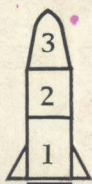


Science with pocket calculators

D. R. Green and J. Lewis



$$n = 3$$

$$\epsilon = 0.8$$

$$\beta = 0.01$$

$$c = 2.5 \text{ km s}^{-1}$$

$$0.01 \div 1.01 = \sqrt[3]{y} \quad 3 \times 0.8$$

$$- 0.8 + 1 = \ln \quad \times 3$$

$$\pm / - \quad \times 2.5 =$$

Maximum final velocity :
 7.4 km s^{-1}

SCIENCE WITH POCKET CALCULATORS

D. R. GREEN

Loughborough University of Technology

J. LEWIS

Understanding British Industry Resource Centre



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Preface

Many people are quite content to use an electronic calculator without bothering how it works or even how it might be used more efficiently. This attitude is quite understandable for the owner of a simple four-function machine who has had little or no formal training in mathematics.

But this book is written for the owners, or intending owners, of the more advanced calculators, which incorporate some scientific and mathematical functions. The various types of calculator, together with the kinds of function they offer, are discussed and some insight is given into how the calculator actually works. After that many examples are given to show how the calculator can be used to make routine mathematics easier and to extend the scope of your work.

There now seems to be little doubt that the calculator represents a major advance in scientific and mathematical techniques. We hope that after reading this book you will agree.

A note on accuracy

We have given the results of the many calculations in this book to varying degrees of accuracy. Your calculations may be slightly different, depending on the machine being used. We would emphasize that although we may give a result to 8 or 10 significant figures we do not claim that to be necessarily entirely accurate, but rather that is what a calculator may well give. Also, we do not wish the reader to uncritically accept or quote many figures in an answer. It is a good habit to round off an answer to an *appropriate* number of figures. What is appropriate will depend upon the problem, the data and the method employed. Often it might have been more exact to have used the 'approximately equals' symbol \approx where $=$ has in fact been used, but the context should indicate to the reader where this is the case. We have restricted the use of \approx to cases of importance.

A note on key-stroke sequences

We have provided many illustrated calculations but your machine may behave slightly differently and key-stroke sequences may need slight

modifications. Again, you may well be able to find *better* ways to perform the calculations demonstrated.

Acknowledgements

Finally we would wish to acknowledge all the help which those in the industry, our colleagues and our friends have given us. In the first group we must list John McDonald of Casio, A. G. Tamone of Hewlett Packard, Ian Jennings formerly of Texas Instruments, and others from CBM, General Instrument Microelectronics, Novus and Broughtons. Those in the last two groups are too numerous to mention individually and include staff of Loughborough University of Technology from all the following departments: Chemistry, Computer Studies, Education, Electrical and Electronic Engineering, Engineering Mathematics, Management Studies, Mathematics, Physics, Transport Technology and also a number of undergraduates who made many valuable comments on the manuscript. However we must express particular thanks to Robin Bradbeer, Jack Banks, John Lee and of course Avi Bajpai, for all the invaluable advice and encouragement which they have given us.

DAVID GREEN
JOHN LEWIS

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1. The types of calculator

Originally electronic calculators were aimed at the commercial office user where they would be competing with the traditional mechanical adding machines. To compete in this market the electronic calculator had its functions limited to the four basic ones of addition, subtraction, multiplication and division, and the entry logic used was chosen so that it would be familiar to the potential operators.

As the versatility of calculators has increased their use has spread into many fields and the range of calculators available has grown, with the result that the buyer is often confused as to which one to purchase. In general, though, calculators tend to fall into six main groups, each of which is available as either desk-top or portable machines. Desk-top machines give a larger display, have larger, well spaced keys, run solely off the mains and are more expensive.

The six main groups are:

1. *Basic calculator* Aimed at the consumer market, costing under £10 in 1978, they offer the basic operations of addition, subtraction, multiplication and division and usually percentage. Often they are provided with a 'constant function' to allow repetitive calculations, and occasionally with a memory. The display is limited to about 6 or 8 digits and the calculator is usually powered by disposable batteries.

2. *Enhanced basic calculator* These are the first of the second-generation machines which have ousted the more limited early models. Their functions are the four basic ones with the addition of: x^2 , $1/x$, a fully addressable memory, constant function and even in most cases \sqrt{x} , and sometimes π and brackets.

Displays tend to be 8 digit with green digitrons in preference to the original red light-emitting diodes (LEDs). Most of the major manufacturers produce models of this type at prices ranging from £7 depending on the power supply, those with disposable batteries being cheapest.

