Jean Dieudonné

A History of Algebraic and Differential Topology 1900–1960

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Jean Dieudonné
120, avenue de Suffren
75015 Paris
France

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Preface

Although concepts that we now consider as part of topology had been expressed and used by mathematicians in the nineteenth century (in particular, by Riemann, Klein, and Poincaré), algebraic topology as a part of rigorous mathematics (i.e., with precise definitions and correct proofs) only began in 1900. At first, algebraic topology grew very slowly and did not attract many mathematicians; until 1920 its applications to other parts of mathematics were very scanty (and often shaky). This situation gradually changed with the introduction of more powerful algebraic tools, and Poincaré's vision of the fundamental role topology should play in all mathematical theories began to materialize. Since 1940, the growth of algebraic and differential topology and of its applications has been exponential and shows no sign of slackening.

I have tried in this book to describe the main events in that expansion prior to 1960. The choice of that terminal date does not correspond to any particular occurrence nor to an inflection in the development of the theory. However, on one hand, I wanted to limit the size of this book, which is already a large one; and, on the other hand, it is difficult to have a bird's eye view of an evolution that is still going on around us at an unabated pace. Twenty years from now it will be much easier to describe what happened between 1960 and 1980, and it will probably fill a book as large as this one.

There is one part of the history of algebraic and differential topology that I have not covered at all, namely, that which is called "low-dimensional topology." It was soon realized that some general tools could not give satisfactory results in spaces of dimension 4 at most, and, conversely, methods that were successful for those spaces did not extend to higher dimensions. I feel that a description of the discovery of the properties of these spaces deserves a book by itself, which I hope somebody will write soon.

The literature on algebraic and differential topology is very large, and to analyze each paper would have been unbearably boring. I have tried to focus the history on the emergence of ideas and methods opening new fields of research, and I have gone into some details on the work of the pioneers, even when their methods were later superseded by simpler and more powerful ones. As Hadamard once said, in mathematics simple ideas usually come last.

I assume that the reader is familiar with the elementary part of algebra and "general topology." Whenever I have had to mention striking applications of algebraic topology to other parts of mathematics, I have summarized the notions necessary to understand these applications.

Notations

General Notations

$\mathbf{N}$, $\mathbf{Z}$, $\mathbf{Q}$, $\mathbf{R}$, $\mathbf{C}$, $\mathbf{H}$, $\mathbf{F}_q$: integers $\geqslant 0$, rational integers, rational numbers, real numbers, complex numbers, quaternions, finite field with q elements.

$\mathbf{S}_n$: sphere $\sum_j |\xi_j|^2 = 1$ in $\mathbf{R}^n$.

$\mathbf{D}_n$: ball $\sum_j |\xi_j|^2 \leqslant 1$ in $\mathbf{R}^n$.

$\mathrm{e}_1, \mathrm{e}_2, \ldots, \mathrm{e}_n$: canonical basis of $\mathbf{R}^n$ (also written $\mathrm{e}_0, \mathrm{e}_1, \ldots, \mathrm{e}_{n-1}$).

$\mathbf{T}^n$ n-dimensional torus $\mathbf{R}^n/\mathbf{Z}^n$.

$\mathbf{P}_n(\mathrm{F})$: n-dimensional projective space over a field F.

$\mathrm{GL}(n, \mathrm{F})$: general linear group, i.e., group of automorphisms of the vector space F^n; or group of invertible $n \times n$ matrices with entries in F.

$\mathrm{SL}(n, \mathrm{F})$: for F a commutative field, subgroup of $\mathrm{GL}(n, \mathrm{F})$, consisting of matrices of determinant 1.

$\mathrm{O}(n)$: orthogonal group, subgroup of $\mathrm{GL}(n, \mathbf{R})$ leaving invariant the euclidean scalar product.

$\mathrm{SO}(n)$: group of rotations $\mathrm{O}(n) \cap \mathrm{SL}(n, \mathbf{R})$.

$\mathrm{U}(n)$: unitary group, subgroup of $\mathrm{GL}(n, \mathbf{C})$ leaving invariant the hermitian scalar product.

$\mathrm{U}(n, \mathbf{H})$, $\mathrm{Sp}(n)$: unitary group over the skew field of quaternions, leaving invariant the hermitian scalar product in $\mathbf{H}^n$.

