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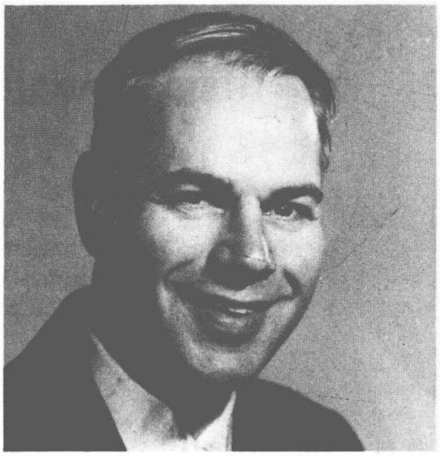
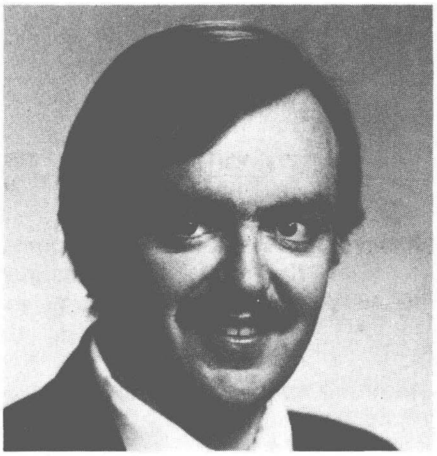
FOREWORD





The Industrial Electronics and Control Instrumentation Society welcomes you to IECI '81 in historic San Francisco. We have changed the format of our Industrial Applications of Mini and Microcomputers Conference by extending the length of time between conferences and by moving the conference to the Bay Area. We hope these changes will make the Conference more accessible to the industrial users of microcomputers, and thus foster greater participation and exchange of technical information.

Our Program Committee has done an excellent job and we hope you enjoy the technical and vendor sessions and panel discussions. We extend our gratitude to the members of the Steering and Program Committees for their dedicated volunteer service to the IECI Society. We would also like to express our thanks to the Authors and Speakers who made this conference possible through their reporting of timely technical aspects of industrial applications of microcomputers.

Finally, we would like to thank the IECI Society for giving us the opportunity to serve as Conference General Chairmen. It has been an honor and privilege we will never forget. We offer each of you a warm invitation to join with us when we again convene this meeting in the Bay Area.

J. David Irwin

H. Troy Nagle, Jr.

Welcome to the 1981 IECI conference with another excellent technical program.

Once again the IECI has received more good papers than can be presented at a three day conference. With three parallel sessions, two sessions per day, and five papers per session, a maximum of 90 papers can be presented. This year we received nearly 150 papers.

Each paper is sent to three reviewers. This year the reviewers included IECI Program Committee members in Japan and Canada. Following receipt of the reviewers comments, there is a meeting of the Program Committee, which makes the final accept or reject decision. This year there were seven members attending this meeting. This committee also assigns the papers to the appropriate technical session and determines the timing for each session.

The most critical task required for an excellent technical program is reviewing the papers and the IECI is interested in locating persons who are willing to review papers in their area of expertise. Contact R. Begun, FMC Corporation, MD 070, 1105 Coleman Avenue, P.O. Box 1201, San Jose, California 95108, Area Code (408) 289-2728.

This year, Victor Huang and I shared the Technical Program Chair. Sharing the workload was much appreciated. Working with professionals such as Victor and the other members of the Program Committee was very rewarding.

R. A. Begun
1981 IECI Technical Program Co-Chairman

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*Manuscript not available for publication.

AN APPLICATION OF A MINI-COMPUTER TO A COMPUTER NUMERICAL CONTROL SYSTEM OF A MACHINE TOOL

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Abstract

A control system of a machine tool is presented in which a mini-computer performs numerical control, servo control and adaptive control for metal cutting process.

A method to decrease computing time of interpolation algorithm of numerical control is discussed. A long linear path is divided into small segments. Each segment is interpolated by using a single precision multiplication instead of double precision multiplications. A circular arc is also approximated by small linear segments. A center angle to each segment is set at 2^{-C_R} (C_R:integer) in order that co-ordinates of end points of the segment are calculated by shifting data on arithmetic registers instead of multiplications.

It is verified that the accuracy of a tool path and the ratio of computing time of the interpolation to total CPU time are improved.

