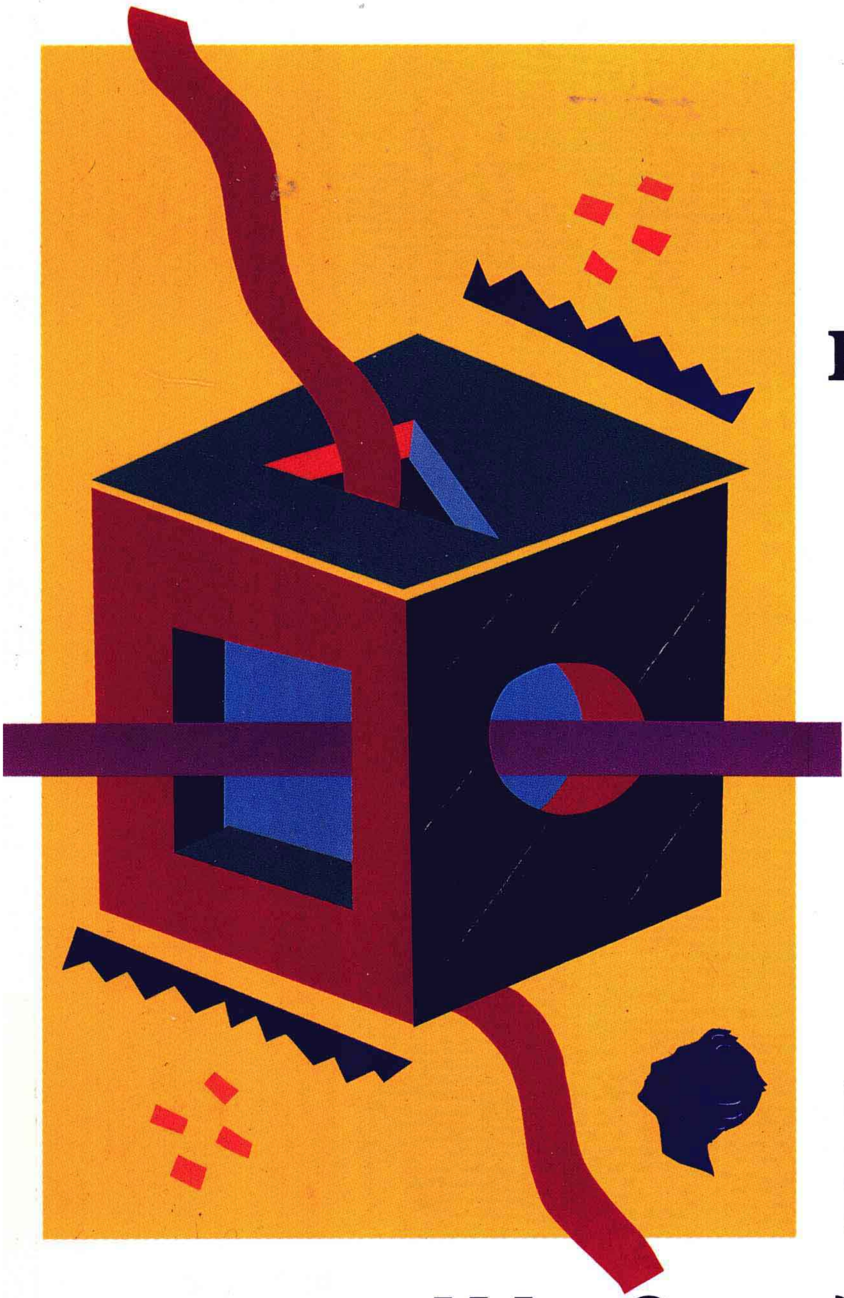

Rosario Urso

Calculus with Applications

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Preface

This book has been designed for a one-term calculus course for students in business, economics, management, and the social and biological sciences. Paced leisurely, it can be used also in a two-term version of such a course. The topics were selected and organized to allow for a maximum flexibility in the book's use.

The book's goal is to make the powerful and interesting subject of calculus as accessible as possible for the intended audience. Among the strategies used to achieve this goal, motivation is foremost: Every section is carefully motivated and includes numerous problems that demonstrate how calculus can be used in a variety of fields.

