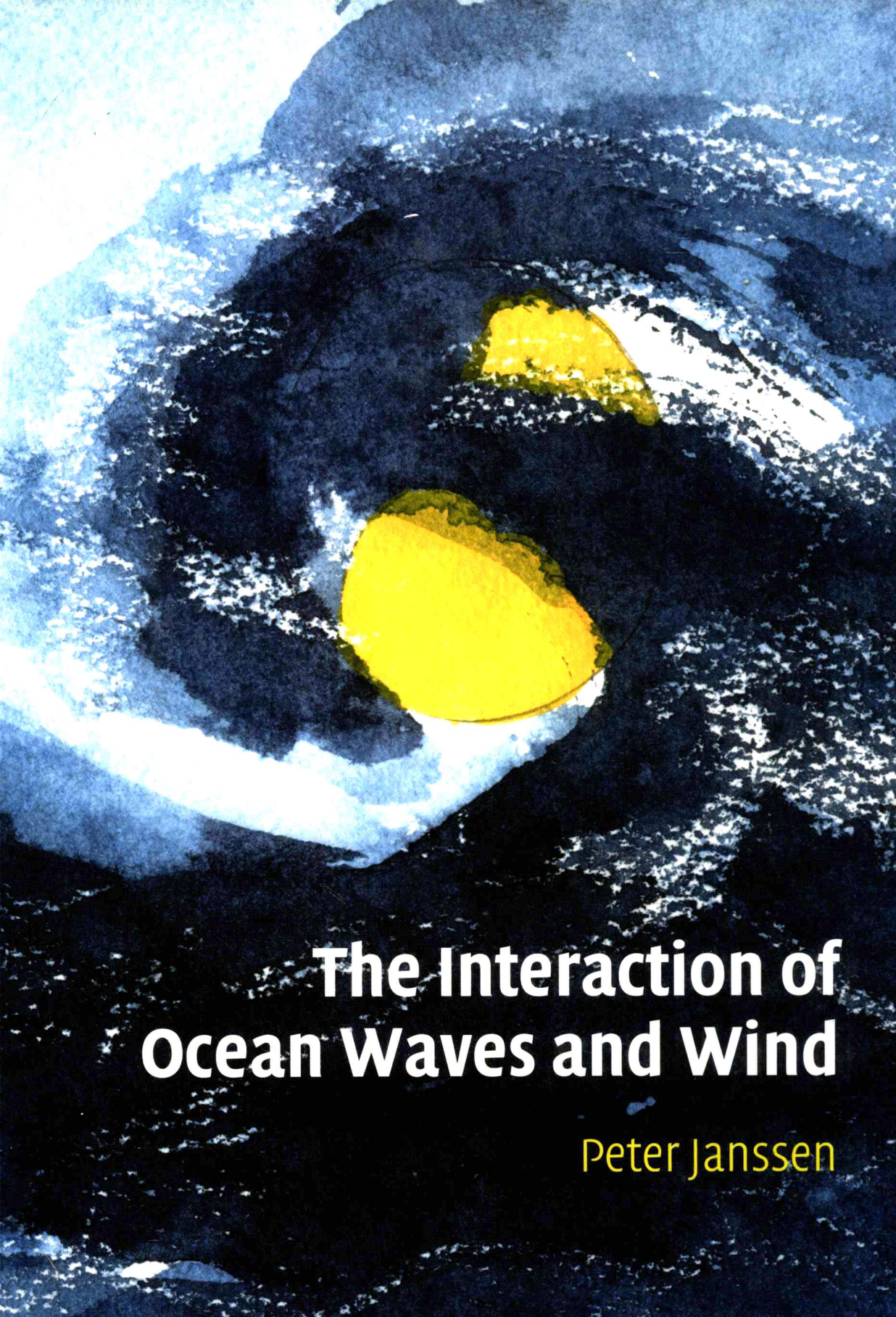


An abstract painting depicting the interaction of ocean waves and wind. The composition features dark, swirling blue and black areas representing the ocean, with bright yellow and white highlights suggesting the crests of waves and the spray of wind. The overall effect is dynamic and textured, with visible brushstrokes and a sense of movement.