By using the CPU time saved by the method, compensation for irregularity of the feedrate of the servo, and adaptive control varying the feedrate according to the change of cutting conditions are executed.

1. Introduction

In this paper, an application of a mini-computer to a numerical control system is presented. In the system, the computer is used to perform numerical control, servo control, and adaptive control for metal cutting process as shown in Fig.1.

How to compose interpolation algorithm which generates a tool path in numerical control is a big problem, because the algorithm frequently requires computation with extreme accuracy using double precision data and, that is, a large potion of CPU time is occupied by the interpolation algorithm.

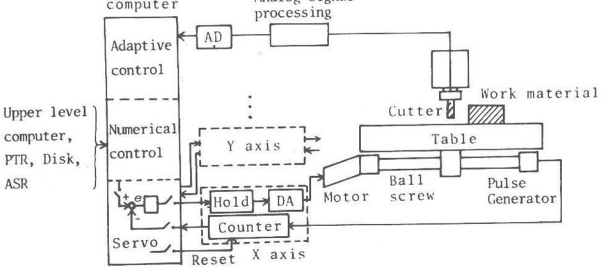


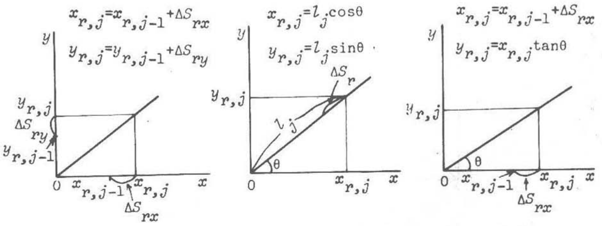
Fig.1. A CNC system with software servo mechanisms and an adaptive control system for milling process.

Ordinal interpolation methods as like as DDA (Digital Differential Analyzer) generates only one pulse to a servo at every calculation (one pulse corresponds to one unit length of position of the servo), therefore, the calculation of interpolation must be executed with very short cycle time in order to get high feedrate of work material (speed of the table on which work material is). Y.Koren improved the algorithm of DDA composed of software.[1] But, the algorithm still occupies a large portion of CPU time.

Then, a multiprocessor system has begun to be used as a numerical controller in which a micro processor is allotted exclusively for interpolation lately.[2] But, the design of the multi processor system is not so easy.

Another approach to decrease the computing time of interpolation is to use software servo system as shown in Fig.1. In this case, it is admissible that interpolation is calculated with long sampling period, because the sampling period of servo control is ordinarily long, and produces output pulses in a lamp. Ordinarily the co-ordinates on the path are calculated by the additions of increments ΔS_{rx} , ΔS_{ry} as shown in Fig.2(a).[3] But, the method is not convenient for adaptive control because ΔS_{rx} , ΔS_{ry} must be calculated by time consumable multiplications of trigonometric functions with accuracy more than double precision (32bits) in order to keep the accuracy of the tool path when the feedrate is varied by adaptive control.

A.E.Middleditch presented another sampling interpolation algorithm[4] to vary the feedrate easily in which co-ordinates on the path are calculated by a double precision addition and multiplications (double precision datum: l × single precision data: $\cos\theta$, $\sin\theta$) as shown in Fig.2(b). But, it needs compensation for the path error due to single precision expression of $\cos\theta$ and $\sin\theta$.



(a) (b) (c)

Fig.2. Linear interpolation in sampling method.

The co-ordinates of the point on a circular arc are calculated from following equations,

$$\left.\begin{aligned} X^*_{i+1} &= (\cos \Delta\varphi)^* \cdot X^*_i - (\sin \Delta\varphi)^* \cdot Y^*_i \\ Y^*_{i+1} &= (\cos \Delta\varphi)^* \cdot Y^*_i + (\sin \Delta\varphi)^* \cdot X^*_i \\ &\Delta\varphi\text{: rotational angle at the center} \end{aligned}\right\} \quad (1)$$

A.E.Middleditch also presented a method to decrease the computing time of circular interpolation by the way to represent $\cos\Delta\varphi$, $\sin\Delta\varphi$ with single precision datum.4)

In this paper, more effective methods to decrease computing time of sampling interpolation algorithm and to increase the accuracy of a tool path are presented. A long linear path is divided into small segments. Each segment is interpolated by using single precision multiplication:single precision datum × single precision datum. A circular arc is also approximated by linear segments. Co-ordinates of end points of each segment are calculated by Eq.(1). In order to avoid multiplications in Eq.(1), center angle $\Delta\varphi$ to each segment is set at 2^{-C_R} rad; C_R :integer. Eq.(1) is computed by shifting of X^*_i and Y^*_i . Each linear segment is interplolated by the upper mentioned method.