Categories

C: arbitrary category.

$\boldsymbol{C}^0$: category dual to ***C***.

Set, ***PSet***: category of sets, of pointed sets.

Gr, ***Ab***: category of groups, of commutative groups.

$\boldsymbol{Mod}_\Lambda$, $\boldsymbol{Vect}_k$, $\boldsymbol{Alg}_k$: categories of modules over a ring Λ, of vector spaces over a field k, of algebras over a field k.

T, ***PT***: category of topological spaces, of pointed spaces.

$\boldsymbol{T}'$: any subcategory of ***T***.

$\boldsymbol{T}_1$: category of pairs (X, A), X topological space, A subspace of X.

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Notations of Set Theory and General Topology

pt.: set or space with only one element.
Id, Id_E, 1_E: identity map of the set E.
$A \coprod B$: disjoint union of A and B (sets or spaces).
$\bar{A}$, $\mathring{A}$, Fr(A): closure, interior, frontier of a subset A of a topological space.
$\varinjlim X_n$: direct limit of a sequence of sets, spaces, or groups (relative to morphisms $\varphi_{n,n+1}: X_n \to X_{n+1}$).
$\varprojlim X_\alpha$: inverse limit of an ordered family $(X_\alpha)_{\alpha \in I}$ (I ordered set) relative to morphisms $\varphi_{\beta\alpha}: X_\beta \to X_\alpha$ for $\alpha < \beta$.

Quotient Spaces (Part 2, chap. V)

X/A: space obtained by collapsing the subset A of X.
$\tilde{C}X$, CX: cone, reduced cone of X.
$\tilde{S}X$, SX: suspension, reduced suspension of X.
$X \vee Y$, $\bigvee_\alpha X_\alpha$: wedges of pointed spaces.
$X \wedge Y = (X \times Y)/(X \vee Y)$: smash product.
$X * Y$: join of two spaces X, Y.
$X \bigcup_f Y$: attachment of X to Y by means of a continuous map $f: A \to Y$, for a subspace $A \subset X$.
Z_f, $\tilde{Z}_f$: mapping cylinder of $f: X \to Y$, reduced mapping cylinder.
C_f, $\tilde{C}_f$: mapping cone of $f: X \to Y$, reduced mapping cone.

General Notations in Algebra and Homological Algebra

Ker f, Coker f, Im f: kernel, cokernel, image of a homomorphism $f: A \to B$ of modules.
$A \oplus B$: direct sum of two Λ-modules.
$A \otimes_\Lambda B$, $A \otimes B$: tensor product of two Λ-modules.
Hom(A, B): module of homomorphisms $A \to B$.
End(A) = Hom(A, A), ring of endomorphisms of A.
Tor(A, B): Part 1, chap. IV, § 5,B.
Ext(A, B): Part 1, chap. IV, § 5,D.
$H_n(\Pi; G)$, $H^n(\Pi; G)$: homology and cohomology groups of the group Π with coefficients in the group G: Part 3, chap. V, § 1,D.

Homology of Chain Complexes and Cohomology of Cochain Complexes (Part 1, chap. IV, § 5)

$C_. = (C_j)_{j \geq 0}$: chain complex.
$\mathbf{b}_p: C_p \to C_{p-1}$: boundary operator (also written **b**).
Z_p, $Z_p(C_.) = \ker \mathbf{b}_p$: module of p-cycles.

