

MATHEMATICS
FOR TECHNICAL STUDENTS
JUNIOR COURSE

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MATHEMATICS FOR TECHNICAL STUDENTS

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JUNIOR COURSE

BY

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WISHAW HIGH SCHOOL

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PREFACE

Mathematics treated as an abstract science appeals strongly to very few, but the applications of Mathematics to problems of common life and physical science are of almost universal interest. It has been the author's aim in the present book to illustrate the subject wherever possible by problems designed to maintain the student's interest, and to give him confidence in the use of what are to him new methods of calculation.

The problems are chosen from a wide range of subjects, the only limit being the amount of general knowledge that the student may reasonably be expected to possess. Where technical knowledge is required sufficient explanation is given by the context. Due attention has been paid to the grading of the examples, so as to avoid the discouragement which is the result of failure at the beginning of a set.

As the groundwork of mathematics is the same for all classes of students, the author's previous books on *Mining Mathematics* have been freely used in the preparation of the text, and also as a source of numerical examples ; but the majority of the problems are printed here for the first time. The treatment throughout is necessarily concise, but the endeavour has been made to omit nothing that is essential and to lay a firm foundation for more advanced work.

The book contains a full two-years' course for students who have already done the work of an elementary school. The work of the first year might consist of the Arithmetic, Algebra up to Simple Equations, the first chapter of Geometry and the easier parts of Mensuration. The second year would be occupied by Algebra, beginning with Logarithms, the second chapter on Geometry, the remainder of the Mensuration, and the chapter on Trigonometry. The applications discussed in the final chapter are placed there in order to avoid interruption of the

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mathematical argument by long digressions. They have been selected on account of their interest or practical importance, and most of them may be taken at any stage after Simple Equations.

The answers to the examples are in many cases approximate, the degree of accuracy being suited to that of the data which are supplied.

The table on p. 282 is reprinted from Professor J. Perry's *Practical Mathematics* by permission of the Controller of H.M. Stationery Office.

PREFACE TO THE THIRD EDITION

The main change in this edition is the extension of Trigonometry to include solution of triangles by the Sine and Cosine rules. This was required because some of the courses for which the book is adapted include more trigonometry than formerly. Methods of solution without these rules are also indicated, as exercises.

There are, however, other considerable additions to the text. Problems solved by simple and simultaneous equations have been supplemented by two additional sets of examples, XXXVA and XLIA. These are intended to be, on the whole, rather easier than the 'B' sets, which remain as they were. This makes up a generous allowance of such problems, justified, probably, by their importance as an interesting and valuable exercise.

Chapter XII, on algebraical graphs, has been remodelled, the subject being introduced by a study of graphs of functions rather than, as before, graphs of equations. For the experienced this distinction is only a matter of emphasis on one aspect rather than another, but beginners often have difficulty with the graph of an equation, where x and y flow on together, while they readily understand the graph of a function as a picture of successive values of the dependent. Linear formulæ obtained from experimental data are fully discussed.

PREFACE TO THIRD EDITION

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Two new sets of examples precede those on the practical use of logarithms. These, besides giving some practice in indices, will make more sure that the fundamental operations are well understood. Similarity also receives more extended treatment than previously, and at the end of the last chapter come further exercises—many of which may be taken earlier—on transformation of formulæ.

The majority of the sets of examples found in previous editions have not been interfered with, although some data are altered and a few examples have been added here and there. Transition in the case of classes who use the book will therefore be easy.

S. N. F.

December, 1945.

TABLES

LENGTH (LINEAR MEASURE)

12 inches = 1 foot.
 3 feet = 1 yard.
 220 yards = 1 furlong.
 1760 yards
 (8 furlongs) = 1 mile.

 1 chain = 100 links = 22 yards
 1 pole = $\frac{1}{4}$ chain = $5\frac{1}{2}$ yards.
 1 fathom = 6 feet.

