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Evaluation of Mineral Reserves
A Simulation Approach

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Preface

This book was initiated by a survey that indicated the limitation of the 1990s literature with regard to published reports of applications of stochastic simulation in the mining industry. Many studies have used numerically simulated grade distributions to evaluate dilution factors, but almost none were published, because of lack of time or perhaps dedication. This statement may fire up those mining geostatisticians who are doing innovative work but did not care about publishing. They owe it to their ingenuity, to their clients if they are consultants, and to their students if they are teachers to document their thoughts and share their experience through publications with substance. Science progresses with wide open disclosure and debate, not with confidential notes or communications to forgettable forums, and certainly not with patented tradecrafts.

This book (re)introduces the concept of stochastic simulation as applied to evaluation of mineral reserves, more specifically evaluation of the impact of the support and information effects on mining dilution. The support effect relates to the difference in size between the volumes of the ore/waste units and the much smaller volumes of rock actually sampled that provide the sample data. The information effect relates to the difference in data quality and quantity between the time of reserves evaluation and that of grade control and actual mining. There are many other aspects of mine planning that affect the recoverable reserves, most prominently data accuracy and correct geological interpretation; these aspects are not covered in this book. Our goal is not to attempt a definitive essay on the science and art of mineral reserves evaluation, but to (re)introduce the stochastic simulation toolkit that provides a numerical framework for estimating many of the factors affecting mining dilution and reserves recovery.

Perhaps a good analogy to convey the potential of simulation is that of a wind tunnel where various designs of an aircraft wing are tested for resistance to turbulence. Our "wind tunnel" is a computer that stores various realizations of the spatial variability of mineral grades; our "aircraft design" is the suite of data acquisition, mine planning, and ore grade control processes that affect profitability; our "turbulence" is the set of unknowns: geological setting, grades, mining dilution, and so forth; and our "resistance" is the robustness of our planned mining operation in the face of all the above uncertainties. Stochastic simulation of mineral grades distribution followed by simulation of mining recovery processes has the potential of testing the adequacy of a mining project and tuning some of its parameters before committing a lot of time and money. The cost of conducting such a simulation is so minute compared to that involved in developing a new mining venture. We therefore submit that banks and regulatory agencies should require such an exercise before delivering any green light.

Chapter I introduces the aspects of mining dilution that will be addressed with the simulation approach developed in Chapter II. To provide a yardstick for comparison, the traditional volume-variance correction for difference of support volumes is recalled in Chapter III. Short of an exhaustive "real" mining data set, two reference data sets have been generated from actual quasi-exhaustive topographic data in Chapter IV, and the ore selection process has been mimicked on both of them. We have tried to remain as general as possible, documenting methodology rather than presenting real case studies; this explains our decision not to introduce explicit grade units. In Chapter V, the proposed simulation approach is applied to the previous reference data sets. The availability of an exhaustive reference, which is sparsely sampled to mimic actual mining data, allows comparison of predictions with actual reference values. In Chapter VI, the volume-variance correction algorithm is applied to the same sample data used in the simulation approach. In Chapter VII, some conclusions and recommendations are suggested.

All the data (reference and sampled) used in the case studies of this book can be downloaded over the Internet from the following Web site:

`http://ekofisk.stanford.edu/SCRFweb/mineral_reserves/index.html`

All kriging and stochastic simulation programs used can be found in the Geostatistical Software Library.

Finally, we would like to acknowledge the many friends who reviewed the numerous preliminary drafts of this book: Clayton Deutsch, Roussos Dimitrakopoulos, Jaime Gómez-Hernández, Pierre Goovaerts, Sia Khorowshahi, Ricardo Olea, Harry Parker, and Mohan Srivastava. This book could not have been written without their help; their numerous constructive criticisms indicate that the book shortcomings are ours only.

Contents

EVALUATION OF MINERAL RESERVES

Chapter I

Introduction

The problem of evaluation of future recoverable reserves is critical to the mining industry, in that reserves condition both investment and profitability associated with any mining venture. Recoverable reserves relate to a specific area/volume of the deposit, delineated by a mining plan to be economically accessible. This is as opposed to geological (in situ) resources, which refer to the amount of mineral available in the general area to be mined irrespective of whether this amount can be recovered (mined out) economically.