The path error of interpolation algorithm is analyzed and the ratio of CPU time is compared with other methods.

By using the algorithm, the computer distributes enough time for servo control and for adaptive control. Compensation for a stick slip phenomenon in servo control is presented, and adaptive control to vary the feedrate according to the change of cutting conditions is presented in this paper.

2. Linear Interpolation and its Path Accuracy

2.1 Preprocessing of linear interpolation

Before interpolation is carried out, the commanded path is divided into short segments as shown in Fig.3. The length X_Δ of the projection of each segment on the co-ordinate axis the nearest to the direction of the feedrate (x axis in the case of Fig.3) is represented with a single precision datum as follows,

$$\left.\begin{aligned} &X_\Delta = 2^{B-1-C_L}\Delta X \\ &\Delta X\text{:unit length of control} \\ &C_L\text{:integer to adjust the length } X_\Delta \\ &B\text{:word length} \end{aligned}\right\} \quad (2)$$

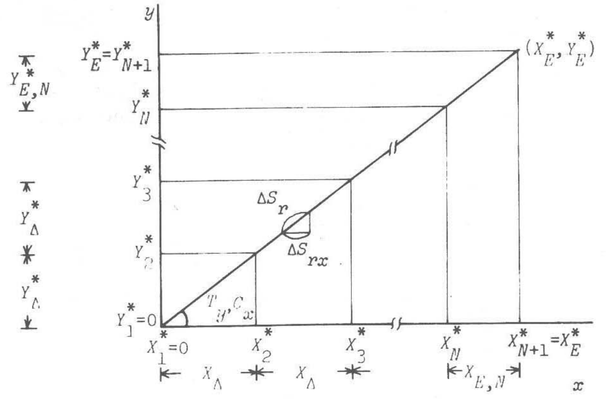


Fig.3. Division of a linear path.

The length of the projection of each segment on the other axis, Y_Δ^* in the case of Fig.3, is also calculated. In this paper, symbols $*$, $\#$ on the shoulder of variables mean that the vaiables are to be expressed with double precision datum.

Tangent and cosin (T_y and C_x in the case of Fig.3) of the angle between the direciton of feedrate and the co-ordinate axis the nearest to the direction are also calculated with single precision accuracy. Fig.4 shows the relation among X_Δ , ΔX , $\Delta X'$ (the length expressed by the LSB of double precision datum), Y^*_Δ and T_y , where accuracies of T_y and Y^*_Δ are 2^{-B} and $\Delta X'/2$ respectively.

2.2 Processing at a point dividing the path

When the interpolation of the segment between the point (X^*_{i-1} , Y^*_{i-1}) and the point (X^*_i , Y^*_i) is finished, the next y co-ordinates Y^*_{i+1} , round value $Y^\#_{i+1}$ of Y^*_{i+1} , and co-ordinate increments of the following segment $X_{E,i}$ and $Y_{E,i}$ are calculated (in the case that x axis is the nearest to the direction of the movement of the work material as shown in Fig.3). A value y^*_0 which compensates the error between Y^*_i and $Y^\#_i$ and makes a round off instead of a truncation at a calculation of interpolation is also computed at this stage.

$$y^*_0 = Y^*_i - Y_i^\# + 0.5\,\Delta X \quad (3)$$

2.3 Calculation of linear interpolation

At first, the co-ordinate value (on the co-ordinate axis being the nearest to the direction of the movement of work material : x axis in the case of Fig.3) of the point on the path is calculated as follows(ref. Fig.5),

$$x^*_{r,j} = x^*_{r,j-1} + \Delta S_x \quad (4)$$

ΔS_x is increment of x co-ordinates for the sampling period and is re-calculated at every time when the feedrate is varied by adaptive control or acceleration decceleration control. The accuracy of ΔS_x does not cause the path error as mentioned in 2.4, therefore, it is not necessary to express ΔS_x with double precision datum. ΔS_x is easily calculated by multiplication of single precison data C_x to the feedrate.

