# FOUNDATIONS OF MODERN PHYSICS

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## TO FLOYD KARKER RICHTMYER "THE PROF"

#### **PREFACE**

What is meant by "modern physics"? Certainly it means those fields in the realm of physics which are at present greenest with new growth. But from the point of view of a book such as this one it includes also those older parts of physics which are necessary to the understanding of the work in these frontier fields, and it also properly includes the more important and more fundamental of the practical applications which have followed discoveries whose primary importance is to science itself.

For some years now a course in modern physics has been given at the George Washington University as the fourth unit of a two-year general course in physics, the scope and content of this course being essentially as defined above. This book grows out of the experience of the author in teaching this course, and is the successor to a briefer lithoprinted book which has been used in it for several years. Emphasis throughout is laid upon the experimental aspects of modern physics, and upon the evidence which these experiments give in support of the new theories. Emphasis is given also to the close relationships which exist between modern physics and the older physics out of which the recent advances have grown. The development of each topic follows the order which may be interpreted as logical from our present vantage point, rather than the historical order. Although at times this may seem incongruous to those already familiar with the topics involved, it simplifies the approach for the beginner.

It is hoped that this book may assist students in all fields of learning to obtain some acquaintance with the fundamental discoveries of modern physics, together with some understanding of the theories whereby these discoveries are explained and interpreted. To further this purpose, technical terms and phraseology have been avoided except where such terms may be fully explained, and only elementary mathematics is used. This does not imply that the treatments are only qualitative but, rather, that the quantitative developments are made in simple terms, with careful interpretations of the mathematical processes involved, instead of by means of the advanced mathematical methods so necessary to advanced theory. At the same time every effort is made to keep these treatments exact. This is accomplished,

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where necessary, by limiting their scope to specific cases. The problems assist by further illustrating the principles and by providing concrete examples of the magnitudes involved in the various phenomena.

#### ACKNOWLEDGMENTS

The author takes great pleasure in acknowledging here his deep indebtedness to all those who have assisted him in so many ways. First, he wishes to thank all those who have kindly sent him photographs and other materials from which illustrations have been made, as is noted elsewhere. Many thanks for assistance and advice in the preparation of the manuscript are then due to his colleagues at the George Washington University, Dr. G. Gamow, Dr. E. Teller, and Dr. W. L. Cheney, and likewise to Dr. T. H. Johnson, of the Bartol Research Foundation of the Franklin Institute, who assisted him similarly with the chapter on cosmic rays. Finally, he wishes to acknowledge a debt of longer standing to Dr. P. Scherrer, who gave a series of demonstration lectures on modern physics at Massachusetts Institute of Technology, in the winter of 1930–31. These lectures first revealed to the author the possibility of presenting modern physics in both an exact and an elementary manner.

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#### MODERN PHYSICS

#### CHAPTER I

### THE ELEMENTARY ELECTRIC CHARGE AND THE ELECTRON

1. The Granular Nature of Electric Charges. Roughly speaking, modern physics is concerned with three major questions: (1) What is the nature of matter? (2) What is the nature of "light"? (3) What is the nature of electricity? As everyone knows, these questions are not capable of independent answers. They do not indeed represent separate problems, but rather three interrelated aspects of one great problem which is presented to us by all of the physical universe—a problem which we must attack piecemeal, obtaining a partial answer here, another partial answer elsewhere, etc., with an occasional glimpse of the unity of the whole problem to encourage us to further effort. Scientific research is thus being pushed forward simultaneously on many fronts, and where we shall begin consideration of the results to date is largely a matter of choice. The fact that we are here beginning with a consideration of the granular nature of electric charge indicates merely that this is a convenient starting point.