Statistical techniques for the evaluation of in situ resources on a panel-by-panel basis were proposed some 50 years ago by Krige (1951) and Sichel (1952). These pioneering works were later developed formally by Matheron (1962) into what is known as geostatistics, the application of which has now extended much beyond the mining arena. A few years later, Koch and Link (1970–71) and Davis (1973) published their seminal works on statistical analysis of geological data.

The difference between resources and reserves in geostatistical terms was clearly established by David (1977). A year later, Journel and Huijbregts (1978) proposed a formal approach to the problem of estimation of open pit reserves, addressing the respective influence of:

(i) The support of selection: ore is mined out as selective mining units of volumes vastly larger than the volumes of cores or composite grade data available at the time of the prediction study.

(ii) The information available at the time of selection, which is usually different from that available at the time of prediction.

This book addresses solely the two previous "predictable" dilution factors: support and information effect. It does not address unpredictable dilution and ore loss caused by numerous imponderables, such as geotechnical or hydrogeological conditions, the skill of miners in following diglines, political turmoil, and so forth. In any future reconciliation exercise, it is essential to distinguish between deviations from a mine plan resulting from predictable factors and those resulting from unpredictable factors.

A selective mining unit (SMU) is the smallest volume of ore that can be practically segregated from waste. The SMU is usually represented as a rectangular volume and, as such, is very tractable for numerical computations. In most deposits, however, ore selection is made using irregular areas/volumes that may be significantly larger than the SMU. The previous SMU definition implies that the volume recovered as ore will be an aggregate of SMUs; the definition of that volume cannot have resolution finer than the SMU size. In an open pit operation, the SMU could be the volume of influence of two to four blastholes.

One important aspect of reserves estimation, recognized but not addressed by the above early formalisms, was the "accessibility effect": a high-grade SMU may not be recoverable if the costs to reach it or to remove it selectively are prohibitive. In other words, there is rarely free selection of each SMU independent of the other SMUs which surround it. The amount of recoverable reserves depends on the mine plan (pit design, mine layout), which itself depends on the spatial distribution of mining unit grades.

The early solutions proposed by David (1977) and Journel and Huijbregts (1978) are still used for the evaluation of reserves some 25 years later. Except from a few notable works, for example, Journel (1974), Deraisme and Dumay (1979), Dagbert (1980), Kim et al. (1982), Chilès (1984), Deraisme et al. (1984), Journel and Isaaks (1984), Ravenscroft (1992), Nowak et al. (1993), Srivastava (1994a, 1994b), Guardiano et al. (1995), Caers et al. (1997), Dowd (1997), Glacken (1997), Nowak and Srivastava (1997), Dimitrakopoulos (1998), and Rossi (1999), the solution provided by conditional stochastic simulation is not widely used or is poorly published (as of 2003). In contrast, usage of conditional simulations in the petroleum industry and environmental sciences is better documented; see for example, Yarus and Chambers (1995), Baafi and Schofield (1997), Goovaerts (1997), Goméz-Hernández et al. (1999), Chilès and Delfiner (1999), Deutsch (2002), and the numerous references listed therein.

Mine planning is still typically performed on estimated maps of the grade spatial distribution; these maps provide a smooth and often biased image of the actual spatial distribution of the future SMU grades. Estimation of reserves is done either before mine planning, assuming free selection of each SMU, or after mine planning within the mine layout and again assuming free selection. A mine dilution/recovery factor is then applied to the estimated reserves. In fact, reserves evaluation and mine planning are intertwined issues, posing a difficult optimization problem (see Kennedy, 1990; Dowd, 1994). Last but not least, the traditional practice does not provide any formalism for assessing the uncertainty related to reserves estimation.