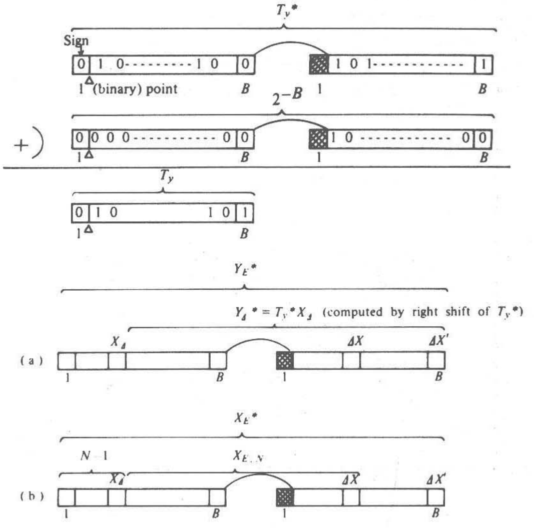


Fig.4. Relation among X_Δ, ΔX, $\Delta X'$, Y^*_Δ and T_y.

The other co-ordinates $y^*_{r,j}$ calculated as follows,

$$y^*_{r,j} = T_y \cdot x_{r,j} + y^*_0 \tag{5}$$

where $x_{r,j}$ is a variable with digits below $\varDelta X$ of $x^*_{r,j}$ truncated.

The outputs of the interpolation algorithm, that is, the inputs of the servo control algorithm are given as follows,

$$\varDelta x_{s,j} = x_{r,j} - x_{r,j-1}, \quad \varDelta y_{s,j} = y_{r,j} - y_{r,j-1} \tag{6}$$

where, $y_{r,j}$ is a variable with digits below $\varDelta X$ of $y^*_{r,j}$ truncated. But, the addition of $y_0{}^*$ to $T_y \cdot x_{r,j}$ in Eq.(5) makes $y_{r,j}$ the round value of $T_y \cdot x_{r,j}$ and removes the effect of the round off error contained in $Y^\#_i$.

By the upper mentioned method, the linear interpolation is done with two double precision additions and only one multiplication of single datum to single datum, therefore, the computing time of interpolation is made shorter than those of other methods.

2.4 Path error of linear interpolation

A path error ε_L is defined as the distance of the shift of the point ($x_{r,j}$, $y_{r,j}$) normal to the desired path. In the presented method, the truncation error of $x_{r,j}$ to $x^*_{r,j}$ does not affect the path error, and the error of $y_{r,j}$ due to the single precision expression of T_y does not affect the path error so much either because the x co-ordinates $x_{r,j}$ is made small by the dividing of the path.

But, a small path error is generated due to errors contained in $Y^\#_i$, T_y and a round off error at calculation of $y_{r,j}$. The range of the path error is given as follows,

$$|\varepsilon_L| \leq C_x \left(2^{2M-3B+2+C_L} + 2^{-C_L-1} + 2^{-1} \right) \varDelta X \equiv \varepsilon_{L,\max} \tag{7}$$

Fig.6 shows the relation between $\varepsilon_{L,\max}$ and index C_L, when C_L represents the fineness of the division of the path. The path error becomes the maximum when the angle of the movement is near to 0°, but not 0°. The path error is large at the part of small C_L, because the divided segment length becomes large with small C_L, and then the error of T_y which is multiplied by the x co-ordinates of the segment influences largely on it.

The path error decreases according to the increase of C_L. It has the minimum value at a value of C_L. After that, it increases according to the increase of C_L because the number of additions of $Y_\varDelta$ increases according to the increase of C_L.

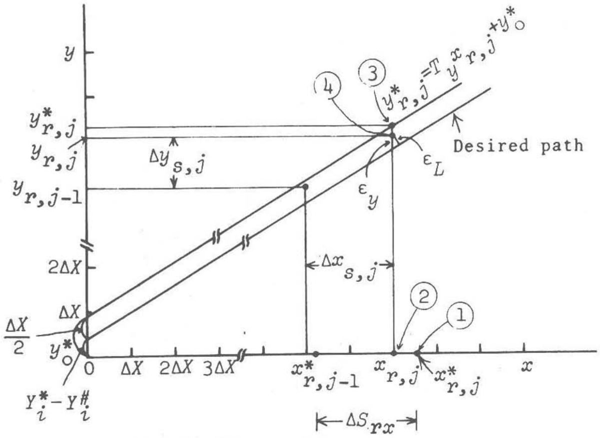


Fig.5. Linear interpolation.