B_p, $B_p(C_.) = \mathrm{Im}\,\mathbf{b}_{p+1}$: module of p-boundaries.
$H_p(C_.) = Z_p/B_p$: p-th homology module.
$C^{\cdot} = (C^j)_{j \geqslant 0}$: cochain complex.
$\mathbf{d}_p$: $C^p \to C^{p+1}$: coboundary operator (also written $\mathbf{d}$).
Z^p, $Z^p(C^{\cdot}) = \mathrm{Ker}\,\mathbf{d}_p$: module of p-cocycles.
B^p, $B^p(C^{\cdot}) = \mathrm{Im}\,\mathbf{d}_{p-1}$: module of p-coboundaries.
$H^p(C^{\cdot}) = Z^p/B^p$: p-th cohomology module
∂_n: $H_n(C_.) \to H_{n-1}(A_.)$ connecting homomorphism of the homology exact sequence for the exact sequence $0 \to A_. \to B_. \to C_. \to 0$ of chain complexes (also written ∂).
$\mathrm{rk}(M)$: rank of a finitely generated $\mathbf{Z}$-module.
$\chi(C_.) = \sum_j (-1)^j \mathrm{rk}(C_j)$: Euler-Poincaré characteristic of a finitely generated chain complex of $\mathbf{Z}$-modules.
$f_. = (f_p)$: $C_. \to C'_.$: chain transformation (Part 1, chap. IV, § 5,F).
$H_p(f_.) = f_{p*}$: $H_p(C_.) \to H_p(C'_.)$: homomorphism in homology corresponding to a chain transformation $f_.$.

Axiomatic Homology and Cohomology (Part 1, chap. IV, § 6,B)

$H_p(X)$, $H_p(X, A)$: homology modules.
$H_p(f)$: $H_p(X, A) \to H_p(Y, B)$: homomorphism corresponding to the morphism f: $(X, A) \to (Y, B)$ in $\boldsymbol{T}_1$ (also written f_*).
$\partial_q(X, A)$: $H_q(X, A) \to H_{q-1}(A)$: connecting homomorphism (written ∂).
$H^p(X)$, $H^p(X, A)$: cohomology modules.
$H^p(f)$: $H^p(Y, B) \to H^p(X, A)$: homomorphism corresponding to the morphism f: $(X, A) \to (Y, B)$ (also written f^*).
∂: $H^{q-1}(A) \to H^q(X, A)$: connecting homomorphism.
$\tilde{H}_0(X)$, $\tilde{H}^0(X)$: reduced 0-homology and 0-cohomology modules.

Singular Homology and Cohomology (Part 1, chap. IV, § 2 and § 3)

Δ_p: standard p-simplex.
$S_.(X; \mathbf{Z}) = (S_p(X; \mathbf{Z}))$: singular complex of X.
$H_.(X; \mathbf{Z}) = (H_p(X; \mathbf{Z}))$: singular homology of X.
$S_p(X, A; \mathbf{Z}) = S_p(X; \mathbf{Z})/S_p(A; \mathbf{Z})$ for a subspace $A \subset X$.
$H_p(X, A; \mathbf{Z})$: relative singular homology.
$u_* = H_p(u)$: $H_p(X; \mathbf{Z}) \to H_p(Y; \mathbf{Z})$: homomorphism in homology deduced from a continuous map u: $X \to Y$.
$b_p(X)$, b_p: p-th Betti number of X.
$S^p(X; G) = \mathrm{Hom}(S_p(X; \mathbf{Z}), G)$: group of singular p-cochains.
$H^p(X; G)$: p-th singular cohomology group of X with coefficients in G.

$H^p(X, A; G)$: p-th relative singular cohomology group, for $A \subset X$.
$u^* = H^p(u)$: $H^p(Y; G) \to H^p(X; G)$: homomorphism in cohomology deduced from a continuous map u: $X \to Y$.
$H_c^p(X; G)$: p-th singular cohomology group of X with compact supports.
$a \smile b$: cup-product of two cohomology classes in $H^{\bullet}(X; \Lambda)$: Part 1, chap. IV, § 4.
$a \frown u$: cap-product of a homology class a and a cohomology class u: Part 1, chap. IV, § 4.
u/c', $c''\backslash u$: right (left) slant product of a cohomology class u and a homology class $c'(c'')$: Part 1, chap. IV, § 5,H.

Čech and Alexander–Spanier Cohomology (Part 1, chap. IV, § 3)

$\check{H}^p(X; G)$: p-th Čech cohomology group of X with coefficients in G.
$C^p(X; G)$: group of maps of X^{p+1} into G.
$C_0^p(X; G)$: subgroup of $C^p(X; G)$ consisting of maps vanishing in a neighborhood of the diagonal.
$\bar{C}^p(X; G) = C^p(X; G)/C_0^p(X; G)$: group of Alexander–Spanier p-cochains.
$\bar{\delta}_p$: $\bar{C}^p(X; G) \to \bar{C}^{p+1}(X; G)$: p-th coboundary operator.
$\bar{H}^p(X; G)$: p-th Alexander–Spanier cohomology group.

