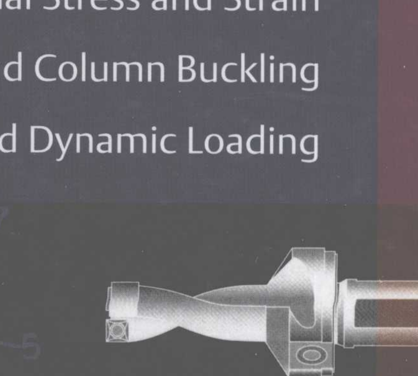
Marks'
STANDARD CALCULATIONS FOR MECHANICAL ENGINEERS

Marks' Calculations for Machine Design

- Foundational Loading
- Thermal Stress and Strain
- Static Design and Column Buckling
- Fatigue and Dynamic Loading



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MARKS' CALCULATIONS FOR MACHINE DESIGN

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To Miriam and Paulie

FOREWORD

Once the design and components of a machine have been selected there is an important engineering analysis process the machine designer should perform to verify the integrity of the design. That is what this book is about.

The purpose of *Marks' Calculations for Machine Design* is to uncover the mystery behind the principles, and particularly the formulas, used in machine design. All too often a formula found in the best of references is presented without the necessary background for the designer to understand how it was developed. This can be frustrating because of a lack of clarity as to what assumptions have been made in the formula's development. Typically, few if any examples are presented to illustrate the application of the formula with appropriate units. While these references are invaluable this companion book presents the application.

In *Marks' Calculations for Machine Design* the necessary background for every machine design formula presented is provided. The mathematical details of the development of a particular design formula have been provided only if the development enlightens and illuminates the fundamental principles for the machine designer. If the details of the development are only a mathematical exercise, they have been omitted. For example, in Chapter 9 the steps involved in the development of the design formulas for helical springs are presented in great detail since valuable insight is obtained about the true nature of the loading on such springs and because algebra is the only mathematics needed in the steps. On the other hand, in Chapter 3 the formulas for the tangential and radial stresses in a high-speed rotating thin disk are presented without their mathematical development since they derive from the simultaneous integration of two differential equations and the application of appropriate boundary conditions. No formula is presented unless it is used in one or more of the numerous examples provided or used in the development of another design formula.

Why has this approach been taken? Because a formula that remains a mystery is a formula unused, and a formula unused is an opportunity missed—forever.

It is hoped that *Marks' Calculations for Machine Design* will provide a level of comfort and confidence in the principles and formulas of machine design that ultimately produces a successful and safe design, and a proud designer.

THOMAS H. BROWN, JR., PH.D., P.E.

PREFACE

As the title of this book implies, *Marks' Calculations for Machine Design* was written to be a companion to *Marks' Standard Handbook for Mechanical Engineers*, providing detailed calculations to the important problems in machine design. For each of the over 175 examples presented, complete solutions are provided, including appropriate figures and diagrams, all algebra and arithmetic steps, and using both the U.S. Customary and SI/Metric systems of units. It is hoped that *Marks' Calculations for Machine Design* will provide an enthusiastic beginning for those just starting out in mechanical engineering, as well as provide a comprehensive resource for those currently involved in machine design projects.

Marks' Calculations for Machine Design is divided into two main parts: Part 1, Strength of Machines, and Part 2, Application to Machines. Part 1 contains seven chapters on the foundational principles and equations of machine design, from basic to advanced, while Part 2 contains three chapters on the most common machine elements based on these principles and equations.

Beginning Part 1, Chapter 1, *Fundamental Loadings*, contains the four foundational loadings: axial, direct shear, torsion, and bending. Formulas for stress and strain, both normal and shear, along with appropriate examples are presented for each of these loadings. Thermal stress and strain are also covered. Stress-strain diagrams are provided for both ductile and brittle materials, and the three engineering properties, (E), (G), and (ν), are discussed.