The path error when the angle of the tool movement is nearly 45° but not 45° is about $1/\sqrt{2}$ of the path error at angle of nearly 0°. The path error of the method presented in Fig.2(b) when the angle is near 45° is $\sqrt{2}$ times as large as that of the method presented in this paper when the angle is near 0°. The path error of DDA is equal to that of the method presented in this paper when C_L is equal to 0.

As the result, it may be said that the path error of the method presented in this paper with adequate value of C_L is smaller than that of other mehtods.

3. Circular Interpolation and its Path Error

3.1 Preprocessing of circular interpolation

In this paper, a circular arc is approximated with linear segments as shown in Fig.7. Each segment is interpolated by the upper mentioned linear interpolation method. In order to shorten the time to compute the co-ordinates (X^*_i , Y^*_i) at the end of interpolation of each segment, the angle $\varDelta\varphi$ at the center of the arc corresponding with each segment is selected as follows,

$$\left.\begin{array}{l} \varDelta\varphi = \pm 2^{-C_R} \\ +\text{:counter clockwise rotation} \\ -\text{:clockwise rotation} \\ C_R\text{:integer} \end{array}\right\} \tag{8}$$

The maximum path error due to the approximation of the arc with linear segment is,

$$\left.\begin{array}{l} \varepsilon_\varDelta = \pm 2^{-2C_R-4} R^* \\ R^*\text{:radius of the arc} \end{array}\right\} \tag{9}$$

The co-ordinates (X^*_1 , Y^*_1) of the starting point of the first segment have to be calculated by using $\varepsilon_\varDelta$ before the interpolation is carried out.

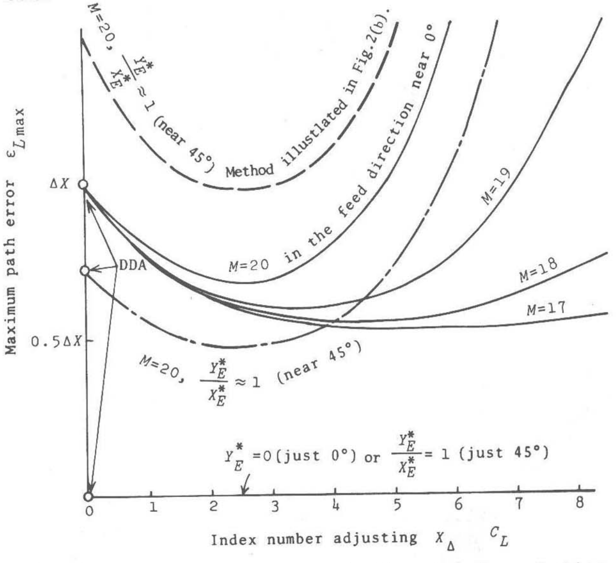


Fig.6. Maximum path error of linear interpolation.
$X_\varDelta = 2^{B-1-C_L} \varDelta X$, $X^*_{E,\max} = (2^M - 1)\varDelta X$.

3.2 Coarse interpolation

At the end of interpolation of a linear segment, co-ordinates of the end point of the following linear segment is computed from a equation similar to Eq.(1).

Several ways to approximate trigonometric functions have been analyzed in order to decrease the time and the error of computation to get co-ordinates of a point on a arc.[1,4,5] Here cos and sin are approximated as follows,

$$\cos \Delta\varphi \fallingdotseq 1 - \Delta\varphi^2/2, \quad \sin \Delta\varphi \fallingdotseq \Delta\varphi\,(1 - \Delta\varphi^2/8) \qquad (10)^{4)}$$

By substituting Eqs.(8),(10) into Eq.(1), the equation to compute the co-ordinates (X^*_{i+1} , Y^*_{i+1}) is given as follows,

$$\left.\begin{aligned} X^*_{i+1} &= X^*_i \mp 2^{-C_R}\left[Y^*_i \pm 2^{-C_R-1}\left\{ X^*_i \mp 2^{-C_R-2}(Y^*_i + 2^{C_R+1}\Delta X') + 2^{C_R}\Delta X' \right\} + 2^{C_R-1}\Delta X' \right] \\ Y^*_{i+1} &= Y^*_i \pm 2^{-C_R}\left[X^*_i \mp 2^{-C_R-1}\left\{ Y^*_i \pm 2^{-C_R-2}(X^*_i + 2^{C_R+1}\Delta X') + 2^{C_R}\Delta X' \right\} + 2^{C_R-1}\Delta X' \right] \end{aligned}\right\}$$