3. *Basic scientific calculator* Realizing the potential school market—especially in science and mathematics—several manufacturers, epitomized by CBM with their 7919 machine (fig. 1.1), introduced models which offered trigonometrical functions, logarithms and



Fig. 1.1 Example of a 'semi-scientific' calculator.

powers. These must not be confused with the earlier so-called 'slide rule calculators' which were of little use because they did not offer exponential notation, also known as standard notation or scientific notation. With exponential notation it is possible to display numbers as large as 9.9999×10^{99} or as small as 1.0000×10^{-99} , on an 8-digit display. Depending on the make, prices will be in the bracket £10 to £20.

4. *Advanced scientific/mathematical calculators* Aimed at the serious student or professional user, these machines are an extension of the semi-scientific models, offering many more functions. The Texas Instruments SR 51 II, for example (fig. 1.2), has the statistical functions mean, standard deviation, factorials, hyperbolic functions and linear regression. There are also three separate memories and numerous conversions from imperial to metric units and vice versa as well as polar—rectangular coordinate conversion. A comparable calculator, the CBM 5190, is available with the added facilities of being able to deal with Gaussian (or normal), Poisson and binomial distributions, numerical integration and even complex numbers.

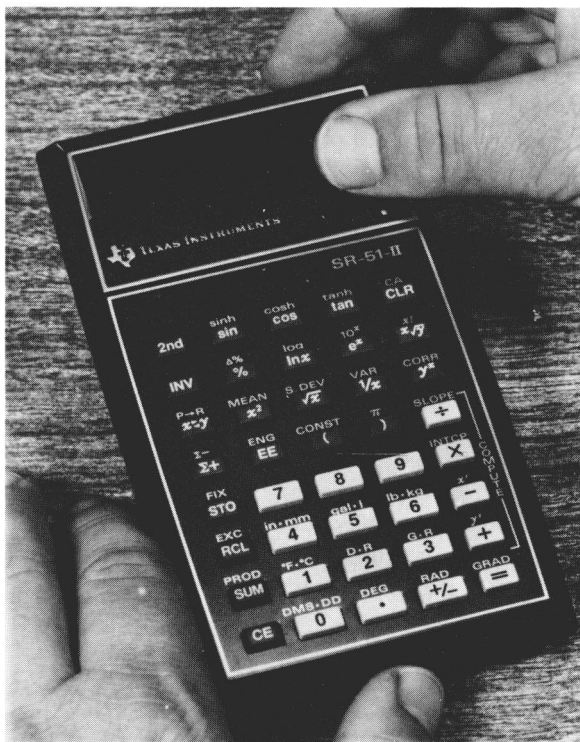


Fig. 1.2 The Texas Instruments SR 51 II.

5. *Specialist calculators* The first specialist calculators were aimed at the financial experts, allowing them to quickly work out such things as discounted cash flows, interest and loan rates plus all the other things which are so important in commerce. It is likely that we shall see more of this type of calculator in the future, mainly because of the ways in which the workings of a calculator are organized (which is explained more fully in Chapter 3). Since the available functions depend only on the preprogrammed Read-Only-Memory (ROM) which controls the workings of the arithmetic unit, it is possible to switch ROMs around to alter functions.

6. *Programmable calculators* This class is dealt with in more detail in Chapter 16. They do allow one to do repetitive complicated calculations without the necessity of performing all the key-strokes every time since these are remembered by the machine.

It is useful to subdivide this particular type into those which can retain the programs in some way even when switched off, and those

which cannot. In the former category are the very powerful HP 67, the Texas Instruments TI 59 (which replaces the SR 52) and the Novus 7100.

These are computer-like in the programming facilities offered. The programs may be stored either on small magnetic cards which are 'read' by the machine when required or alternatively in special semiconductor memories contained in a tiny cartridge which is inserted into the calculator. When not in use the cards or cartridges can form part of an extensive software library.

In the second category (those which cannot retain the program when the power is turned off) are machines such as the Texas Instruments TI 58 (which replaces the SR 56). A relatively new development by Casio and Hewlett-Packard enables the program to be stored in the calculator's *internal* memories even though the power has been switched off. Casio do this by providing separate silver oxide batteries for use with the program storage chips only, which means that 256 program steps can be stored for up to a year. However once a new program is written, the old one is lost and can only be re-entered by going through all the key-strokes again. In point of fact, the large amount of program storage often makes it possible, by careful management, to store more than one program.



Fig. 1.3 Texas Instruments PC 1000 printer cradle with an SR 52 in place.

DISPLAYS

Manufacturers are now offering calculators in most categories which give a print-out in addition to the more usual illuminated display. With a printer—usually a thermal one where a matrix of small heaters is arranged in a similar fashion to the individual diodes of the LED display—a permanent ‘hard copy’ of calculations can be obtained. A major advantage in having a print-out is that the user can check through the working.