The idea that the electric "fluid" or "juice" is really a stream of electrons flowing in the wires of an electric circuit is now as familiar to everyone as the idea that water, although it appears to the eye as an infinitely divisible continuum, is really made up of molecules of finite though tiny size. If, however, this idea of the granular nature of electric charge is not to be accepted without question, merely because it is a familiar idea, or because it has the authority of textbook writers behind it, how may its verity be proved? The existence of an elementary or smallest quantity of electric charge was made evident by the researches in electrolysis of Faraday, and was firmly established before Townsend and H. A. Wilson made the first rough measurements of the value of this charge. The outstanding features of this earlier experimental evidence will be presented later on in this

chapter. Disregarding the historical order, the evidence which will first be considered is that given by Millikan's method for measuring the elementary quantity of electric charge. (This is in keeping with the policy outlined in the preface, that each topic be developed in the order which, from the point of view of our present knowledge concerning it, represents the simplest and most direct presentation of it.)

2. Millikan's Oil-Drop Experiment. The ordinary laboratory instruments for measuring electric charge, such as the ballistic galvanometer or even the most sensitive electroscope, are none of them sensitive enough to give any evidence that electric charges are not infinitely divisible. And the method which does prove to be sensitive enough to measure the elementary quantity of electric charge proves also to be the simplest and the most direct method possible.

This method is to measure the force exerted upon the charge by a known electric field. It is illustrated in Fig. 1. A and B represent

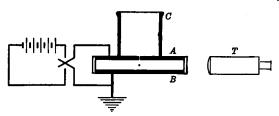


FIG. 1. MILLIKAN'S OIL-DROP EXPERIMENT.

two horizontal metal plates, seen in cross section; in the original apparatus of Millikan they were about 10 inches in diameter and several centimeters apart. In the laboratory apparatus shown in the figure they

are smaller and spaced about 5 millimeters apart. A potential difference of several hundred volts between these plates will produce a uniform electric field in the space between them, and any charged particle in this space will be acted upon by this field, with a force which is proportional to the charge on the particle. The most suitable particle is a tiny oil droplet. A fog of oil droplets is sprayed into the cup, C, and ultimately one of these slowly settling droplets may fall through the small hole made for that purpose in the center of the top plate and come into view in the field of the telescope.

If this droplet then possesses an electric charge, q, the electric field, E, will exert upon it a force equal to Eq, which may be made to act upward by throwing the reversing switch in the proper direction. (If the droplet does not acquire a "frictional" charge as a consequence of being sprayed from the atomizer, it may be given a charge with the aid of a bit of radioactive material.) The droplet may be observed through the short-range telescope, T, and the electric field may then be varied in strength by varying the potential difference across the condenser

until the electric force just balances w, the effective weight of the droplet, so as to hold it stationary. (The effective weight equals the true weight less the buoyant force of the displaced air.) When the droplet is stationary

$$Eq = w$$
, or  $q = \frac{w}{E}$  (1·1)

Actually, it is found more expedient to make the field stronger, so that the drop moves slowly upward when the field is acting, and then falls slowly downward with zero field. The mathematical reasoning is longer, but the principles are the same.

3. Weighing the Oil Drop. Before this charge may be computed, the oil droplet must be weighed; and no ordinary means of weighing will suffice for so small a mass, which is rarely greater than  $10^{-8}$  mg.

A new method of weighing had to be devised, and again a very fundamental physical principle was used. Any body moving through the air is opposed by forces due to the internal friction or *viscosity* of the air; if the body is one of spherical shape, such as a balloon, soap bubble, or fog droplet, and is moving slowly, the force is that given by Stokes' law

$$F = 6\pi \eta v r \tag{1.2}$$

See Fig. 2. v is the speed of the sphere, r is its radius, and  $\eta$  is the coefficient of viscosity for the air. A soap Fig. 2. Stokes' bubble or a fog droplet will soon reach a steady speed Law. where the forces acting upon it are balanced, i.e., such that the friction force equals the effective weight. Since the effective weight for an oil droplet may be computed from its volume and the densities of the oil and of the displaced air (d and d', respectively)

$$6\pi \eta v r = w = \frac{4}{3}\pi r^3 (d - d')g \tag{1.3}$$

for an oil droplet. This equation, when solved for r, the radius of the oil droplet, gives <sup>1</sup>

$$r = \sqrt{\frac{9\eta v}{2(d-d')g}} \tag{1.4}$$

The droplet may thus be weighed by first observing its speed of fall under gravity (zero electric field), next computing its radius

<sup>1</sup> Millikan discovered that, for such small spheres as these oil droplets, Stokes' law requires a small correction term to give sufficiently accurate results. See reference at the end of the chapter to his work. For simplicity it is neglected here.

by equation  $(1\cdot 4)$ , and then computing its effective weight from equation  $(1\cdot 3)$ . Once this weight is gotten, the charge on the droplet may be computed as indicated by equation  $(1\cdot 1)$ .