Chapter 2, *Beams*, provides the support reactions, shear and bending moment diagrams, and deflection equations for fifteen different beam configurations. There are ten simply-supported beam configurations, from end supported, single overhanging, and double overhanging. There are five cantilevered beam configurations. Loadings include concentrated forces and couples, as well as uniform and triangular shaped distributed loadings. Almost 45% of the total number of examples and over 30% of the illustrations are in this single chapter. Nowhere is there a more comprehensive presentation of solved beam examples.

Chapter 3, *Advanced Loadings*, covers three such loadings: pressure loadings, to include thin- and thick-walled vessels and press/shrink fits; contact loading, to include spherical and cylindrical geometries; and high-speed rotational loading.

Chapter 4, *Combined Loadings*, brings the basic and advanced loadings covered in Chapters 1, 2, and 3 together in a discussion of how loadings can be combined. Seven different combinations are presented, along with the concept of a plane stress element.

Chapter 5, *Principal Stresses and Mohr's Circle*, takes the plane stress elements developed in Chapter 4 and presents the transformation equations for determining the principal stresses, both normal and shear, and the associated rotated stress elements. Mohr's circle, the graphical representation of these transformation equations, is also presented. The Mohr's circle examples provided include multiple diagrams in the solution process, a half dozen on average, so that the reader does not get lost, as typically happens with the more complex single solution diagrams of most other references.

Chapter 6, *Static Design and Column Buckling*, includes two major topics: design under static conditions and the buckling of columns. The section on static design covers both ductile and brittle materials, and a discussion on stress concentration factors for brittle materials with notch sensitivity. In the discussion on ductile materials, the maximum-normal-stress theory, the maximum-shear-stress theory, and the distortion-energy theory are presented with examples. Similarly, for brittle materials, the maximum-normal-stress theory, the Coulomb-Mohr theory, and the modified Coulomb-Mohr theory are presented with examples. The discussion on stress concentration factors provides how to use the stress-concentration factors found in *Marks' Standard Handbook for Mechanical Engineers* and other references. In the discussion on column buckling, the Euler formula is presented for long slender columns, the parabolic formula for intermediate length columns, the secant formula for eccentric loading, as well as a discussion on how to deal with short columns.

Chapter 7, *Fatigue and Dynamic Design*, contains information on how to design for dynamic conditions, or fatigue. Fatigue associated with reversed loading, fluctuating loading, and combined loading is discussed with numerous examples. The Marin equation is provided with examples on the influence of its many modifying factors that contribute to establishing an endurance limit, which in turn is used to decide whether a design is safe. Extensive use of the Goodman diagram as a graphical approach to determine the safety of a design is presented with appropriate examples.

Beginning Part 2, Chapter 8, *Machine Assembly*, discusses the two most common ways of joining machine elements: bolted connections and welded connections. For bolted connections, the design of the fastener, the members, calculation of the bolt preload in light of the bolt strength and the external load, static loading, and fatigue loading are presented with numerous examples. For welded connections, both butt and fillet welds, axial, transverse, torsional, and bending loading is discussed, along with the effects of dynamic loading, or fatigue, in shear.

Chapter 9, *Machine Energy*, considers two of the most common machine elements associated with the energy of a mechanical system: springs and flywheels. The extensive discussion on springs is limited to helical springs, however these are the most common type used. Additional spring types will be presented in future editions. In the discussion on flywheels, two system types are presented: internal combustion engines where torque is a function of angular position, and electric motor driven punch presses where torque is a function of angular velocity.

Chapter 10, *Machine Motion*, covers the typical machine elements that move: linkages, gears, wheels and pulleys. The section on linkages includes the three most famous designs: the four-bar linkage, the quick-return linkage, and the slider-crank linkage. Extensive calculations of velocity and acceleration for the slider-crank linkage are presented with examples. Gears, whether spur, helical, or herringbone, are usually assembled into gear trains, of which there are two general types: spur and planetary. Spur gear trains involve two or more fixed parallel axles. The relationship between the speeds of these gear trains, based on the number of gear teeth in contact, is presented with examples. Planetary gear trains, where one or more planet gears rotate about a single sun gear, are noted for their compactness. The relative speeds between the various elements of this type of design are presented.