±,∓:upper side;counter clockwise rotation
lower side;clockwise rotation

····(11)

All coefficients of Eq.(11) are exponential functiuons of 2. Then, multiplications of these coefficients to variables X_i^*, Y_i^* are computed by just right shifts of X_i^*, Y_i^* on the registers of the CPU in stead of time consumptive multiplication operations of double precision data. Additions of $2^{C_R+1}\Delta X'$, $2^{C_R}\Delta X'$, $2^{C_R-1}\Delta X'$ are done in order each corresponding right shift. The additions change the truncation error with shift into round off error whose expected value is zero. Then, the expected value of the cumulative path error produced by repetitions of Eq.(11) is able to be kept zero. Round off errors $\varepsilon_{x,i}$ and $\varepsilon_{y,i}$ produced in X^*_{i+1} and Y^*_{i+1} respectively by the calculation of Eq.(11) are in the following range,

$$|\varepsilon_{x,i}|,\ |\varepsilon_{y,i}| \le \varepsilon_{xy} \equiv (2^{-1} + 2^{-C_R-1} + 2^{-2C_R-2})\,\Delta X' \qquad (12)$$

Compensation constant y^*_0 used in Eq.(5) for the linear interpolation is given as follows (ref. Fig.8),

$$y^*_0 = -(X^*_i - X^{\#}_i)\,T_y + Y^*_i - Y^{\#}_i + 0.5\,\Delta X \qquad (13)$$

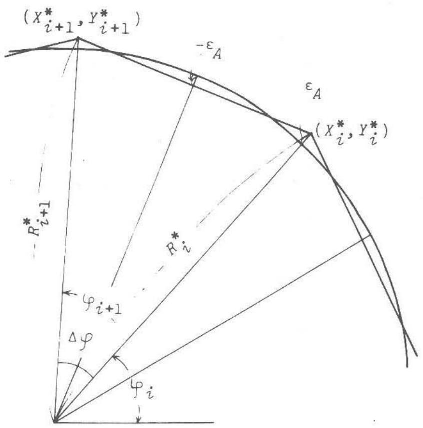


Fig.7. Path error of linear approximation of a circular arc.

3.3 Path error of circular interpolation

The path error (radius error) in circular interpoaltion is generated by the approximation of a circular arc with linear segments, the error of coarse interpolation (error of Eq.(11)) and the error of linear interpolation of each segment.

Radius R^*_{i+1} of the terminal of a segment to the center of the arc is given as follows from the coarse interpolation Eq.(11),[4]

$$\left.\begin{aligned} R^*_{i+1} &\fallingdotseq R^*_i + \frac{2^{-6C_R}}{128} R^*_i \\ &\quad + (\cos\varphi_i \mp 2^{-C_R}\sin\varphi_i)\,\varepsilon_{x,i} + (\sin\varphi_i \pm 2^{-C_R}\cos\varphi_i)\,\varepsilon_{y,i} \end{aligned}\right\}$$

φ_i:ref. Fig.7
∓,±:upper side;counter clockwise rotation
lower side;clockwise rotation

····(14)

The second term of the right hand side of Eq.(14) is due to the approximation of cos and sin in Eq.(10). The path error due to this error has the maximum value when interpolation is carried out around a whole circle and the maximum value is given as follows,

$$\pi \cdot 2^{C_R+1} \cdot 2^{-6C_R-7} \cdot R^* = \pi \cdot 2^{-5C_R-6} \cdot R^* \qquad (15)$$

The third and the forth terms in the right hand side of Eq.(14) are due to the round off errors. The path error caused by these errors becomes the maximum when a whole circle is interpolated and the maximum path error is given as follows,

$$\left.\begin{aligned} &\sum_{i=1}^{N} \pm\varepsilon_{xy}\left[(\cos\varphi_i \mp 2^{-C_R}\sin\varphi_i)\,\mathrm{sgn}(\cos\varphi_i \mp 2^{-C_R}\sin\varphi_i) \right. \\ &\left. + (\sin\varphi_i \pm 2^{-C_R}\cos\varphi_i) \times \mathrm{sgn}(\sin\varphi_i \pm 2^{-C_R}\cos\varphi_i) \right] \\ &\fallingdotseq \pm 2^{C_R+3}\,\varepsilon_{xy} \end{aligned}\right\} (16)$$

The path error generated by the linear interpolation of segments approximating the arc is due to mainly round off error (for example, the error between $y^*_{r,j}$ and $T_y \cdot x_{r,j}$ in Eq.(5)) because the length of segments is short enough to neglect the effect of the error of T_y which is a problem in the case of linear interpolation.