The Interaction of Ocean Waves and Wind

Peter Janssen

THE INTERACTION OF OCEAN WAVES AND WIND

PETER JANSSEN

European Centre for Medium-Range Weather Forecasts



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THE INTERACTION OF OCEAN WAVES AND WIND

The Interaction of Ocean Waves and Wind is the first book to describe in detail the two-way interaction between wind and ocean waves and shows how ocean waves affect weather forecasting on time scales of 5 to 90 days.

Winds generate ocean waves, but at the same time airflow is modified due to the loss of energy and momentum to the waves; thus, momentum loss from the atmosphere to the ocean depends on the state of the waves. This volume discusses ocean-wave evolution according to the energy balance equation, which describes the rate of wave-spectrum change by advection and by physical processes such as wind input, dissipation and nonlinear interactions. An extensive overview of nonlinear transfer is given, and as a by-product the role of four-wave interactions in the generation of extreme events, such as freak waves, is discussed. Effects on ocean circulation are described. Coupled ocean-wave–atmosphere modelling gives improved weather and wave forecasts.

This volume will interest ocean-wave modellers, physicists and applied mathematicians, and engineers interested in shipping and coastal protection.

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Cover: water colour by Danielle Mérelle.

Preface

This is a book about ocean waves, their evolution and their interaction with the environment. It presents a summary and unification of my knowledge of wave growth, nonlinear interactions and dissipation of surface gravity waves, and this knowledge is applied to the problem of the two-way interaction of wind and waves, with consequences for atmosphere and ocean circulation.

The material of this book is, apart from my own contributions, based on a number of sources, ranging from the works of Whitham and Phillips to the most recent authoritative overview in the field of ocean waves, namely the work written by the WAM group, *Dynamics and Modelling of Ocean Waves*. Nevertheless, the present book is limited in its scope because it will hardly address interesting issues such as the assimilation of observations, the interpretation of satellite measurements from, for example, the radar altimeter, the scatterometer and the synthetic-aperture radar, nor will it address shallow-water effects. These are important issues but I felt that the reader would be served more adequately by concentrating on a limited number of subjects, emphasizing the role of ocean waves in practical applications such as wave forecasting and illuminating their role in the air–sea momentum exchange.

I started working on this book some 8 years ago. It would never have been finished had it not been for the continuous support of my wife Danielle Mérelle. Her confidence in my ability to complete this work far exceeded my own. I thank my parents, Aloysius Janssen and Rosa Burggrave, for supporting me to follow a university education. I am indebted to my Ph.D. advisor Martin Weenink and L. J. F. Broer for their introduction into the field of nonlinear physics. Also, it is a pleasure to acknowledge the contributions of P. G. Saffman and G. B. Whitham to my education in ocean waves. Things really started to happen when I joined the WAM group (WAM) group. Most of the members of the WAM group thought that this was a unique opportunity for collaboration, and we had the time of our lives. I would like to thank Gerbrand Komen, Klaus and Susanne Hasselmann, Mark Donelan and Luigi Cavaleri for all the fruitful discussions and the collaborations. Furthermore, I would

like to thank Luciana Bertotti, Heinz Günther, Anne Guillaume, Piero Lionello and Liana Zambresky for sharing the burden of the development of a beautiful piece of software, and for all the fun we had.

Last, but not least, I would like to thank Pedro Viterbo and Jim Doyle for disentangling all the intricacies involved in the actual coupling of an atmospheric model and an ocean-wave prediction system. The former and present members of ECMWF's ocean-wave team, Jean Bidlot, Björn Hansen, Saleh Abdalla, Hans Hersbach and Øyvind Sætra are thanked for their dedicated efforts to further develop the WAM model software, while support by Lennart Bengtsson, David Burridge, Anthony Hollingsworth, Adrian Simmons and, in particular, Martin Miller is much appreciated.