This book aims at presenting the conditional simulation toolkit, which allows generating alternative numerical models of the spatial distribution of grades, accounting for both the support effect and the information ef-

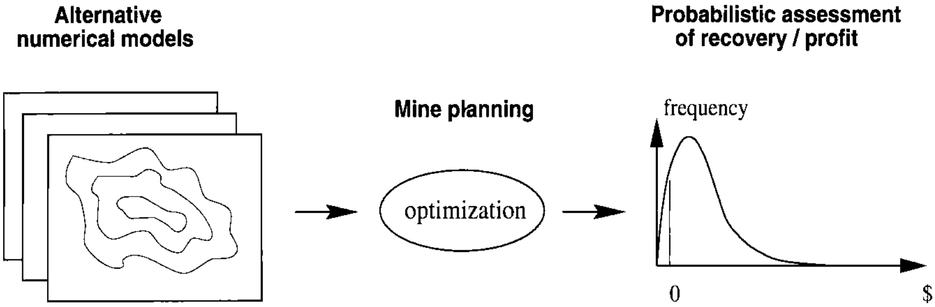

Figure I.1: Mining and recovery evaluation from alternative equiprobable numerical models of the grade(s) spatial distribution. An optimal mine plan and evaluation of corresponding recovery is applied to each numerical model, resulting in a probabilistic distribution of recovery (tonnage, metal, and profit).

fect. SMUs are selected on the basis of future grade estimates, the statistics of which are different from those of the presently available core grades. These numerical models are stochastic, in the sense that they are many, all "equiprobable", or more precisely, equally likely to be drawn. The difference between these alternative models reflects the uncertainty that is left once they are conditioned to the data available at the time of the prediction study. As opposed to maps of estimated values, conditional simulation provides images of the grade spatial distribution that are not smoothed. Such smoothing generally leads to underestimation of the proportion of SMUs with extreme high grades, and overestimation of the proportion of SMUs with low grades. Mine planning aimed at optimizing recoverable reserves and profit can be performed on each (or a selected few) of these alternative numerical models, resulting in not one but a distribution (histogram) of recoverable reserves and expected profit values, thus providing a measure of uncertainty (see Fig. I.1).

The difficult optimization problem underlying the joint definition of an optimal mine layout and the evaluation of the corresponding reserves is not addressed in this book. This problem is not likely to have a general solution: the solution will depend on the type of mine (e.g., underground vs. open pit) and specific technological constraints (e.g., rock mechanics, ventilation, dewatering, stockpiling). In all cases, however, conditional simulations provide a better basis (more realistic numerical models) to perform such an optimization (see Deutsch et al., 2000). For a perspective of the general problem of mine planning and reserves evaluation, readers may refer to King et al. (1982), David (1988), Kennedy (1990), Ravenscroft (1993), and Farrelly and Dimitrakopoulos (2002).

This book addresses the limited problem of evaluation of reserves within a given panel, under the assumption of free selection. This will be done using conditionally simulated models of the grade spatial distribution, and the results will be compared to predictions obtained using routine present-day geostatistical methodology.

The limitation of free selection within a given panel is necessary because, again, evaluation of reserves under technical constraints is an optimization problem specific to each set of such constraints. On a limited number of such panels, recovery under free selection can be simulated; in addition, the lesser recovery under specific technical constraints can be evaluated (see Norrena and Deutsch, 2001). The ratio of these two recovery figures would provide an assessment of these predictable aspects of mining dilution, to be applied to all other recovery figures obtained under free selection.

Section I.5 at the end of this chapter indicates the scope of this book and the spectrum of readers who may benefit from it.

I.1 Statement of the Problem and Notation

Selective mining operates on blocks or selective mining units (SMUs). The domain informed by each SMU; that is, its support, is denoted as v. In an open pit situation, the size of each SMU, denoted as $|v|$, can range anywhere from 3 m $\times$ 3 m to 20 m $\times$ 20 m $\times$ bench height. In an underground operation, the SMU size $|v|$ can be highly variable, ranging from a massive column in a block caving operation down to the volume of one scoop of a front-end loader in a highly selective operation. In the following, we will consider a constant SMU v, assuming that there is no resolution below that support v: each SMU or local aggregate of SMUs is sent entirely to the mill (it is recovered) or to the waste pile (its metal content is lost). One can rightfully argue that the concept of a single SMU size does not apply to an underground operation, in which case the examples given in this book only relate to open pit operations.