The range of the total path error around the whole circle is given from Eqs.(15),(16) as follows,

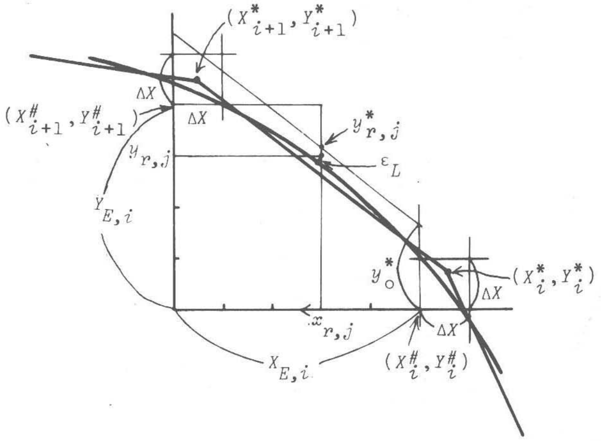


Fig.8. Linear interpolation approximating a circular arc.

$$|\varepsilon_R| < \left\{ 2^{M-2C_R-4} + \pi \cdot 2^{M-5C_R-6} + 2^{M-2B+C_R+4}(1+2^{-C_R}+2^{-2C_R-1}) + 0.5|\cos\varphi_i|) \right\} \Delta X \quad (17)$$

Dot-dash lines in Fig.9 shows the maximum path error vs. the index C_R of the angle $\Delta\varphi$ at the center. C_R means the fineness of the division of the arc. M is the index of coefficient to represent the allowable maximum radius $R^*_{\max}=(2^M-1)\Delta X$ to be commanded. The minimum value of the maximum path error with $M=20$($R^*=1.049$m; $\Delta X=1$um) is about $3\Delta X$ at $C_R=9$ as shown in Fig.9. It might be thought that the value is not small enough for practical use. But, another result is introduced from stochastic characteristics of the path error. It is possible to compute the 0.01 % confidence limit $\varepsilon_{RC0.01}$ of the path error when a whole circle is interpolated,

$$\varepsilon_{RC0.01} \doteqdot \left\{ 2^{M-2C_R-4} + \pi \cdot 2^{M-5C_R-6} + 3.2\sqrt{\pi}\, 2^{M-2B+0.5C_R+1}(1+2^{-C_R}+2^{-2C_R-1}) + 0.5|\cos\varphi_i| \right\} \Delta X \quad \cdots\cdots (18)^*$$

The solid line in Fig.9 shows the confidence limit. The probability that the path error goes over the line is 1/10000. The line is below ΔX in the range: $C_R=9\sim15$. The probability to have a dangerous path error is small enough, then there is no problem of the practical use of the method presented in this paper.

The optimum value of C_R depends on the radius of the arc. For example, the path error has a minimum at $C_R=11$ for the radius $R^*=(2^{20}-1)\Delta X$ ($R^*=1.049$m for $\Delta X=1$um) and at $C_R=7$ for $(2^{10}-1)\Delta X$ (1.024mm) as shown with the solid line with $M_R=10$ on Fig.9.

Circular interpolation by DDA is considered as a kind of calculation of Eq.(1) in which $\cos\Delta\varphi$ and $\sin\Delta\varphi$ are approximated as $\cos\Delta\varphi \doteqdot 1$ and $\sin\Delta\varphi \doteqdot \Delta\varphi$,1) therefore, it may produce a larger path error than the method presented in this paper. Y. Koren's DDA method that is improved to decrease the path error produces the path error of ΔX when a half circle is interpolated.1)

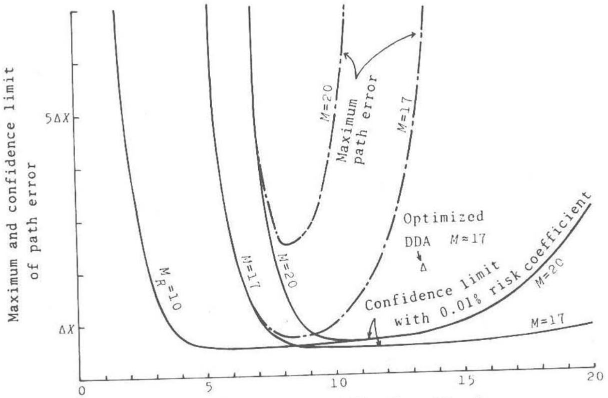


Fig.9. Maximum and confidence limit of path error of circular interpolation around a complete circle.