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1

Introduction

The subject of ocean waves and their generation by wind has fascinated me greatly since I started to work in the Department of Oceanography at the Royal Netherlands Meteorological Institute (KNMI) at the end of 1979. The wind-induced growth of water waves on a pond or a canal is a daily experience for those who live in the lowlands, yet it appeared that this process was hardly understood. Gerbrand Komen, who arrived 2 years earlier at KNMI and who introduced me to this field, pointed out that the most prominent theory explaining wave growth by wind was the Miles (1957) theory which relied on a resonant interaction between wind and waves. Since I did my Ph.D. in plasma physics, I noticed immediately an analogy with the problem of the interaction of plasma waves and electrons; this problem has been studied extensively both experimentally and theoretically. The plasma waves problem has its own history. It was Landau (1946), who discovered that depending on the slope of the particle distribution function at the location where the phase velocity of the plasma wave equals the particle velocity, the plasma wave would either grow or damp. Because of momentum and energy conservation this would result in a modification of the particle-velocity distribution. For a spectrum of growing plasma waves with random phase, this problem was addressed in the beginning of the 1960s by Vedenov *et al.* (1961) and by Drummond and Pines (1962). The principal result these authors found was that because of the growth of the plasma waves the velocity distribution would change in such a way that for large times its slope vanishes in the resonant region, thereby removing the cause of the instability. Thus, a new state emerges consisting of a mixture of stable, finite-amplitude plasma waves and a modified particle-velocity distribution.

Based on this analogy, I realized that the approach by Miles (1957), which relied on linear theory, could not be complete, because energy and momentum were not conserved. Taking nonlinear effects into account would enable me to determine how much momentum transfer there is from the wind to the waves, which would give rise to a wave-induced stress on the airflow. This resulted then in a slowing down

of the airflow, and hence in a modified wind profile. Considering, for simplicity, the two-dimensional problem only (hence wave propagation in one direction), I performed the necessary calculations which were similar in spirit to those of the plasma problem. They indeed confirmed my expectation that in the presence of growing water waves the wind profile would change. The role of the particle-velocity distribution in this problem was played by the vorticity of the mean flow; hence, in the absence of all kinds of other effects (e.g. turbulence) a new state would emerge consisting of stable, finite-amplitude water waves and a mean flow of which the gradient of the mean vorticity would vanish in the resonant region. It should be noted that a number of years earlier, Fabrikant (1976) reached a similar conclusion while Miles (1965) also addressed certain aspects of this problem. This theory has become known as the quasi-linear theory of wind-wave generation.

A number of colleagues at KNMI pointed out to me, however, that my treatment was far from complete if it was to be of practical value. And, indeed, I had neglected lots of complicating factors such as nonlinear wave-wave interactions, dissipation due to white capping, flow separation, air turbulence, water turbulence, etc. For example, it is hard to imagine that in the presence of air turbulence the mean airflow would have a linear dependence on height (corresponding to the vanishing of the gradient of its vorticity), since the turbulent eddies would try to maintain a logarithmic profile. Thus, in general, it is expected that there will be competition between the effect of ocean waves, through the wave-induced stress, and turbulence; presumably, the steeper the waves, the larger the wave effect will be. Nevertheless, it was evident that knowledge of the momentum transfer from air to sea required knowledge of the evolution of ocean waves, which apart from wind input is determined by nonlinear wave-wave interactions and dissipation due to white capping. In short, in order to show the practical value of the idea of the wave effect on the airflow, the running of a wave model was required.

At the beginning of the 1980s a spectral ocean-wave model, including wave-wave interactions, was not considered to be a viable option. The reason for this was that there was not enough computer power available to determine the nonlinear transfer in a short enough time to be of practical value for wave forecasting. This picture changed with the introduction of the first supercomputers and with the work of Hasselmann and Hasselmann (1985) who proposed an efficient parametrization of the nonlinear transfer. Combined with the promise of the wealth of data on the ocean surface from remote-sensing instruments on board new satellites such as ERS-1, ERS-2 and Topex-Poseidon, this provided sufficient stimulus to start a group of mainly European wave modellers who called themselves the WAVE Model (WAM) group. Apart from a keen interest in advancing our knowledge regarding the physics of ocean waves and assimilation of wave observations, the main goal was to develop a spectral wave model based on the so-called ‘energy balance equation’

which included the physics of the generation of ocean waves by wind, dissipation due to white capping and, of course, nonlinear interactions. I joined the WAM group in 1985 because of my interest in wave prediction and, in the back of my mind, with the hope that perhaps I could now study the consequences of the slowing down of the airflow in the presence of ocean waves.