While much has been presented in these ten chapters, some topics had to be left out to meet the schedule, not unlike the choices and tradeoffs that are part of the day-to-day practice of engineering. If there are topics the reader would like to see covered in the second edition, the author would very much like to know. Though much effort has been spent in trying to make this edition error free, there are inevitably still some that remain. Again, the author would appreciate knowing where these appear.

Good luck on your designs. It has been a pleasure uncovering the mystery of the principles and formulas in machine design that are so important to bringing about a safe and operationally sound design. It is hoped that the material in this book will inspire and give confidence to your designs. There is no greater reward to a machine designer than to know they have done their best, incorporating the best practices of their profession. And remember the first rule of machine design as told to me by my first supervisor, "when in doubt, make it stout!"

ACKNOWLEDGMENTS

My deepest appreciation and abiding love goes to my wife, Miriam, who is also my dearest and best friend. Her encouragement, help, suggestions, and patience over the many long hours it took to complete this book is a blessing from the Lord.

I am grateful for the love and understanding of my three children, Sianna, Hunter, and Elliott, who have been so very patient through the many weekends without their Dad, and who are a continual joy and source of immense pride.

To my Senior Editor Ken McCombs, whose confidence and support have guided me throughout this project, I gratefully give thanks. To Sam (Samik RoyChowdhury) and his wonderful and competent staff at International Typesetting and Composition (ITC) in Noida, India—it has been a pleasure and honor to work with you in dealing with the "bzillion" details to bring this book to reality.

And finally, thanks to Paulie (Paul Teutel, Jr.) of Orange County Choppers who embodies the true art of machine design. The unique motorcycles he and the staff at OCC bring to life, particularly the fabulous theme bikes, represents the joy and pride that mechanical design can provide.

THOMAS H. BROWN, JR., PH.D., P.E.

CONTENTS

P · A · R · T · 1

STRENGTH OF MACHINES

CHAPTER 1
FUNDAMENTAL LOADINGS

1.1 INTRODUCTION

The fundamental loadings on machine elements are axial loading, direct shear loading, torsion, and bending. Each of these loadings produces stresses in the machine element, as well as deformations, meaning a change in shape. There are only two types of stresses: normal and shear. Axial loading produces a normal stress, direct shear and torsion produce shear stresses, and bending produces both a normal and a shear stress.

Figure 1.1 shows a straight prismatic bar loaded in tension by opposing forces (P) at each end. (A prismatic bar has a uniform cross section along its length.) These forces produce a tensile load along the axis of the bar, which is why it is called axial loading, resulting in a tensile normal stress in the bar. There is also a corresponding lengthening of the bar. If these forces were in the opposite direction, then the bar would be loaded in compression, producing a compressive normal stress and a shortening of the bar.

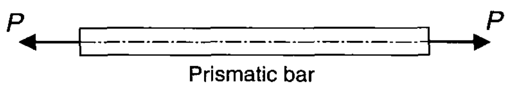

FIGURE 1.1 Axial loading.

Figure 1.2 shows a riveted joint, where a simple rivet holds two overlapping bars together. The shaft of the rivet at the interface of the bars is in direct shear, meaning that a shear stress is produced in the rivet. As the forces (P) increase, the joint will rotate until either the rivet *shears* off, or the material around the hole of either bar pulls out.

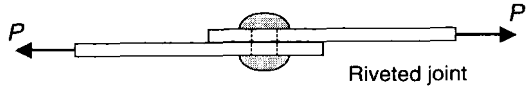

FIGURE 1.2 Direct shear loading.

Figure 1.3 shows a circular shaft acted upon by opposing torques (T), causing the shaft to be in torsion. This type of loading produces a shear stress in the shaft, thereby causing one end of the shaft to rotate about the axis of the shaft relative to the other end.

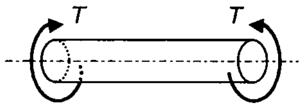

FIGURE 1.3 Torsion.