Some of the newer scientific machines from Texas Instruments are capable of being plugged into a special cradle which contains a printer (fig. 1.3). Thus a pocket calculator can double as a desk-top model with print-out, enabling it to be used in the home, office or laboratory, as well as in the field.

Most displays still employ illuminated digits. The three types of display commonly used are the light emitting diode (LED), the liquid crystal, and the green digitron.

The LED displays are made by a process which is very similar to that used to produce the calculator chips themselves and are completely solid-state, being based on gallium arsenide. The display is invariably red—although it is now possible to produce other colours—and is highly magnified by cylindrical lenses, which can limit peripheral viewing. The digits are formed by illuminating the relevant bars of a seven segment matrix (fig. 1.4).



Fig. 1.4 Seven-segment displays.

The bars may be a single solid block of semiconductor or each bar may be composed of a number of individual diodes.

Liquid crystals are a very promising display since they do not generate light and hence draw very little power. Rather they reflect or transmit incident light (so you cannot use them in the dark). In the ‘field effect’ type of liquid crystal cell, a nematic liquid crystal is sandwiched between two polarizing plates whose optical axes are at right angles. When the display is excited by an electric field the orientation of the liquid crystal molecules changes, effectively rotating the plane of polarization, thus allowing light to pass through or to be reflected from the mirror-like backing plate. By selectively altering the crystal in this way and using a seven-segment matrix, figures can be produced which are either light against a dark background, or else dark against a bright background,

this contrast making the device visible over a wide range of lighting intensities, unlike the LED which cannot be seen in bright sunlight. The life of a liquid crystal display is limited, according to some estimates, to about five years.

A digitron is an assembly of triode valves in a glass envelope. The phosphor-coated anodes form the seven segments of the visible part of the display. A positive potential difference between the anode and cathode accelerates electrons onto the anode causing the phosphor to glow. The controlling grid and the filament wires are in front of the anode and are normally invisible. Their life, too, is reportedly limited. Both liquid crystals and digitrons give larger digits than LEDs and so may be viewed without prior magnification.

None of these displays presents a steady picture. A technique known as multiplexing is used to light each digit for a fraction of the total cycle but the flicker is so rapid that it is not noticeable.

Multiplexing can be used in this way because the data within the calculator is read in *serial* form, that is it comes one digit after another rather than in *parallel* form where all the digits are produced, and need to be displayed, simultaneously.

POWER SUPPLIES

The power supply for a calculator can be quite expensive, especially if disposable batteries are used. These batteries are available either in the traditional Leclanché cell form or as the mercury cell equivalent. The latter are more expensive but offer a longer life. None of these disposable cells should ever be recharged since an explosive build up of gas within the cell may result.

To reduce the running costs, manufacturers often provide a 'mains unit' so that the calculator may be operated directly from the mains. This is not a charger in the strict meaning of the word, since it does not charge up the batteries. The insertion of the jack plug into the calculator disconnects the internal batteries: if this is not so then the disposable batteries must be removed before using the mains unit. One manufacturer prints a warning to this effect on the disposable batteries.

Alternatively, rechargeable nickel-cadmium cells may be fitted and a mains charger unit supplied. Initially more expensive, this combination is cheaper in the long run. Note that if rechargeable cells are fitted they should be in place when the calculator is being run from the mains unit as they often help smooth the rectified voltage.

One or two machines based on disposable batteries have an automatic display turn-off that operates after some 15 sec. The display is recalled simply by pressing one of the function keys, but some people find the disappearance of the displayed number after such a short time interval rather annoying.

It is not always possible to tell that the batteries are low, because the display does not necessarily get dimmer. The first indication is often an erratic display followed by incorrect calculations. One or two machines do provide a few minutes' warning, e.g. all the decimal points light up. Even so this will normally only let you finish the current calculation. In critical situations such as examinations it is always best to ensure that either spare disposable batteries or another battery pack are available.

The mains unit should conform to British Standards in the U.K. and should have a separate mains lead. Some Continental European specifications allow the mains plug to be integral with the charger. In use, the unit may be warm to the touch although this is no cause for alarm. Always check that, if a changeover switch is fitted, it is set to the correct a.c. voltage.

In no circumstances should a calculator ever be operated from a charger other than that recommended for it. Different models from the same manufacturer may require different chargers or mains units.

KEYBOARDS

The keyboard is the interface between the user and the calculator and a lot of design work has gone into producing a suitable ergonomic layout. Originally large reed switches were used, operated by small magnets which were pushed down by the key. Today very thin keyboards have been developed which can cram fifty or more keys onto a pocket calculator. The requirement is for the key depression to have a positive feel so that the user can tell when a digit has been entered. One way of achieving this is to use the 'oil can' or 'clicker' property of a piece of curved metal so that when the key is depressed the metal snaps down to make contact. An alternative is to use elastomer keyboards and yet another way is to use a small coiled spring under each key to overcome the natural springiness of the metal contact. Whichever method is used, the contact is never a clean 'make', it bounces for a few times and this key bounce has to be allowed for electronically, otherwise spurious entries would be made.