4. Value of the Elementary Charge, e. The smallest charge ever measured by this method has a value, as obtained by Millikan, of  $4.774 \times 10^{-10}$  electrostatic unit of charge (e.s.u.). And all other charges measured by him were, within the limits of experimental error, exact integral multiplies of this amount. This smallest quantity of electric charge will hereafter be designated by the letter e. Recent measurements have shown that this value is probably too small, principally because of an error in the value of  $\eta$ , the coefficient of viscosity for air, which was available to Millikan. The best value for e which is available at the present time (see Appendix II) is probably

$$e = 4.800 \times 10^{-10} \text{ e.s.u.}$$

with an uncertainty indicated by a "probable error" of  $\pm 4$  in the last place.

All of the evidence, however, is in agreement in indicating that this is the ultimate unit of electric charge, and that all charges, both positive and negative, are integral multiples of this charge.

5. Cathode Rays. Although this experiment of Millikan's is commonly referred to as measurement of the electron charge, it does of itself give proof only for the existence of elementary charges, and gives no evidence as to how those charges are related to material particles such as the electron. The discovery of the electron had, of course, been made earlier, and is attributable in considerable part to the improvements in vacuum pumps which were made about 1860. At the lowest pressures which were producible before that time (about 10<sup>-4</sup> atmosphere) an electrical discharge in a gas appears principally as a glow throughout the tube. This type of discharge is exemplified by the glowing tubes of luminous signs.<sup>2</sup> The improved vacuum pumps could reduce the pressure to values a hundred times or so lower than this, and at these lower pressures the phenomena of electrical discharge are essentially different. There is now very little glow visible in the residual gas, and the most noticeable phenomenon

<sup>&</sup>lt;sup>2</sup> Although usually called "neon" signs, these tubes may contain a variety of gases, the color of the glow depending chiefly upon the nature of the gas in the tube. Neon gives the red glow; helium, yellow; mercury vapor, blue; and mercury vapor in a yellow glass tube, green. The phenomena involved in this stage of the discharge will be discussed in Chapter IX.

is a fairly bright glow which appears on the inside of the glass walls of the evacuated vessel.

If a discharge tube such as is shown in Fig. 3 is used, the flat electrode, C, being made the negative terminal, or **cathode**, this glow will be confined to the end of the tube opposite to C, and any obstacle set up in the tube between C and the end of the tube will cause a sharp shadow to appear in the midst of the glow. These phenomena strongly suggest that this glow in the glass wall is due to some kind of rays which start out from C in a direction perpendicular to its surface and proceed in more or less straight lines across the tube to the opposite wall; if the pressure within the tube is not too low, and the discharge

is strong enough, the path of this beam of rays may be visible, marked out by a faint glow in the residual gas. Since the negative terminal is called the cathode, these rays, whatever they may be, are called cathode rays. (Curiously enough, the position in this tube of the anode, or positive terminal, is relatively unimportant, and it may be put in a side tube as shown. The cause of this curious behavior of the discharge

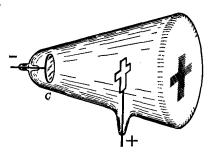


Fig. 3. Cathode-Ray Tube.

may be traced to the presence of the residual gas. The explanation will be given later on, in Sec. 118.)