A prerequisite of intermediate algebra is assumed. In courses where college algebra is the prerequisite, many of the topics in Chapter 1 may be omitted. For classes that need additional work in algebra, an algebra review is provided in Appendix A. The review includes rules, examples, and exercises. It can be used as part of the course or as a reference.

The advent of graphing calculators has made graphical visualization more accessible than ever. Such visualization enhances understanding. Therefore, many sections include graphing calculator material that can easily be woven into the course.

Pedagogical Features

Motivation

In most cases, before defining a concept, the text presents a situation where the concept naturally arises. This approach makes students more receptive to definitions. Concepts are usually motivated through problems of particular interest to the intended audience, rather than problems whose attraction is primarily mathematical. For example, instead of introducing the definite integral through a problem in which the primary goal is to compute an area, the text introduces the definite integral through a problem in which the primary goal is to find a decrease in the size of a timberland.

Problems

Learning occurs mainly through the process of solving problems. So great care has been taken in composing the problem sets. The problem sets have the following features:

(*a.*) They are graded.
(*b.*) They are paired (odd-even) according to similarity.
(*c.*) They are coordinated with the worked-out examples in the text.
(*d.*) Practically every problem set includes a generous supply of applied problems.
(*e.*) A balance exists between the routine drill problems and the more difficult problems.
(*f.*) Review problems grouped by sections appear at the end of each chapter.

Features (*a*) through (*c*) allow instructors to base their lectures on the even (or odd) numbered problems and then assign the odd (or even) numbered problems. Thus, the design of the problem sets enhances the teachability of the book.

Verbalization

Several mathematical organizations recommend that mathematics curricula include more opportunities for students to provide verbalized (written) responses to questions. Therefore, in most problem sets, problems are included that prompt students to respond with an explanation. Problems 35 and 36 in Section 2.7 are examples of such problems, which are signified by the accompanying pencil icon.

Content Features

Few Proofs but Many Plausibility Arguments

The students to whom this book is addressed aspire to become *users* of mathematics, not *makers* of mathematics. Therefore, the text includes few mathematical proofs. Usually, a deducible statement is introduced through a concrete situation that either illustrates the statement or involves steps that parallel those in a general proof of the statement. For instance, the generalized power rule is introduced by computing the derivative of a constant multiple of a power function without using the rule. Then it is shown that the result suggests a shortcut for computing derivatives of such functions. The shortcut, of course, is the generalized power rule.

Graphing Calculator Material

Many sections include graphing calculator problems. A few sections also include a subsection that serves as a basis for some of these problems.

These problems are clearly identified with the icon . Omission of this material will not disturb the continuity of the book.

The approach to the graphing calculator material is generic and, thus, devoid of key pressing discussions. The term "graphing calculator" is also used to mean a computer loaded with graphing software.

Linear regression is introduced early (in Section 1.3). Then problems based on the visual linear regression capability of graphing calculators are included. Problem 36 in Section 4.3 is an example of such a problem.

Of course, graphing calculator problems that are not based on linear regression are also included. For example, Problems 41 and 42 in Section 2.2 ask students to use the graph of a function to approximate the points where the function is not differentiable.

Computer Material

The book includes nine Basic computer programs. The sections where these programs appear also include computer problems. The programs are clearly identified with the icon . As with the graphing calculator material, omission of this material will not disturb the continuity of the book.

EVAL1, in Section 1.1, evaluates a function at consecutive integers in its domain.
EVAL2, in Section 1.1, evaluates a function at any point in its domain.
OPT1, in Section 1.2, finds the minimum and maximum values of a function whose domain is a set of consecutive integers.
OPT2, in Section 2.6, uses the random number generator to approximate the minimum and maximum values of a continuous function on a closed interval.
NEWTON, in Section 3.7, uses Newton's Method to approximate zeros.
PROBE, in Section 3.7, finds the general location of the zeros of a continuous function on a closed interval.
INT, in Section 5.3, uses the random number generator to approximate a definite integral.
SIMP, in Section 6.1, uses Simpson's rule to approximate a definite integral.
LESQ, in Section 7.5, finds least-squares linear functions.