Figure 1.4 shows a simply supported beam with a concentrated force (F) located at its midpoint. This force produces both a bending moment distribution and a shear force distribution in the beam. At any location along the length (L) of the beam, the bending moment produces a normal stress, and the shear force produces a shear stress.

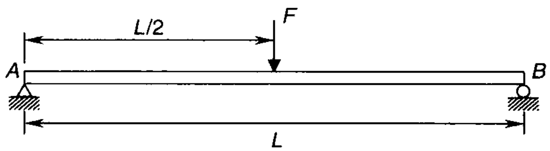

FIGURE 1.4 Bending.

The beam shown in Fig. 1.4 will deflect downward along its length; however, unlike axial loading, direct shear loading, and torsion that have a single equation associated with their deformation, there is not a single equation for the deformation or deflection of any beam under any loading. Each beam configuration and loading is different. A detailed discussion of 15 different beam configurations is presented in Chap. 2, complete with reactions, shear force and bending moment distributions, and deflection equations.

1.2 AXIAL LOADING

The prismatic bar shown in Fig. 1.5 is loaded in tension along its axis by the opposing forces (P) at each end. Again, a prismatic bar has a uniform cross section, and therefore a constant area (A) along its length.

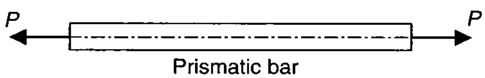

FIGURE 1.5 Axial loading.

Stress. These two forces produce a tensile load along the axis of the bar, resulting in a tensile normal stress (σ) given by Eq. (1.1).

$$\sigma = \frac{P}{A} \tag{1.1}$$

As stress is expressed by force over area, the unit is given in pound per square inch (psi) in the U.S. Customary System, and in newton per square meter, or pascal (Pa), in the metric system.

U.S. Customary	*SI/Metric*
Example 1. Determine the normal stress in a square bar with side (a) loaded in tension with forces (P), where	**Example 1.** Determine the normal stress in a square bar with side (a) loaded in tension with forces (P), where
$P = 12\text{ kip} = 12{,}000\text{ lb}$ $a = 2\text{ in}$	$P = 55\text{ kN} = 55{,}000\text{ N}$ $a = 5\text{ cm} = 0.05\text{ m}$
solution *Step 1.* Calculate the cross-sectional area (A) of the bar.	**solution** *Step 1.* Calculate the cross-sectional area A of the bar.
$A = a^2 = (2\text{ in})^2 = 4\text{ in}^2$	$A = a^2 = (0.05\text{ m})^2 = 0.0025\text{ m}^2$
Step 2. From Eq. (1.1), calculate the normal stress (σ) in the bar.	*Step 2.* From Eq. (1.1), calculate the normal stress (σ) in the bar.
$\sigma = \frac{P}{A} = \frac{12{,}000\text{ lb}}{4\text{ in}^2}$ $= 3{,}000\text{ lb/in}^2 = 3.0\text{ kpsi}$	$\sigma = \frac{P}{A} = \frac{55{,}000\text{ N}}{0.0025\text{ m}^2}$ $= 22{,}000{,}000\text{ N/m}^2 = 22\text{ MPa}$
Example 2. Calculate the minimum cross-sectional area (A_{min}) needed for a bar axially loaded in tension by forces (P) so as not to exceed a maximum normal stress (σ_{max}), where	**Example 2.** Calculate the minimum cross-sectional area (A_{min}) needed for a bar axially loaded in tension by forces (P) so as not to exceed a maximum normal stress (σ_{max}), where
$P = 10\text{ kip} = 10{,}000\text{ lb}$ $\sigma_{max} = 36{,}000\text{ psi}$	$P = 45\text{ kN} = 45{,}000\text{ N}$ $\sigma_{max} = 250\text{ MPa}$
solution *Step 1.* Start with Eq. (1.1) where the normal stress (σ) is maximum and the area (A) is minimum to give	**solution** *Step 1.* Start with Eq. (1.1) where the normal stress (σ) is maximum and the area (A) is minimum to give
$\sigma_{max} = \frac{P}{A_{min}}$	$\sigma_{max} = \frac{P}{A_{min}}$
Step 2. Solve for the minimum area (A_{min}).	*Step 2.* Solve for the minimum area (A_{min}).
$A_{min} = \frac{P}{\sigma_{max}}$	$A_{min} = \frac{P}{\sigma_{max}}$
Step 3. Substitute for the force (P) and the maximum normal stress.	*Step 3.* Substitute for the force (P) and the maximum normal stress.
$A_{min} = \frac{10{,}000\text{ lb}}{36{,}000\text{ lb/in}^2} = 0.28\text{ in}^2$	$A_{min} = \frac{45{,}000\text{ N}}{250 \times 10^6\text{ N/m}^2} = 0.00018\text{ m}^2$