Recoverable reserves relate to the proportion (tonnage) and average grade of those SMUs that will be selected as ore within any given panel V of size $|V|$ (see Fig. I.2A). Prior mine planning would have determined which panel V would be subjected to selective mining. Free selection of SMUs of constant size $|v|$ is assumed within each panel V. However, the panel V could be of any size, at the limit as large as the whole deposit D or as small as to contain only one single SMU; the SMU size may vary from one panel to another.

The panel V should not be so large as to include widely different mineralization zones, each having different statistics (grade continuity and histogram). In geostatistical terms, a decision of stationarity should be reasonable within each panel. The panel size and geometry can vary depending on the extent of each mineralization zone considered. The panel V should not be so small as to include too few SMUs, rendering the histogram of SMU grades within V unreliable. Beware also that the smaller the panel V, the less the error averaging and the larger the uncertainty about the estimate of its recoverable reserves. In most applications, the whole panel V would be mined out, but only part of it will be sent to the mill or to the ore pile.

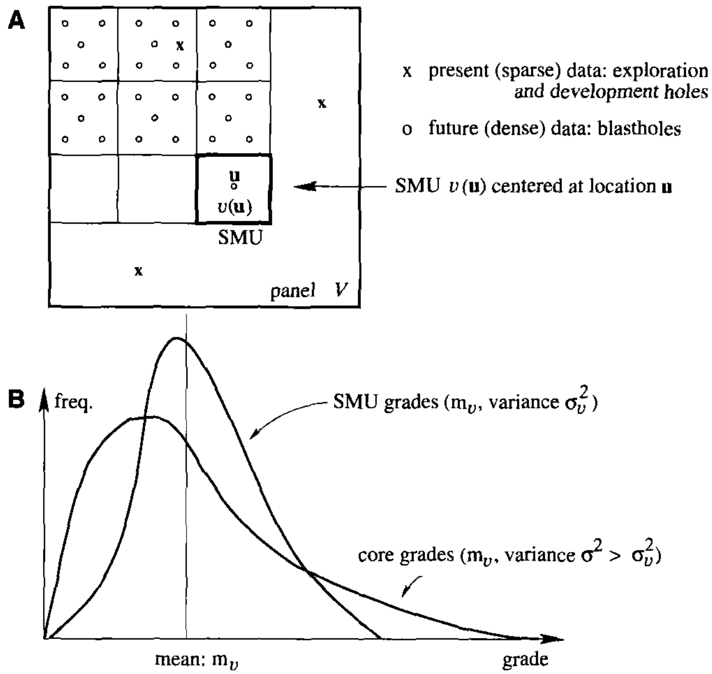

Figure I.2: The support and information effects. **A:** within a large panel V, selection is performed on estimated SMU grades of support volume $|v|$ much greater than the elementary core data volume. The selection estimate is based on future data available at the time of mining. **B:** the histogram of SMU grades (whether estimated or true values) is different from the histogram of sample grades.

Evaluation of recoverable reserves under free selection faces two major problems associated with:

1. **The support effect:** SMU grades relate to a volume of rock, orders of magnitude larger than the support volume of the core or composite grade data available at the time of the prediction study. In particular, the histogram of SMU grades has smaller variance and a different shape than the histogram of sample support grades within the same panel V (see Fig. I.2B). However, because any SMU grade is the linear average of its core grades (under exhaustive sampling), the two histograms will have the same mean, m_v.

2. **The information effect:** Actual selection is not performed on the true SMU grade but on some estimate of it; that estimate is typically based on the latest data available at the time of selection, including possibly a dense grid of grade control data; for example, blasthole samples in an open pit situation. That estimate is *not* the estimated SMU grade available at the time of prediction, when only exploration and development drilling is available.