6. Cathodoluminescence. A good many substances will luminesce, or glow as does glass, under the impact of cathode rays. This phenomenon has acquired a great deal of importance in recent years, in connection with cathode-ray-tube oscillographs (Sec. 13), television (Sec. 71), etc. Willemite (a variety of zinc silicate, containing a trace of manganese), which gives a brilliant green luminescence, is commonly used for cathode-ray-tube screens, although calcium tungstate, which luminesces a bright blue, is also sometimes used for this purpose, especially when the patterns produced on the screen are to be photographed. The luminescent properties depend upon the presence of traces of impurities of suitable kinds (manganese, in willemite), the color as well as the brightness being dependent upon the nature of the impurity. This is shown most remarkably by zinc sulfide, which may luminesce with a number of colors from blue to orange-red when "activated" by different impurities.

Beautiful effects may be produced by powdering luminescent

materials and using them to paint designs upon metal or mica plates which serve as targets in cathode-ray tubes. Sometimes the glow occurs only while the cathode rays are falling upon the target; it is then called fluorescence. Sometimes the glow persists for a greater or less time after the cathode rays are stopped (this is especially true of some of the sulfides), and is then called phosphorescence. These same materials also luminesce under the influence of ultraviolet light (Sec. 76) and X-rays (Sec. 80).

7. Deflection of Cathode Rays. The physical nature of cathode rays is revealed by the deflections which they undergo upon passage through electric and magnetic fields. Deflection by an electric field may be seen by means of a tube of the form shown in Fig. 4. If the plates A and B are uncharged, the narrow beam of cathode rays which

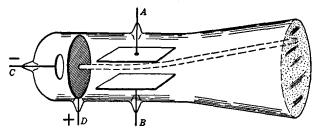


Fig. 4. Electrostatic Deflection of Cathode Rays. The plates A and B are charged + and - respectively by connection to a high voltage source.

comes through the hole in the anode, D, will continue straight across the tube to the opposite end, where it will make a spot of fluorescent light. (If the gas pressure is high enough, a small fraction of the rays will be stopped by molecules of the residual gas, which will then emit light, and the path of the beam through the tube will then appear as a faint line of light.) If, however, a potential difference is set up between the plates A and B, by connecting them to a suitable voltage source, the resultant electric field between them will deflect the cathode-ray beam as shown. The direction of this deflection, away from the negative plate and towards the positive one, is evidence that the beam consists of negative electric charges; and the fact that the beam is not carried directly to the positive plate, but is only deflected somewhat out of its original path, is equally good evidence that the rays have inertia, i.e., that they consist of particles possessing mass.

The same tube may be used to show the deflection produced by a magnetic field. If a horseshoe magnet is held astraddle of the tube, with the north pole in front and the south pole behind so that the lines

of force of its magnetic field are perpendicular to the beam of cathode rays, the beam will be deflected downward, at *right angles* to the direction of the magnetic field. If a larger tube is used, such as the one shown in Fig. 5, together with a magnetic field which is uniform

over the whole tube, the beam may be bent into a complete circle as shown. (Here the anode is a cylinder, A, with a short slit in one side, and the cathode, C, is a hot tungsten wire. The advantage of using a hot cathode will be explained in Sec. 11. The magnetic field is perpendicular to the plane of the figure and directed inward, or away from the reader. Enough gas is left in the tube to reveal the path of the cathode-ray beam by the luminescence of the residual gas.)

The deflection is in exactly the direction in which a conductor would be pushed when a stream of negative electric charges is flowing through it; and a free stream of tiny fastmoving particles, each carrying a negative electric charge would be bent by a magnetic field it.

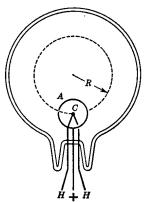


Fig. 5. Magnetic Deflection of Cathode Rays.<sup>4</sup>

electric charge, would be bent by a magnetic field into just such a circular path.

8. The Electron. Sir William Crookes in 1879 advanced the hypothesis that cathode rays are streams of negatively charged particles of matter traveling at very high speeds. And in 1897, Sir

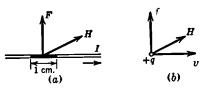


Fig. 6. (a) Force on a Current-Bearing Conductor in a Magnetic
 Field. (b) Force on a Moving Charge in a Magnetic Field.