The interests and backgrounds of the members of the WAM group varied greatly. It brought together experimentalists, theorists, wave forecasters and people with a commercial interest. Nevertheless, owing to the great enthusiasm of the group, the tremendous efforts by Susanne Hasselmann to develop a first version of the WAM model, and, not least, the computer facilities generously provided by the European Centre for Medium-Range Weather Forecasts (ECMWF), developments progressed rapidly. After a number of studies on the limited area of the North Sea and the north-east Atlantic with promising results, a global version of the WAM model was running quasi-operationally at ECMWF by March 1987. Surface windfields were obtained from the ECMWF atmospheric model. The reason for the choice of this date was that by mid March a large experimental campaign, measuring two-dimensional wave spectra, started in the Labrador Sea (Labrador Sea Extreme Waves Experiment (LEWEX)). Results of the comparison between observed and modelled spectra were later reported at the final LEWEX meeting by Zambresky (1991). By August 1987, a first version of an altimeter wave-height data assimilation system had already been tested by Piero Lionello, while a number of verification studies on wave-model performance were well underway by the end of 1987. Zambresky (1989) compared 1 year of WAM model results with conventional buoy observations, while Janssen *et al.* (1989) and Bauer *et al.* (1992) compared results with altimeter wave-height data from the Seasat mission and Romeiser (1993) compared with Geosat altimeter data. Meanwhile the WAM model, which originally was a deep-water model with some simple shallow-water effects, was generalized extensively to include bottom and current refraction effects, while the problem of swell dissipation being too strong (as was evident from the comparison studies with altimeter data) was alleviated by modifying the dissipation source term. Finally, extensive efforts were devoted to beautifying the wave-model code and making it more efficient, and in July 1992 the WAM model became operational at ECMWF. By the end of 1994, the WAM model was distributed to more than 75 institutes, reflecting the success of the WAM group. A more detailed, scientific account of all this may be found in Komen *et al.* (1994).

In the meantime, while taking part in the WAM group, I tried to assess the relevance of my findings on the slowing down of airflow by ocean waves. First of all, observational evidence suggested that the drag coefficient C_D increases with wind speed U_{10} . Here the drag coefficient C_D follows from the kinematic stress τ and the wind speed at 10 m height according to $C_D = \tau / U_{10}^2$. The increase of

C_D with U_{10} for airflow over ocean waves is in contrast with the classical results of airflow over a smooth, flat plate. For such a surface, the slowing down of the airflow is caused by viscous dissipation. As a result, since for larger windspeed, and hence larger Reynolds number, the effect of viscosity becomes less important, the drag coefficient decreases with wind speed. Apparently, in the presence of ocean waves there are additional ways to transfer air momentum; an obvious candidate for such a process is the generation of surface waves by wind. This was realized by Charnock (1955) who suggested that the roughness length of airflow over ocean waves should therefore depend on two parameters, namely acceleration of gravity g and friction velocity $u_* = \tau^{1/2}$. Dimensional considerations then gave rise to the celebrated Charnock relation for the roughness length, and, although in the mid 1950s there was hardly any observational evidence, a realistic estimate for the Charnock parameter was given as well. In Charnock's analysis, it was tacitly assumed that the sea state was completely determined by the local friction velocity u_* . However, observations of the windsea state obtained during the Joint North Sea Wave Project (JONSWAP, in 1973) suggested that the shape of the ocean-wave spectrum depends on the stage of development of the sea state or the so-called 'wave age'. In the early stages of development, called 'young' windsea, the wave spectrum showed a very sharp peak while the high-frequency waves were steep. On the other hand, when the sea state approaches equilibrium, the wind waves were less steep and the spectral peak was less pronounced. This led Stewart (1974) to suggest that the Charnock parameter is not really a constant, but should depend on the stage of development of wind waves.