Two Types of Rational Functions Receive Special Attention

Many applications involve quantities that approach a fixed level either at a decreasing rate or at a rate that increases at first and decreases eventually. With certain restrictions on the coefficients, the rational functions

$$f(x) = \frac{ax + b}{cx + d}$$

and

$$f(x) = \frac{ax^2 + b}{cx^2 + d}$$

have properties that make them excellent models for such quantities. Nevertheless, these two functions seldom, if ever, receive the recognition they deserve. In this book, the importance of these functions is elevated by allocating an entire section to them.

Unambiguous Approach to Exponential Functions

Exponential functions are defined as the functions that can be expressed in the form

$$f(x) = Ae^{rx}$$

where $A > 0$ and $r \neq 0$. Such functions form the set of all positive functions that have a constant percentage rate of change.

General Comments

Chapter 1 deals primarily with algebraic functions and limits. Attention is given to finding formulas for functions in applied situations. Limits are defined intuitively without using ϵ and δ. Limits at infinity are presented before limits as x approaches a fixed number. This approach is used because to a beginner the idea of a limit is more natural in the context of the independent variable increasing without bound than in the context of the independent variable approaching a fixed number. Other topics included in Chapter 1 are average rate of change, composite functions, continuity, slopes, and several special functions used in economics.

Chapters 2 and 3 deal with differentiation of algebraic functions of one independent variable. More specifically, in Chapter 2, the derivative is interpreted as a slope and used to solve optimization problems, to approximate change, and to determine where a function increases and where it decreases. At first, derivatives are found by computing limits. But all the rules for computing derivatives of algebraic functions are eventually presented in the chapter. The idea of a function's elasticity appears in a subsection of Section 2.8.

In Chapter 3, the derivative is interpreted as a rate of change. Then the second derivative is used to determine where a function changes at an increasing rate and where it changes at a decreasing rate. In this chapter, the second derivative is also used to determine concavity. Although the students are exposed to the relationship between derivatives and graphs in Chapter 2, it is in Chapter 3 that they are first asked to use derivatives to sketch graphs. Special attention is given to two types of rational functions that have properties compatible with quantities that approach a fixed level. The topic of related rates is presented in a subsection of Section 3.1. Section 3.6 introduces percentage rate of change. The chapter concludes with a section on Newton's method.

Chapter 4 begins with compound interest. This approach motivates the exponential functions, which are defined in terms of base e. Then the natural logarithmic function is introduced and used to help solve problems that involve exponential functions. The chain rule is used to derive the rule for computing derivatives of functions of the form ln $u(x)$. The new rule is then used to derive the rule for computing derivatives of functions of the form $e^{u(x)}$. Chapter 4 ends with a section on two special functions constructed with exponential functions. These two functions are used to model quantities that experience bounded growth.

Chapters 5 and 6 deal with integration of functions of one independent variable. The first section in Chapter 5 introduces antidifferentiation and presents formulas for the antiderivatives of x^n, e^x, and $1/x$. It also presents formulas for the antiderivatives of sums and multiples of these functions. The next section uses the fundamental theorem of calculus as the definition for definite integrals and interprets definite integrals in terms of change. Then Section 5.3 interprets definite integrals in terms of area. Section 5.4 introduces integration by substitution, while Section 5.5 introduces integration by parts. (Integration by tables is presented in Appendix B.) The last section of Chapter 5 shows that a definite integral can be viewed as the limit of a sequence of Riemann sums. Definite integrals are then used to approximate Riemann sums.

Chapter 6 begins with Simpson's rule. Then definite integrals are used to find consumers' and producers' surpluses, probabilities, and expected values. Antidifferentiation is used to solve differential equations that arise where change is bounded or exponential. Improper integrals are dealt with in a subsection of Section 6.3.

In Chapter 7, partial derivatives are computed by applying the differentiation rules from earlier chapters. Such derivatives are then used to approximate change and solve optimization problems. Constrained optimization problems are solved with and without Lagrange multipliers. The method of least squares is used to fit linear and quadratic functions to data points. Chapter 7 can be studied immediately after Chapter 4.