Although any particular SMU is selected on the basis of its future estimated grade, it is the corresponding true grade (not its estimate) that is sent to the mill. The challenge of recoverable reserves estimation is to predict the number (tonnage) and average grade of those SMUs that will be recovered in the future at the time of mining, accounting for both support and information effects. Once again, free selection of each SMU within the panel V is assumed here; that is, each SMU is selected on the basis of its (future) estimated grade independent of any other SMU or any accessibility consideration within that panel.

We are aware that our definition of recoverable reserves departs somewhat from presently (2004) accepted definitions, such as those proposed by the Australasian Institute of Mining and Metallurgy (Joint Ore Reserves Committee [JORC] 1999). Per JORC, reserves imply recoverable; the only modifiers permitted reflect the degree of confidence—that is, proven or probable. The JORC definition relates to global reserves of either the entire mine or a large section of it. Our definition relates to local reserves over a much smaller area or panel V, and we do need the modifier between ore and waste, recoverable and noneconomical reserves, within that panel.

Although our proposed simulation approach does provide a numerical assessment of uncertainty about recoverable reserves (within a panel V), we did not attempt to extend that assessment into a classification of proven versus probable reserves. There are not yet any general geostatistical guidelines for such a classification at the local (panel) level.

Preliminary Notation (see also Appendix B).

• D: deposit, i.e., an assemblage of large panels V not necessarily all of the same size.

• V: panel, i.e., an assemblage of N selective mining units (SMUs) of equal size; from one panel to another the SMU size may vary. The size of panel V is denoted as $|V|$.

• v: SMU of constant size $|v|$ within any single panel V.

• $\mathbf{u} = (x, y)$: coordinate vector denoting a two-dimensional location within the deposit D. A a three-dimensional location would be denoted as $\mathbf{u} = (x, y, e)$, where e denotes elevation.

• $z(\mathbf{u})$: sample support grade at location $\mathbf{u}$; the core or composite support volume is assumed constant and quasi-point.

• $z(\mathbf{u}_\alpha)$: grade value at sample location $\mathbf{u}_\alpha$.

• n: total number of presently available grade samples for prediction of recoverable reserves. (n) denotes the set of samples.

• $n(\mathbf{u})$: number of presently available grade samples within a neighborhood around location $\mathbf{u}$. Note that $n(\mathbf{u}) < n$.

• nn: number of grade samples available at the future time of actual selection. Note that $nn(\mathbf{u}) > n(\mathbf{u})$.

• $z_v(\mathbf{u}) = \frac{1}{|v|}\int_{v(\mathbf{u})} z(\mathbf{u}')d\mathbf{u}' \simeq \frac{1}{n_v}\sum_{j=1}^{n_v} z(\mathbf{u}_j)$: true average grade of the SMU v centered at $\mathbf{u}$, with n_v being the number of points $\mathbf{u}_j$ discretizing the support v.

• $z_V(\mathbf{u}) = \frac{1}{|V|}\int_{V(\mathbf{u})} z(\mathbf{u}')d\mathbf{u}' = \frac{1}{N}\sum_{j=1}^{N} z_v(\mathbf{u}_j)$: true average grade of the panel V comprised of N SMUs of size $|v|$.

• $z_v^*(\mathbf{u})$: estimated SMU grade using the $n(\mathbf{u})$ data available at the time of prediction.

• $z_v^{**}(\mathbf{u})$: future estimated SMU grade using the $nn(\mathbf{u})$ data available at the time of selection. The three values $z_v(\mathbf{u})$, $z_v^*(\mathbf{u})$, and $z_v^{**}(\mathbf{u})$ are usually different.

• $z^{(l)}(\mathbf{u})$: simulated quasi-point support grade value at location $\mathbf{u}$. The superscript (l) stands for the lth simulated realization out of a total of L: $l = 1, \ldots, L$.

• $z_v^{(l)}(\mathbf{u}) = \frac{1}{|v|}\int_{v(\mathbf{u})} z^{(l)}(\mathbf{u}')d\mathbf{u}' \simeq \frac{1}{n_v}\sum_{j=1}^{n_v} z^{(l)}(\mathbf{u}_j)$: simulated true average grade of the SMU v centered at $\mathbf{u}$.