Thus, the work of Charnock and Stewart suggested that wind-generated gravity waves, which receive energy and momentum from the airflow, should contribute to the slowing down of the airflow. In other words, ocean waves and their associated momentum flux may be important in controlling the shape of the wind profile over the oceans. However, the common belief in the field was that air turbulence was dominant in shaping the wind profile while the effect of surface gravity waves was considered to be small (Phillips, 1977). On the other hand, Snyder *et al.* (1981) found that the momentum transfer from wind to waves might be considerable, therefore the related wave-induced stress may be a substantial fraction of the total stress in the surface layer. This turned out to be the case, particularly for 'young' windseas, which are steep. The consequence is that the momentum transfer from air to ocean and therefore the drag coefficient at 10 m height depend on the sea state. The first experimental evidence for this was found by Donelan (1982), and it was confirmed by Smith *et al.* (1992) during the Humidity Exchange of the Sea (HEXOS) experiment.

It therefore seemed natural to combine results of the quasi-linear theory of wind-wave generation with knowledge on the evolution of wind waves, in order to be able

to determine the sea-state dependence of air–sea momentum transfer. Of course, it should be realized that the quasi-linear theory is strictly speaking not valid because, for example, effects of air turbulence on the wave-induced motion are disregarded, and also effects of flow separation are ignored. Nevertheless, I thought it worthwhile to study whether it was possible to obtain, in the context of this theory, realistic estimates of the air–sea momentum transfer. This turned out to be the case. However, results were found to depend in a sensitive manner on the state of the high-frequency waves because these are the fastest-growing waves and therefore carry most of the wave-induced stress. The close relation between aerodynamic drag and the sea state implied that an accurate knowledge of momentum transfer required a reliable determination of the high-frequency part of the spectrum. It turned out that this could be provided by the WAM model.

The consequence was that a reliable knowledge of momentum transfer required the running of a wave model because of the two-way interaction between wind and waves. I therefore started wondering whether the sea-state dependence of the drag would be relevant in other areas of geophysics such as in storm-surge modelling, weather prediction, the atmospheric climate and gas transfer. Although observations (Donelan, 1982) and theory (Janssen, 1989) did suggest an enhancement of drag by a factor of 2 for young windsea, which is quite significant, it appears that the relevance of this wave effect can only be assessed after performing some numerical experiments. One of the reasons for this is that when a change is being made in one part of a complicated system, (unexpected) compensations may occur that are induced by other parts of the system. Consider, as an example, the impact of the sea state on the evolution of a depression. When the wind starts blowing, the young sea state will give an increased roughness which on the one hand may result in an enhanced filling up of the pressure low; on the other hand, however, the enhanced roughness may lead to an increased heat flux which, through vortex stretching, results in a deeper depression. The final outcome can, therefore, only be determined in the context of a coupled ocean-wave–atmosphere model.

To date, a number of studies have shown the relevance of the sea-state-dependent momentum transfer for storm-surge modelling (Mastenbroek *et al.*, 1993), weather prediction (Doyle, 1995; Janssen *et al.*, 2002), the atmospheric climate (Janssen and Viterbo, 1996) and ocean circulation (Burgers *et al.*, 1995). These studies suggest that the modelling of momentum transfer (and also of heat and moisture) can only be done adequately in the context of a coupled model. Ideally, one would therefore imagine one grand model of our geosphere, consisting of an atmospheric- and an ocean-circulation model, where the necessary interface between ocean and atmosphere is provided by an ocean-wave model.

This book is devoted to the problem of the two-way interaction of wind and waves and the possible consequences for air–sea interaction. I therefore start with an

introduction to the subject of ocean waves. First, important concepts and tools such as dynamical equations, the dispersion relation, the role of the group velocity and the Hamiltonian and the Lagrangian for ocean waves are introduced. This is followed by an emphasis on the need for a statistical description of ocean waves by means of the wave spectrum. The evolution equation for the wave spectrum, called the 'energy balance equation', is derived from Whitham's averaged Lagrangian approach. The energy balance equation describes the rate of change of the wave spectrum due to advection and refraction on the one hand and, on the other hand, the rate of change due to physical processes such as wind input, nonlinear interactions and dissipation by white capping. After a brief discussion of advection and refraction I will give a thorough discussion of the energy transfer from wind to ocean waves, the consequent slowing down of the airflow and of nonlinear interactions. This is followed by a brief discussion of the least-understood aspect of wave dynamics, namely dissipation due to white capping.