Order of Presentation

The following sections are *not* prerequisites for other sections and, thus, may be omitted without any loss of continuity: 2.8, 3.7, 4.6, 5.5, 5.6, 6.2, 6.4, 6.5, 6.6, 7.5, and 7.7. In addition, Section 6.1 may be omitted if Section 6.2 is omitted, Section 6.3 may be omitted if Section 6.4 is omitted, and Section 7.6 may be omitted if Section 7.7 is omitted.

Student Supplements

1. A **Solutions Manual** is available at a nominal cost through the bookstore. The manual contains detailed solutions to the end-of-section odd exercises as well as solutions to all of the end-of-chapter review exercises.
2. **Environmental and Life Science Applications manual** by Anthony Barcellos (American River College)

3. **Interactive Tutorial Software** package is available for use with IBM, IBM compatibles, and Macintosh computers.
4. A **Videodisk** contains real-life simulations of several applications of calculus concepts.

Instructor Supplements

1. A unique **Computer-Generated Testing System** is available to instructors. This system allows the instructor to create tests using algorithmically generated test questions and those from a standard testbank. This testing system enables the instructor to choose questions either manually or randomly by section, question type, difficulty level, and other criteria. This system is available for IBM, IBM compatibles, and Macintosh computers.
2. A **Printed and Bound Testbank** is also available. This bank is a hard-copy listing of the questions found in the standard testbank.
3. An **Instructor's Manual** is also available. This manual contains all answers and solutions to the exercises in the text. Sample tests, transparencies, and teaching hints and suggestions are also included.
4. **Environmental and Life Science Applications manual** by Anthony Barcellos (American River College)
5. A **Videodisk** contains real-life simulations of several applications of calculus concepts.

For further information about these supplements, please contact your local McGraw-Hill College Division sales representative.

Error Check

Because of careful checking and proofing by a number of mathematics instructors, the author and publisher believe this book to be substantially error-free. For any errors remaining, the author would be grateful if they were sent to Mathematics Editor, College Division, 27th floor, McGraw-Hill, 1221 Avenue of the Americas, New York, NY 10020.

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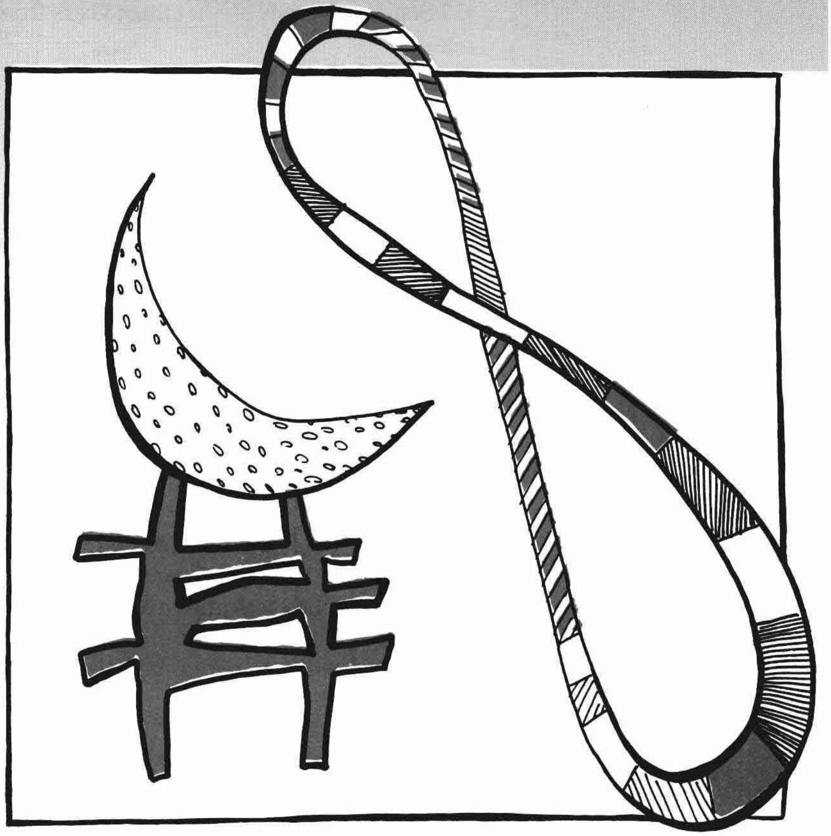
Rosario Urso

Contents

CHAPTER 1

Functions and Limits



This book is mainly about *derivatives*. Derivatives are *functions*. They are defined in terms of *limits* and used to analyze other functions. It is fitting, therefore, that we begin with a chapter on functions and limits.

