widely available in the famous monograph of Artin, Nesbitt and Thrall [5]. Structural results on modular group algebras were given by Lombardo-Radice [179] and Jennings [148].

In 1941, Brauer [33] also encouraged interest in the problem of determining algebras of finite and infinite representation type, which, as previously noted, had already been posed by Nakayama [203]. In this abstract, Brauer asserted that he had found some sufficient conditions for an algebra to have infinite representation type. Several years later, Thrall [259] announced further results, which he said were similar to Brauer's. Conditions for infinite representation type were formulated in terms of the Cartan matrices of factors of the algebra by powers of its radical. These conditions can be recognized as constraints on the quiver of the algebra. Unfortunately, Brauer and Thrall never published the details of their investigations, but the work of Jans [144] includes the announced results, and Jans indicated that he made use of Thrall's calculations.

Thrall also deserves much credit for his paper [260], published in 1948, one year after the abstract [259]. In this work, Thrall introduced generalizations of quasi-Frobenius rings by requiring various properties of modules. This approach to describing classes of rings was relatively new; previously, internal ideal-theoretic notions were dominant. Such a point of view indicated the trend towards viewing a ring in terms of its representations, and eventually led to such useful concepts as the Morita theory of equivalence and duality. Thrall's QF types have themselves been the subject of much interest; see Tachikawa's survey [256].

In the early 1950's, ring theory was revolutionized by the introduction of homological methods (which had been developing for some time, especially in the context of cohomology of groups). In particular, the role of the principal indecomposable modules was finally clarified in terms of the concept of projectiveness, while the components of the "second regular representation" were revealed as the injective indecomposable modules. Also, the work of Rosenberg and