Strain. The axial loading shown in Fig. 1.6 also produces an axial strain (ε), given by Eq. (1.2).

$$\varepsilon = \frac{\delta}{L} \tag{1.2}$$

where (δ) is change in length of the bar and (L) is length of the bar.

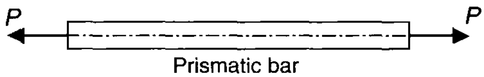

FIGURE 1.6 Axial loading.

Strain is a dimensionless quantity and does not have a unit if the change in length ε and the length (L) are in the same units. However, if the change in length (δ) is in inches or millimeters, and the length (L) is in feet or meters, then the strain (ε) will have a unit.

U.S. Customary	SI/Metric
Example 3. Calculate the strain (ε) for a change in length (δ) and a length (L), where	**Example 3.** Calculate the strain (ε) for a change in length (δ) and a length (L), where
$\delta = 0.015$ in $L = 5$ ft	$\delta = 0.038$ cm $L = 1.9$ m
solution *Step 1.* Calculate the strain (ε) from Eq. (1.2).	**solution** *Step 1.* Calculate the strain (ε) from Eq. (1.2).
$\varepsilon = \frac{\delta}{L} = \frac{0.015\text{ in}}{5\text{ ft}}$	$\varepsilon = \frac{\delta}{L} = \frac{0.038\text{ cm}}{1.9\text{ m}}$
$= 0.003\text{ in/ft} \times 1\text{ ft}/12\text{ in}$	$= 0.02\text{ cm/m} \times 1\text{ m}/100\text{ cm}$
$= 0.00025\text{ in/in} = 0.00025$	$= 0.0002\text{ m/m} = 0.0002$

Stress-Strain Diagrams. If the stress (σ) is plotted against the strain (ε) for an axially loaded bar, the stress-strain diagram for a ductile material in Fig. 1.7 results, where A is proportional limit, B elastic limit, C yield point, D ultimate strength, and F fracture point.

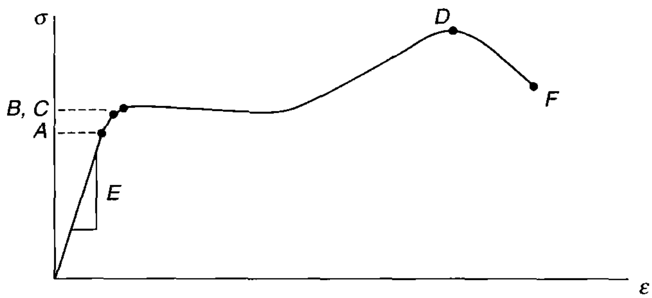

FIGURE 1.7 Stress-strain diagram (ductile material).

The stress-strain diagram is linear up to the proportional limit, and has a slope (E) called the modulus of elasticity. In this region the equation of the straight line up to the proportional limit is called Hooke's law, and is given by Eq. (1.3).

$$\sigma = E\,\varepsilon \tag{1.3}$$

The numerical value for the modulus of elasticity (E) is very large, so the stress-strain diagram is almost vertical to point A, the proportional limit. However, for clarity the horizontal placement of point A has been exaggerated on both Figs. 1.7 and 1.8.