Next, the role of the various source terms in shaping the wave spectrum is studied, resulting in an understanding of the evolution of the windsea spectrum. At the same time the sea-state dependence of the air-sea momentum transfer is considered and its sensitive dependence on the high-frequency part of the wave spectrum is emphasized.

Because air-sea interaction depends in a sensitive way on the quality of the sea state, the present status of ocean-wave forecasting needs to be addressed. This is done by presenting a validation of ECMWF wave forecast and analysis results against conventional buoy data and against altimeter wave-height data obtained from the ERS-2 satellite.

Having established the role of an ocean waves in the field of air-sea interaction, it is suggested that the standard model of the geosphere, which usually consists of an atmospheric- and an ocean-circulation model, should be extended by means of an ocean-wave model that provides the necessary interface between the two. The role of ocean waves in air-sea interaction is then illustrated by studying the impact of the sea-state-dependent momentum transfer on storm surges, and by showing that ocean waves also affect the evolution of weather systems such as depressions. Finally, ocean waves are also shown to affect, in a systematic manner, the atmospheric climate on a seasonal time scale.

2

The energy balance of deep-water ocean waves

In this chapter we shall try to derive, from first principles, the basic evolution equation for ocean-wave modelling which has become known as the energy balance equation. The starting point is the Navier–Stokes equations for air and water. The problem of wind-generated ocean waves is, however, a formidable one, and several approximations and assumptions are required to arrive at the desired result. Fortunately, there are two small parameters in the problem, namely the steepness of the waves and the ratio of air density to water density. As a result of the relatively small air density, the momentum and energy transfer from air to water is relatively small so that, because of wind input, it will take many wave periods to have an appreciable change of wave energy. In addition, the steepness of the waves is expected to be relatively small. In fact, the assumption of small wave steepness may be justified *a posteriori*. Hence, because of these two small parameters one may distinguish two scales in the time–space domain, namely a short scale related to the period and wavelength of the ocean waves and a much longer time and length scale related to changes due to small effects of nonlinearity and the wind-induced growth of waves.

Using perturbation methods, an approximate evolution equation for the amplitude and the phase of the deep-water gravity waves may be obtained. Formally, in lowest order one then deals with free surface gravity waves while higher-order terms represent the effects of wind input, nonlinear (four-) wave interactions and dissipation. In this manner the problem of wind-generated surface gravity waves (shown schematically in Fig. 2.1) may be solved.

After Fourier transformation, a set of ordinary differential equations for amplitude and phase of the waves is obtained which may be solved on a computer. This approach is followed in meteorology. The reason for its success is that the integration period (between 5 and 10 days) is comparable to the period of the long atmospheric waves. For water waves this approach is not feasible, however, because of the disparity between a typical wavelength of ocean waves (in the range of 1 to

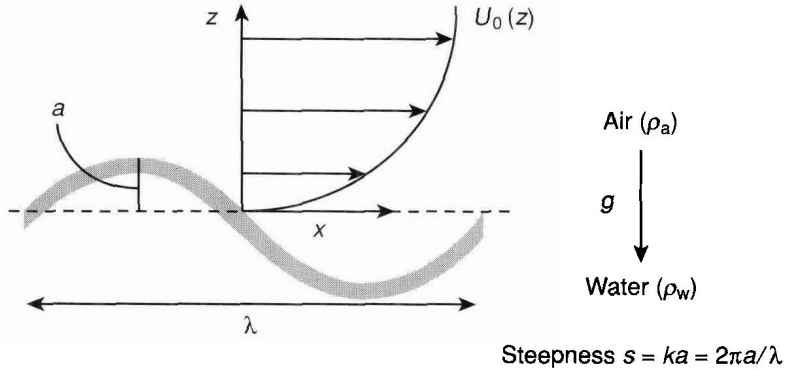


Fig. 2.1. Schematic of the problem in two dimensions.