AREA (SQUARE MEASURE)

144 square inches = 1 square foot.
 9 square feet = 1 square yard.
 4840 square yards = 1 acre.
 640 acres = 1 square mile.

 1 acre = 4 roods.
 1 rood = 40 poles (or square poles).

 1 acre = 10 square chains.
 1 square chain = 10,000 square links.

VOLUME (CUBIC MEASURE)

1728 cubic inches = 1 cubic foot. 27 cubic feet = 1 cubic yard.

CAPACITY

4 gills = 1 pint.
 2 pints = 1 quart.
 4 quarts = 1 gallon.
 2 gallons = 1 peck.
 4 pecks = 1 bushel.
 8 bushels = 1 quarter.

WEIGHT (AVOIRDUPOIS)

16 ounces = 1 pound (lb.).
 28 pounds = 1 quarter.
 4 quarters
 (112 lbs.) = 1 hundredweight
 (cwt.).
 20 cwts. (2240 lbs.) = 1 ton.

 1 stone = 14 lbs.
 1 pound = 7000 grains.

1 cubic foot of water at 62° F. weighs 62.4 lbs. (nearly) or 1000 ounces (roughly).

1 gallon of water at 62° F. weighs 10 lbs., and contains about $277\frac{1}{2}$ cubic inches.

ANGULAR MEASURE

60 seconds (60") = 1 minute (1').
 60 minutes = 1 degree (1°).
 90 degrees = 1 right angle.
 360 degrees = 1 complete revolution.

CONVERSION TABLE FOR METRIC SYSTEM

1 metre = 39.37 inches (approximately).	
1 centimetre = .3937 inch.	1 inch = 2.540 centimetres
1 metre = 1.094 yards.	1 yard = .9144 metre.
1 kilometre = .6214 mile.	1 mile = 1.609 kilometres.
1 kilogram = 2.205 pounds.	1 pound = .4536 kilogram.
1 litre = .2200 gallon.	1 gallon = 4.546 litres.

Also see Art. 32, page 26.

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MATHEMATICS FOR TECHNICAL STUDENTS

CHAPTER I

FACTORS AND MULTIPLES

FACTORS

1. A factor of a number is a number which will divide the given number without leaving any remainder. Thus 2, 3, 4 and 6 are factors of 12.

Every number has at least two factors, viz. itself and 1. A number that has no other factors than itself and unity is called a **prime number**. Thus 2, 3, 5, and 7 are prime numbers.

If a factor is a prime number, it is called a **prime factor**. The prime factors of a number may be found by guessing them one at a time and dividing them out in succession.

Ex. Find the prime factors of 3960.

2	3960
2	1980
2	990
3	495
3	165
5	55
11	11
	1

$$\begin{aligned}\therefore 3960 &= 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 11 \\ &= 2^3 \times 3^2 \times 5 \times 11.\end{aligned}$$

In the latter form of the result, 2^3 (read 'two cubed,' or 'two to the third power') means $2 \times 2 \times 2$; and 3^2 (read 'three squared') means 3×3 .

EXAMPLES I

1. Write down all the prime numbers between 1 and 30.

Find the prime factors of the numbers given in examples 2-13.

2. 105.

3. 210.

4. 418.

5. 546.

6. 990.

7. 2310.

8. 1386.

9. 18,018.

10. 40,425.

11. 83,160.

12. 213,444.

13. 1,944,000.

14. Write down all the factors of 30.

15. Is 401 a prime number?

16. Is 1369 a prime number?

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2. A **common factor** of two or more numbers is a number which will divide each of the given numbers without leaving any remainder. Thus 3 is a common factor of 12 and 18.

The **highest common factor (H.C.F.)** of two or more numbers is the greatest number which will divide each of the given numbers without leaving any remainder. Thus 6 is the highest common factor of 12 and 18.

The highest common factor is often known as the **greatest common measure (G.C.M.)**.