NEW DESIGNS

Prices seem to be stable. Bargains are likely therefore to be the ends of ranges or 'new' machines built around an obsolete chip. In 1978 the life of a calculator range to a manufacturer is a year to eighteen months, after which a new design takes its place having more features and functions. Fig. 1.5 shows a possible development for the future.

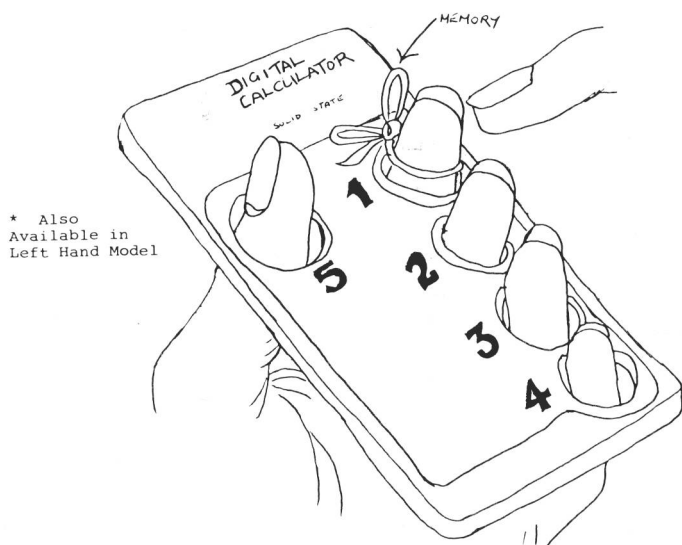


Fig. 1.5 A rough drawing of the prototype of the next generation of calculators.

2. Calculator features

The various features found on a calculator depend on which of the six categories it falls into (see pp. 1–4). Most buyers choose a calculator with too many functions, even for advanced work, and find that some of the functions are superfluous—but it is nice to know that they are available!

TYPES OF LOGIC

One major decision in the purchase of a scientific/mathematical calculator is the method of entry—otherwise known as the *language* or the *logic*. Originally arithmetic logic was available, identified by there being

two $\boxed{=}$ keys on the calculator. One was $\boxed{\begin{smallmatrix} + \\ = \end{smallmatrix}}$ and the other, usually

in red, was $\boxed{\begin{smallmatrix} - \\ = \end{smallmatrix}}$. This system has now become almost extinct, leaving

algebraic logic and Reverse Polish logic. Both these forms have their advantages and disadvantages though the balance is slightly towards algebraic logic because it appears more natural.

It was the Polish logician Jan Lukasiewicz who first demonstrated, in 1951, how expressions could be specified by placing operators immediately *before* their operands. As an example take the expression

$$(a+b) \times (c-d)$$

This can be re-written as

$$\times + ab - cd$$

and is as unambiguous in the second form as in the first, mathematically that is. It would be read as ‘multiply the sum of a and b by the difference of c and d ’. Alternatively the operators may come *after* the operands in this form

$$ab + cd - \times$$

which also has the same meaning.

In honour of Lukasiewicz, the two notations have become known as Polish and Reverse Polish respectively.

Computer compiler writers have been quick to realize the advantages of using Reverse Polish logic since it means that as an expression is scanned from left to right every operator can be executed immediately it is encountered. It does not have to be stored as in algebraic logic where the operator is sandwiched between the operands. Thus the Reverse Polish Notation has advantages in needing less storage space on a calculator chip. Hewlett Packard were the first manufacturers to produce a scientific machine using Reverse Polish and they have stuck to it. Other manufacturers have gradually swung over to algebraic logic, which corresponds more closely to the written format. To show the differences let us try some problems using both methods:

$$a + b \times c$$

For algebraic logic the key strokes:

a + b × c =

will give the incorrect answer (unless you are using a Texas Instruments machine) because the expression to be evaluated really means

$$a + (b \times c)$$

in mathematical terms but the key strokes described would work out $(a + b) \times c$. So the key strokes would need to be:

b × c + a =

which is inconvenient for the user, who has to rearrange the expression. Using a Reverse Polish machine, however, the key strokes are quite simply:

a enter b enter c × +

but some thought is still required of the user.

Another example might be

$$(a \times b) + (c \times d)$$

and this becomes much more difficult on the algebraic machine since, if it does not have memories or parentheses, the expression has to be rearranged to become

$$\left(\frac{a \times b}{d} + c \right) \times d$$

for which the key strokes are:

a × b ÷ d + c × d =

or, if a memory is available:

a × b = STO c × d + RCL =