Sheaves, Sheaf Cohomology (Part 1, chap. IV, § 7)

$f_*(\mathscr{F})$: direct image of a sheaf by a continuous map.
$\Gamma(\mathscr{F})$: sections of a sheaf.
$H^{\bullet}(X; \mathscr{F})$: cohomology of X with coefficients in a sheaf $\mathscr{F}$.
$H_{\Phi}^{\bullet}(X; \mathscr{F})$: cohomology of X with supports in Φ and coefficients in $\mathscr{F}$.

Spectral Sequences (Part 1, chap. IV, § 7,D and Part 3, chap. IV, § 3,C)

$M^{\bullet} = \bigoplus_q M_q$, differential graded module, with $d(M_q) \subset M_{q+1}$, and decreasing filtration F, with $F^p(M^{\bullet}) = \bigoplus_q M_q \cap F^p(M^{\bullet})$.
Z_r^{pq}: set of $z \in F^p(M^{\bullet}) \cap M_{p+q}$ with $dz \in F^{p+r}(M^{\bullet})$, $Z_r^p = \bigoplus_q Z_r^{pq}$, $B_{r-1}^{pq} = M_{p+q} \cap dZ_{r-1}^{p+1-r}$; $E_r^{pq} = Z_r^{pq}/(Z_{r-1}^{p+1,q-1} + B_{r-1}^{pq})$;
d_r: $E_r^{pq} \to E_r^{p+r,q-r+1}$ $d_r \circ d_r = 0$
If $E_r^{\bullet} = \bigoplus_{p,q} E_r^{pq}$, $H^{\bullet}(E_r^{\bullet}) = E_{r+1}^{\bullet}$ for the coboundary d_r.
$E^{pq} = F^p H^{p+q}(M^{\bullet})/F^{p+1} H^{p+q}(M^{\bullet})$.
For a graded differential module $M^{\bullet} = \bigoplus_q M^q$ with $d(M^q) \subset M^{q-1}$ and an increasing filtration F, with $F_p(M^{\bullet}) = \bigoplus_q M^q \cap F_p(M^{\bullet})$, one writes Z_{pq}^r: set of $z \in F_p(M^{\bullet}) \cap M^{p+q}$ with $dz \in F_{p-r}(M^{\bullet})$; $Z_p^r = \bigoplus_q Z_{pq}^r$, $B_{pq}^r = M^{p+q} \cap dZ_{p+r}^r$, $E_{pq}^r = Z_{pq}^r/(Z_{p-1,q+1}^{r-1} + B_{pq}^{r-1})$.

De Rham Cohomology (Part 1, chap. III, § 3)

$\mathscr{E}_p(M)$: smooth p-forms on the manifold M.
$\mathscr{D}_p(M)$: smooth p-forms on M with compact support.
$H^p(M)$: De Rham cohomology, i.e., cohomology of the cochain complex ($\mathscr{E}_p(M)$) for the exterior differential $\omega \mapsto d\omega$.
$H_c^p(M)$: De Rham cohomology with compact support, i.e., cohomology of the subcomplex ($\mathscr{D}_p(M)$).
$\mathscr{E}'_p(M)$: p-currents on M with compact support.
$H'_p(M)$: homology of ($\mathscr{E}'_p(M)$) for the boundary operator $b = {}^t d$.

Fundamental Classes (Part 2, chap. I, § 3,A and chap. IV, § 3,A)

[M]: fundamental homology class of an oriented pseudomanifold M.
e_M^*: cohomology fundamental class, or orientation class of an oriented smooth compact n-dimensional manifold M, class of the n-form ω such that $\int_M \omega = 1$.
s_n: orientation class of the sphere $\mathbf{S}_n$.
$\mu_{M,K}$: for an oriented smooth n-dimensional manifold M and a nonempty compact subset K, fundamental class relative to K, element of $H_n(M, M - K; \mathbf{Z})$.