4. Time Occupation Rate of Interpolation

Fig.10 shows the relation between the time occupation rate of interpolation (computing time of interpolation/total computing time) and the feedrate (the speed of the table). The time occupation rate of various kinds of interpolation methods are compared.** The sampling period of servo control and interpolation may be chosen in the range of 1∼50ms according to the dynamic characteristics of the servo. Here 2ms is selected as the sampling period.

Computation of the ordinal DDA interpolation should be done with a period of $\Delta X/S_{r\max}$ ($S_{r\max}$: the maximum feedrate). The computation of Koren's DDA interpolation is done with a period of $\Delta X/S_r$ (S_r: feedrate), but the judgement if the computation of interpolation should be done or not must also be done with the period of $\Delta X/S_{r\max}$. When these DDA methods are used, the feedrate is limited as shown in Fig.10.

The sampling interpolation method presented in this paper has lower time occupation rate than both DDA method and Middleditch's sampling interpolation method.

The increase of the time occupation rate according to the increase of one linear interpolated axis in servo system is 0.04 in the case of the method presented in this paper, which is smaller than 0.174 of Middleditch's method.

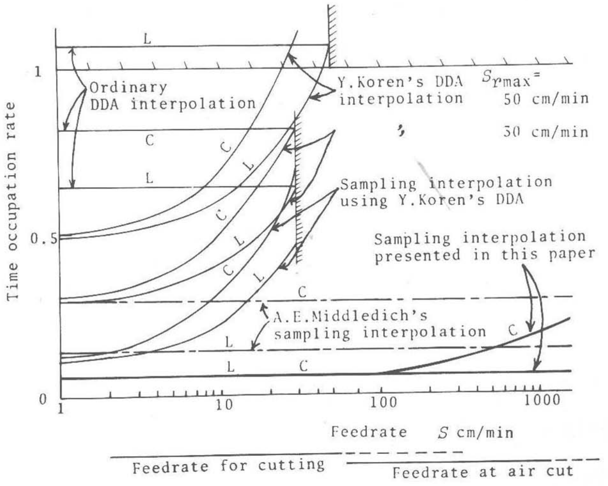


Fig.10. Time occupation rate of interpolation.
L:Linear interpolation, $X^*_{E,\max}=2^{20}\Delta X$,
C:Circular interpolation, $R^*_{\max}=2^{20}\Delta X$,
$\Delta X=1\mu$m.

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* The confidence limit is calculated mainly from the varience σ_N of the cumulative path error caused by round off errors as shown in Eq.(12).4)

$$\sigma_N{}^2 = \frac{1}{(2\varepsilon_{xy})^{2N}} \int_{-\varepsilon_{xy}}^{\varepsilon_{xy}} \cdots \int_{-\varepsilon_{xy}}^{\varepsilon_{xy}} \left[\sum_{i=1}^{N} \{ (\cos\varphi_i \mp 2^{-C_R}\sin\varphi_i)\varepsilon_{x,i} + (\sin\varphi_i \pm 2^{-C_R}\cos\varphi_i)\varepsilon_{y,i} \} \right]^2 d\varepsilon_{x,1}\, d\varepsilon_{y,1} \cdots d\varepsilon_{x,N}\, d\varepsilon_{y,N} \doteqdot 2^{C_R+1}\pi \frac{\varepsilon_{xy}{}^2}{3}$$

** Mini-computer NEAC3200-30 is used for the composition of controller. Its calculation speed is as follows; memory cycle time: 1.96μs, simple precision addition: 3.2μs, simple precision multiplication: 8.8μs, double precision addition: 4.8μs, a shift of data on registers:0.6μs, double precision multiplication using software: 114μs, interrupt with storing and restoring of data on arithmetic registers: 40μs.