1000 m) and the size of a typical ocean basin (of the order of 10 000 km). A way of circumventing this problem is to employ a multiple-scale approach. Since there are two scales in the problem at hand, and since the solution for the free gravity waves is known, we only have to consider the evolution of the wave field on the long time and space scale, thus making the wave forecasting problem on a global scale a tractable one.

Furthermore, in practice there is no need for detailed information regarding the phase of the ocean waves. In fact, there are no observations of the phase of ocean waves on a global scale. Usually, we can content ourselves with knowledge about the distribution of wave energy over wavenumber $\mathbf{k}$. In other words, only knowledge of the wave spectrum $F(\mathbf{k})$ is required. A statistical description of the sea state, giving the wave spectrum averaged over a finite area, seems therefore the most promising way to proceed. From the slow time evolution of the wave field it follows that the wave spectrum F is a slowly varying function of time as well. Its evolution equation, called the energy balance equation, is the final result of this chapter. We conclude the chapter by giving a brief overview of our knowledge on observations of wave evolution. This will be accompanied by the introduction of a number of relevant physical parameters, all derived from the wavenumber spectrum F , that are frequently used in the remainder of this work.

2.1 Preliminaries

Referring to Fig. 2.1 for the geometry, our starting point is the usual evolution equation for an incompressible, two-layer fluid consisting of air and water. Consider a fluid with density ρ that flows with a velocity $\mathbf{u}$. In general, density and velocity depend on position $\mathbf{x} = (x, y, z)$ and time t . A right-handed coordinate system is chosen in such a way that the coordinate z points upwards while the acceleration

of gravity $\mathbf{g}$ points in the negative z -direction. The rate of change of the velocity is caused by the Coriolis force, by the pressure, p , gradient, by acceleration of gravity and by the divergence of the stress tensor τ . Denoting the interface between air and water by $\eta(\mathbf{x}, t)$, we then have

$$\begin{aligned} \nabla \cdot \mathbf{u} &= 0, \\ \left(\frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla \right) \mathbf{u} + \mathbf{f} \times \mathbf{u} &= -\frac{1}{\rho} \nabla p + \mathbf{g} + \nabla \cdot \tau, \end{aligned} \quad (2.1)$$

where

$$\rho = \begin{cases} \rho_a, & z > \eta, \\ \rho_w, & z < \eta, \end{cases}$$

and the subscripts a and w refer to air and water respectively. For surface gravity waves, the Coriolis acceleration may be ignored because the frequency of the waves is much higher than the Coriolis parameter $\mathbf{f}$. Velocities and forces, such as the normal and tangential stresses, are continuous at the interface. A particle on either side of the surface, described by $z = \eta(\mathbf{x}, t)$ will move in a time Δt from $(\mathbf{x}, z = \eta)$ to $(\mathbf{x} + \Delta \mathbf{x}, z + \Delta z = \eta(\mathbf{x} + \Delta \mathbf{x}, t + \Delta t))$ with $\Delta \mathbf{x} = \mathbf{u} \Delta t$ and $\Delta z = w \Delta t$. Thus, by Taylor expansion of $z + \Delta z$ and by taking the limit $\Delta t \rightarrow 0$ one obtains the kinematic boundary condition

$$\frac{\partial \eta}{\partial t} + \mathbf{u} \cdot \nabla \eta = w. \quad (2.2)$$

Here, $\mathbf{u}$ is the horizontal velocity at the interface while w is its vertical velocity. In order to complete the set of equations, one has to express the stress tensor τ in terms of properties of the mean flow. The stress contains the viscous stress and in addition may contain contributions from unresolved turbulent fluctuations (the Reynolds stress).

Finally, boundary conditions have to be specified. In deep water one imposes the condition that for $z \rightarrow \pm\infty$ the wave motion should vanish. However, for waves in water of finite depth the normal component of the water velocity should vanish at the bottom.

In order to derive the energy balance equation we shall discuss the properties of pure gravity waves. Thus the following approximations are being made.

- Neglect viscosity and stresses. This gives the Euler equations. Continuity of the stress at the interface of air and water is no longer required. The parallel velocity at the interface may now be discontinuous.
- We disregard the air motion altogether because $\rho_a/\rho_w \ll 1$. In our discussion on wave growth, the effects of finite air–water density ratio are, of course, retained.