1.1 Functions

Many situations involve a variable quantity whose values *depend* on the values of another variable quantity. For instance, the monthly demand for a product depends on the price. In such a situation, there often exists a relatively simple mathematical formula that expresses the dependency, at least approximately. In this section, we deal with formulas that can be used in this manner. We begin with an example that involves a variable quantity whose values depend on time.

Social scientists predict that t years from now (up to $t = 10$) there will be

$$\frac{8t + 10}{t + 2}$$

million single-parent families in a Third World country. With this formula, we can predict the number of such families for any time during the next 10 years. For example, by substituting 2 for t in the formula we get

$$\frac{8(2) + 10}{2 + 2} = 6.5$$

This result tells us that 2 years from now there will be 6.5 million single-parent families in the country.

Let s represent the number of single-parent families (in millions) t years from now. Then

$$s = \frac{8t + 10}{t + 2}$$

for $0 \le t \le 10$. The above formula is a rule that associates with each admissible value of t exactly one corresponding value of s. For example, 6.5 is the only value the formula assigns to s when $t = 2$. Table 1.1 shows the values of s that correspond to several values of t. The table suggests that the number of single-parent families will increase during the next 10 years.

Table 1.1

t	0	1.2	2	3	4.4	6	8
s	5	6.125	6.5	6.8	7.0625	7.25	7.4

Definition 1.1

Suppose x and y are variables. A **function** is a rule that associates with each value of x *exactly one* corresponding value of y.

In the context of Definition 1.1, we say "y is a function of x according to the rule." We refer to x as the **independent variable** and to y as the **dependent variable**. The set of admissible values for the independent variable is the **domain** of the function, while the set of corresponding values of the dependent variable is the **range** of the function. In this book, we deal primarily with functions that can be expressed with mathematical formulas.

In the single-parent families situation, s is a function of t defined by the rule

$$s = \frac{8t + 10}{t + 2}$$

For this function, the independent variable is t, and the dependent variable is s. According to the social scientists, the formula for s is valid up to $t = 10$. Therefore, the domain of the function is the set of real numbers t such that $0 \leq t \leq 10$, which is the closed interval $[0, 10]$. (Real numbers and intervals are discussed in Section A.1 of Appendix A.) Although we will not show the details, the range consists of the closed interval $[5, 7.5]$. Thus, the number of single-parent families will not exceed 7.5 million during the next 10 years.

Symbols for Functions

We often use letters as names for functions. If, for instance, we use the letter f as the name of a function whose independent variable is x, the symbol $f(x)$ (read "f of x") represents the dependent variable. So there is an important difference between the symbols f and $f(x)$: f is the name of the function, while $f(x)$ represents the dependent variable. Nevertheless, in this book, we take the liberty of using the symbol $f(x)$ to represent *both* the function and the dependent variable. Which of the two it represents will be clear from the context.

According to our convention, we could use the symbol $g(n)$ (read "g of n") to represent a function whose independent variable is n. Our convention has the advantage of exhibiting the letter that represents the independent variable.

We like to represent functions with symbols that reflect the meaning of the variables. For example, since "single" begins with "s" and "time" begins with "t," we will use $s(t)$ to represent the function that predicts the number of single-parent families t years from now. Then

$$s(t) = \frac{8t + 10}{t + 2}$$

for $0 \leq t \leq 10$. According to this notation, $s(2)$, for example, represents the value of $s(t)$ when $t = 2$. Thus, $s(2) = 6.5$.

Assumption about Domains

If we remove the function

$$s(t) = \frac{8t + 10}{t + 2}$$

from its applied role and consider it from a strictly mathematical point of view, its domain is the set of all real numbers except -2. We exclude -2 from the domain because the denominator is 0 if -2 is substituted for t, and $s(-2)$, therefore, is not a real number.