- J. J. Thomson showed that this hypothesis is correct, and that all of these cathode-ray particles are identical in kind, no matter what the metal of the cathode, or what the gas in the tube. He named these particles electrons. To show that they were all alike he "weighed" them by means of their deflections by electric and by magnetic fields.
- 9. Magnetic Force on a Moving Charged Particle. Before Thomson's method of weighing electrons may be explained, we must first obtain the equation for the force exerted upon a moving electric charge by a magnetic field. As is well known, if a current-bearing conductor is placed at right angles to a magnetic field, each unit length of it is acted upon by a force,  $F = HI_m$ , which is perpendicular both to the

field and to the conductor. See Fig. 6-a. H is the magnetic field intensity,<sup>3</sup> and  $I_m$  is the current strength in *electromagnetic units*. Or, if electric charge and current are measured in *electrostatic units* (as will be the practice throughout this book; see Appendix I), the force equation becomes

$$F = \frac{HI}{c} \tag{1.5}$$

where I is the current in electrostatic units and c is the ratio between the two systems of units.

If now we think of the current as being a stream of electrically charged particles, each having the same charge, q, and speed, v, the current is then I = nqv, where n is the number of such charged particles per unit length of the conductor, and the force equation then becomes

$$F = \frac{Hnqv}{c} \tag{1.6}$$

It follows that the force on each charged particle is

$$f = \frac{F}{n} = \frac{Hqv}{c} \tag{1.7}$$

10. Mass of the Electron. The mass of the electron may be measured by apparatus<sup>4</sup> such as is shown in Fig. 5. The principles used are much the same as those employed by Thomson's method, although the procedure is somewhat simpler. First we shall assume that the charge on the electron is the elementary negative charge,  $e = 4.800 \times 10^{-10}$  e.s.u. The validity of this assumption is usually assumed without question. Indeed, as has been stated earlier, Millikan's measurement of the elementary charge is usually referred to as measurement of the charge of the electron. It is nevertheless

<sup>4</sup> This form of apparatus was devised by K. T. Bainbridge for use in the student laboratory. It is more fully described in *American Physics Teacher*, Vol. 6, pp. 35-36, February, 1938.

<sup>&</sup>lt;sup>3</sup> More exactly speaking, H here represents the flux density of the magnetic field, the quantity usually represented by B; and equation 1.5 would be more exact if written as F = BI/c. In air, however, both flux density and field intensity have equal numerical values (the one in gauss, the other in oersteds); and H is used in equation 1.5 et seq. to conform with the notation found in corresponding equations in most textbooks and research papers. It must be remembered, however, that H, as so used, is measured in gauss.

an assumption, and its justification comes from the consistency of all the many results which follow from its use.

Since the electrons travel in a circle of radius R, in the magnetic field H, the magnetic force must produce an acceleration of  $v^2/R$ , or

$$f = \frac{Hev}{c} = \frac{mv^2}{R} \tag{1.8}$$

Since both the mass, m, and the speed, v, are unknown, this equation is insufficient to give us the value of either quantity.

A second equation involving these two quantities may, however, be obtained from the fact that the electrons acquire their speed as a result of the action upon them of the potential difference, U, between the cathode, C, and the anode, A. Hence the kinetic energy of each electron equals the work, Ue, done upon it by this potential difference. I.e.,

$$\frac{1}{2}mv^2 = Ue \tag{1.9}$$

Solving these equations simultaneously, one obtains

$$v = \frac{2U}{HR}c \tag{1.10}$$

and

$$m = \frac{H^2 R^2 e}{2Uc^2} \tag{1.11}$$

A large number of different methods have been used to determine m (or e/m; see below), all of them now being in substantial agreement with each other. From all these measurements the best value for the mass of the electron may be taken to be

$$m = 9.10 \times 10^{-28} \, \text{gram}$$

As a matter of history, Thomson performed his experiments before any experiments had yet been devised with which to measure the elementary charge; all that he then could compute was the specific charge, or the ratio of the electron charge to its mass. In terms of the best values obtainable today, this is

$$\frac{e}{m} = 5.273 \times 10^{17} \text{ e.s.u. per gram}$$

with an uncertainty indicated by a probable error of  $\pm 2$  in the last place. See Appendix II.