Degree and Fixed Points (Part 2, chap. I, chap. III, and chap. IV, § 1,B)

$\deg f$: degree of a continuous map $f: M \to M'$, where M, M′ are compact connected oriented pseudomanifolds of the same dimension.
$d(f, M, p)$: degree of f relative to M and p, for a continuous map $f: \overline{M} \to \mathbf{R}^n$ and a point $p \notin f(\mathrm{Fr}(M))$.
$\deg_a f$: local degree at the point $a \in X$, for a C^∞ map: $X \to Y$ of smooth oriented manifolds of the same dimension, a being isolated for f.
$\mathrm{lk}(A, B)$: linking number of two chains A, B with no common points, of dimensions k and $n - k$ in a connected oriented n-dimensional combinatorial manifold X, when αA and βB are boundaries, for integers α, β.
$\mathrm{Fix}(f)$: set of fixed points of a map $f: X \to X$.
$\Lambda(f)$: Lefschetz number of f, when X is a finite simplicial complex.

Homotopy (Part 3, chap. I and II)

$\mathscr{C}(Y, y_0; X, x_0)$: set of continuous maps $(Y, y_0) \to (X, x_0)$ of pointed spaces, with the compact-open topology.
$[Y, y_0; X, x_0] = \pi_0(\mathscr{C}(Y, y_0; X, x_0))$: set of arcwise connected components of

$\mathscr{C}(Y, y_0; X, x_0)$, or, equivalently, homotopy classes of maps $(Y, y_0) \to (X, x_0)$ of pointed spaces.

$[f]$: homotopy class of a continuous map f of pointed spaces.

$*$: the point $\mathbf{e}_1 \in \mathbf{S}_n$

$[0, 1]$, I: the interval $0 \leqslant t \leqslant 1$ in $\mathbf{R}$.

$\Omega(X, x_0) = \mathscr{C}(\mathbf{S}_1, *; X, x_0)$: space of loops in X of origin x_0.

$\pi_n(X, x_0) = \pi_0(\mathscr{C}(\mathbf{S}_n, *; X, x_0)$: n-th homotopy group of the pointed space (X, x_0).

$u_*: \pi_n(X, x_0) \to \pi_n(Y, y_0)$: homomorphism $[f] \mapsto [u \circ f]$ (written $[u] \circ [f]$, for a continuous map u of pointed spaces).

$(\Omega^p(X), x_p)$: pointed iterated loop space of (X, x_0).

E: $[Y, y_0; X, x_0] \to [SY, y_0; SX, x_0]$: homotopy suspension.

$E(f)$ or Sf: natural map $(SY, y_0) \to (SX, x_0)$ deduced from $f: (Y, y_0) \to (X, x_0)$, so that $E([f]) = [Sf]$.

$[u, v] \in \pi_{m+n-1}(X, x_0)$: Whitehead product of $u \in \pi_m(X, x_0)$ and $v \in \pi_n(X, x_0)$.

ε_n: relative homology class in $H_n(\bar{\Delta}_n, \bar{\Delta}_n - \Delta_n; \mathbf{Z})$ of the identity map of $\bar{\Delta}_n$.

h_n: $[f] \to f_*(\varepsilon_n)$: Hurewicz homomorphism $\pi_n(X, x_0) \to H_n(X; \mathbf{Z})$.

$\mathscr{C}(Y, B, y_0; X, A, x_0)$: subspace of $\mathscr{C}(Y, y_0; X, x_0)$ consisting of maps f such that $f(B) \subset A$ (with $y_0 \in B$, $x_0 \in A$).

$[Y, B, y_0; X, A, x_0]$: set of relative homotopy classes in $\mathscr{C}(Y, B, y_0; X, A, x_0)$.

$\Omega^n(X, A, x_0) \simeq \mathscr{C}(\mathbf{D}_n, \mathbf{S}_{n-1}, *; X, A, x_0)$: iterated space of paths.

$\pi_n(X, A, x_0) = \pi_0(\mathscr{C}(\mathbf{D}_n, \mathbf{S}_{n-1}, *; X, A, x_0))$: relative homotopy set for $n \geqslant 1$ (group for $n \geqslant 2$).

$u_*: \pi_n(X, A, x_0) \to \pi_n(Y, B, y_0)$: map $[f] \to [u \circ f]$ for u a map $(X, A) \to (Y, B)$ such that $u(x_0) = y_0$.