From now on, we make the following assumption about domains:

> The domain of a function $f(x)$ is the set of all real numbers x such that $f(x)$ is also a real number, unless additional restrictions on x are specified or implied by an application.

Example 1.1

Find the domain of the function $f(x) = \sqrt{5 - x}$.

Solution

The square root of a negative number is not a real number. Therefore, if $x > 5, f(x)$ is not a real number. Otherwise, $f(x)$ is a real number. Thus, the domain of the function is the set of real numbers x such that $x \leq 5$, which is the half-open interval $(-\infty, 5]$.

As we saw, the domain of the function in Example 1.1 is the half-open interval $(-\infty, 5]$. We may require additional restrictions on the domain if we use the function to express the relationship between two variables in an application. For instance, if x represents a product's price, we would exclude negative numbers from the domain, because prices are not represented by negative numbers.

Graphs of Functions

Functions generate **ordered pairs** of real numbers. To illustrate, we return to the function

$$s(t) = \frac{8t + 10}{t + 2}$$

used in the single-parent families example. This function generates the collection of ordered pairs of real numbers of the form $(t, s(t))$, where t is in the closed interval $[0, 10]$. Since $s(2) = 6.5$, one ordered pair generated by the function $s(t)$ is $(2, 6.5)$. [*Warning*: Here $(2, 6.5)$ does not represent

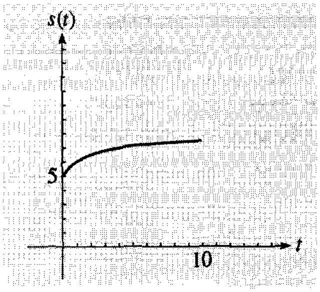


Figure 1.1
$s(t) = (8t + 10)/(t + 2)$ for t in [0, 10].

an interval.] Of course, since there are infinitely many numbers in the interval [0, 10], the function generates infinitely many ordered pairs.

The ordered pairs generated by a function can be represented by points in a **rectangular coordinate plane**. We establish such a plane by selecting a horizontal number line (**horizontal axis**) and a vertical number line (**vertical axis**) that meet at their respective origins. Then, the ordered pair (2, 6.5), for instance, can be represented by the point located 6.5 units above the point with coordinate 2 on the horizontal axis (t axis). The infinitely many points that correspond to the infinitely many ordered pairs generated by the function $s(t)$ form the curve in Figure 1.1.

We could produce the curve in Figure 1.1 by plotting several ordered pairs generated by the function $s(t)$ and then sketching a curve that passes through the points. Later, however, we will learn a method that will enable us to produce this curve more efficiently.

The **graph** of a function is the set of points in a rectangular coordinate plane that corresponds to the set of ordered pairs generated by the function. Thus, the graph of the function $s(t)$ is the curve in Figure 1.1.

Graphs reveal important information about the functions they represent. For example, the curve in Figure 1.1 reveals that the number of single-parent families will increase during the next 10 years. Actually, the curve reveals that this number will increase at a slower and slower rate. Later we will realize how the curve reveals this information.

Rectangular coordinate planes are sometimes called **cartesian planes** in recognition of the Frenchman René Descartes (1596–1650), who pioneered the use of coordinate systems.

The first number in an ordered pair of numbers is the **abscissa** of the corresponding point in a cartesian plane. The second number is the **ordinate** of the point. Thus, 2 is the abscissa and 6.5 is the ordinate of the point that represents the ordered pair (2, 6.5). If the horizontal axis is labeled the x axis and the vertical axis is labeled the y axis, the abscissa could be called the ***x* coordinate** and the ordinate could be called the ***y* coordinate**.

Constant Functions

In the single-parent families situation, the value of $s(t)$ varies whenever the value of t varies. This condition is not necessary for a rule to be a function. If the value of the dependent variable of a function does not change whenever the value of the independent variable varies, we call the function a **constant function**. An example of such a function is expressed by the formula

$$g(x) = 0.24$$

Here the value of $g(x)$ is 0.24 regardless of the value of x. The range of this constant function consists of the single number 0.24.