• $z_v^{**(l)}(\mathbf{u})$: simulated estimated SMU grade using the $nn(\mathbf{u})$ simulated data $\{z^{(l)}(\mathbf{u}'_\alpha),\ \alpha = 1, \ldots, nn(\mathbf{u})\}$, mimicking the information available at the time of selection.

Knowledge of the pair of values $\{z_v(\mathbf{u}), z_v^{**}(\mathbf{u})\}$, $\forall v \in V$, would solve fully the problem, as one would know the actual average grade $z_v(\mathbf{u})$ of any SMU v selected on the basis of its future estimated value $z_v^{**}(\mathbf{u})$. At the time of prediction, however, both $z_v(\mathbf{u})$ and $z_v^{**}(\mathbf{u})$ are unknown. The known simulated pairs $\{z_v^{(l)}(\mathbf{u}), z_v^{**(l)}(\mathbf{u})\}$, $l = 1, \ldots, L$, provide a suitable proxy.

Conditioning the simulated grade realizations ensures that the present data are reproduced:

$$z^{(l)}(\mathbf{u}_\alpha) = z(\mathbf{u}_\alpha), \quad \forall l, \alpha = 1, \ldots, n$$

• z_c, or simply z: cutoff grade above which a SMU is declared as ore, i.e., sent to the mill.

Perfect selection corresponds to the cutoff grade applied to the actual true SMU grade $z_v(\mathbf{u})$.

Actual selection corresponds to the cutoff grade applied to the estimate $z_v^{**}(\mathbf{u})$ available at the time of selection.

- $i_v^0(\mathbf{u}; z_c) = \begin{cases} 1 & \text{if } z_v(\mathbf{u}) > z_c \\ 0 & \text{if not} \end{cases}$ indicator of perfect selection

- $i_v^{**}(\mathbf{u}; z_c) = \begin{cases} 1 & \text{if } z_v^{**}(\mathbf{u}) > z_c \\ 0 & \text{if not} \end{cases}$ indicator of actual selection

- $t_v^0(z_c)$, $q_v^0(z_c)$, $m_v^0(z_c)$, $p_v^0(z_c)$: ore tonnage, quantity of metal, mean ore grade, and profit under perfect selection as functions of the cutoff grade z_c.

- $t_v(z_c)$, $q_v(z_c)$, $m_v(z_c)$, $p_v(z_c)$: recovery functions under actual selection performed on the estimates $z_v^{**}(\mathbf{u})$ available at the time of mining.

I.2 Perfect Selection

Ideal recovery would correspond to a perfect, faultless, selection based on the true SMU grades $z_v(\mathbf{u})$; this is *never* achievable in practice.

Let $i_v^0(\mathbf{u}; z_c)$ denote the indicator associated with perfect selection of a SMU v centered at $\mathbf{u}$, defined as:

$$i_v^0(\mathbf{u}; z_c) = \begin{cases} 1 & \text{if } z_v(\mathbf{u}) > z_c \\ 0 & \text{if not} \end{cases} \tag{I.1}$$

where z_c denotes the cutoff grade determined from economic considerations, and the superscript 0 indicates perfect selection obtained by applying the cutoff z_c to the true SMU grade $z_v(\mathbf{u})$.

Ore tonnage: The corresponding tonnage is defined as the number of profitable SMUs within panel V, recovered under perfect selection:

$$t_v^0(z_c) = \sum_{j=1}^{N} i_v^0(\mathbf{u}_j; z_c) \quad \in [0, N]\ ,\ \mathbf{u}_j \in V \tag{I.2}$$

where N is the number of SMUs in the panel, with $|V| = N \times |v|$. The tonnage of one SMU is assumed unit (tonnage factor equal to 1).

Quantity of metal: Because the SMU tonnage is unity, the corresponding quantity of metal is defined as the sum of profitable SMU grades, again recovered under perfect selection:

$$q_v^0(z_c) = \sum_{j=1}^{N} i_v^0(\mathbf{u}_j; z_c) \cdot z_v(\mathbf{u}_j) \tag{I.3}$$