$\partial: \pi_n(X, A, x_0) \to \pi_{n-1}(A, x_0)$: connecting map of the homotopy exact sequence.

Fibrations (Part 3, chap. III)

$\lambda = (E, B, F, \pi)$ [or (E, B, F), or (E, B, π)]: fibration with total space E, base space B, typical fiber F, projection π.

$E_b = \pi^{-1}(b)$: fiber at the point $b \in B$.

(f, g): $(E, B, \pi) \to (E', B', \pi')$: morphism of fibrations, with $g: B \to B'$ and $f: E \to E'$ continuous, such that $\pi'(f(x)) = g(\pi(x))$.

$f_b = f | E_b$: continuous map $E_b \to E'_{g(b)}$.

$g^*(\lambda)$: pull-back of a fibration (E, B, π) by a continuous map $g: B' \to B$; $g^*(\lambda) = (E', B', \pi')$ with $E' = E \times_B B' \subset E \times B'$, $\pi' = \mathrm{pr}_2 | E'$.

$\lambda' \times \lambda''$: product fibration $(E' \times E'', B' \times B'', \pi' \times \pi'')$ of two fibrations $\lambda' = (E', B', \pi')$, $\lambda'' = (E'', B'', \pi'')$.

$E' \oplus E''$: direct (or Whitney) sum of two vector bundles E', E'' with base space B.

$E' \otimes E''$: tensor product of two vector bundles E', E'' with base space B.

$P \times^G F$: fiber space associated to a principal fiber space (P, B, G) and to an action of G on F.

$\Omega(X, A, x_0) = \mathscr{C}([-1, 1], \{-1, 1\}, *; X, A, x_0)$: space of paths in X with origin at x_0 and extremity in $A \subset X$.

$P_{x_0}X = \Omega(X, X, x_0)$: space of paths in X with origin x_0.

$(P_{x_0}X, X, \Omega(X, x_0), \pi)$: fibration with total space the space of paths $P_{x_0}X$, base space X and fiber $\pi^{-1}(x_0) = \Omega(X, x_0)$, space of loops of origin x_0.

$G_{n,p}(F)$: grassmannian of p-dimensional vector subspaces of the vector space F^n over the field (or skew field) F.

$G'_{n,p}(\mathbf{R})$: special real grassmannian of oriented p-dimensional vector spaces in $\mathbf{R}^n$.

$S_{n,p}(F)$: Stiefel manifold of orthonormal frames of p vectors in F^n, for $F = \mathbf{R}$, $\mathbf{C}$ or $\mathbf{H}$.

(E_G, B_G, G): principal fiber space with n-universal total space E_G, n-classifying base space B_G, structural group G.

Characteristic Classes (Part 3, chap. IV)

$w_r(\xi)$ (or $w_r(E)$): Stiefel–Whitney class $w_r(\xi) \in H^r(B; \mathbf{F}_2)$ of a real vector bundle $\xi = (E, B, \mathbf{R}^n)$.

$w(E; t) = \sum_r w_r(E)t^r$ (or $w(E) = \sum_r w_r(E)$), total Stiefel–Whitney class of E.

$p_k(\xi) \in H^{4k}(B; \mathbf{Z})$: Pontrjagin class of the vector bundle ξ.

$e(\xi) \in H^n(B; \mathbf{Z})$: Euler class of an oriented vector bundle $\xi = (E, B, \mathbf{R}^n)$.

$c_k(\xi)$: Chern class of a complex vector bundle $\xi = (E, B, \mathbf{C}^n)$.

$c(E; t) = \sum_r c_r(E)t^r$ (or $c(E) = \sum_r c_r(E)$), total Chern class of E.

$s_k(c(\xi))$: polynomial expressing $t_1^k + t_2^k + \cdots + t_n^k$ in terms of the elementary symmetric polynomials σ_j in the t_j, where σ_j is replaced by $c_k(\xi)$.

$\mathrm{ch}(\xi)$: Chern character of ξ, equal to $\sum_{k=0}^{\infty} \frac{1}{k!} s_k(c(\xi))$.

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Part 2. The First Applications of Simplicial Methods and of Homology

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