The H.C.F. of two or more numbers may be found by resolving the numbers into their prime factors.

Ex. Find the H.C.F. of 1890 and 2340.

$$1890 = 2 \times 3^3 \times 5 \times 7;$$

$$2340 = 2^2 \times 3^2 \times 5 \times 13;$$

Therefore both numbers are divisible by 2, by 3^2 , and by 5;
 \therefore the H.C.F. $= 2 \times 3^2 \times 5 = 90$.

3. The H.C.F. of two numbers may also be found by a process of division. The method depends on the fact that a common factor of two numbers is also a factor of their difference. Hence by subtracting one number from the other time after time we arrive finally at the H.C.F. Division takes the place of a series of subtractions.

Ex. Find the H.C.F. of 495 and 1440.

495)1440(2	1	495	1440	2
990		450	990	
<hr style="width: 100px; margin: 0;"/> 450)495(1		45	450	10
450			450	
<hr style="width: 100px; margin: 0;"/> 45)450(10				
450				
<hr style="width: 100px; margin: 0;"/>				

1440 is divided by 495; the remainder is 450, and therefore the required H.C.F. must be the H.C.F. of 495 and 450. 495 is divided by 450, and the process is repeated until an exact divisor is reached; this divisor must be the required H.C.F.

Therefore the H.C.F. of 495 and 1440 is 45.

Two methods of putting down the work are shown: the second is more compact.

EXAMPLES II

1. Write down all the common factors of 24 and 36.
2. Write down the highest common factor of 24 and 36.
3. Write down the common factors of 32 and 48.
4. Write down the highest common factor of 32 and 48.

Find by the method of prime factors the H.C.F. of the numbers given in examples 5-10.

5. 84 and 96.
6. 190 and 798.
7. 24, 42, and 51.
8. 78, 182, and 195.
9. 84, 140, and 616.
10. 204, 595, and 714.

Find by the method of division the H.C.F. of the numbers given in examples 11-18.

11. 2117 and 4745.
12. 296 and 703.
13. 2051 and 29,995.
14. 867 and 5967.
15. 900 and 3996.
16. 9317 and 16,335.
17. 8255 and 160,303.
18. 306, 810, 2214.

19. Find the greatest number that will divide 12,900 and 23,908 without leaving a remainder.

20. A floor 19 ft. 10 ins. long and 14 ft. broad is to be laid with square tiles. What is the largest size of tile that can be laid in order that no tiles may have to be cut?

MULTIPLES

4. A **multiple** of a number is a number that can be divided by the given number without leaving any remainder. Thus 12 and 18 are multiples of 6.

A **common multiple** of two or more numbers is a number that can be divided by each of the given numbers without leaving any remainder. Thus 12 is a common multiple of 4 and 6.

The **least common multiple** (L.C.M.) of two or more numbers is the smallest number that can be divided by each of the given numbers without leaving a remainder.

If the numbers are small, their L.C.M. is easily found by inspection, e.g. the L.C.M. of 2, 3, and 8 is 24.

If large numbers occur, the L.C.M. may be found by resolving them into their prime factors.

Ex. Find the L.C.M. of 12, 8, 16, and 30.

$$\begin{array}{lcl}
 12=2^2 \times 3; & & \\
 8=2^3; & & \\
 16=2^4; & & \\
 30=2 \times 3 \times 5; & \left. \begin{array}{l} \\ \\ \\ \end{array} \right\} & \begin{array}{l} \text{Hence, in order to be divisible by each of} \\ \text{the above numbers, the L.C.M. must contain} \\ \text{the factors } 2^4, 3, \text{ and } 5. \\ \therefore \text{ the L.C.M.} = 2^4 \times 3 \times 5 = 240. \end{array}
 \end{array}$$

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Another method of writing down the work is as follows :

2	12, 8, 16, 30	
2	6, 8, 15	∴ the L.C.M. = $2^3 \times 3 \times 4 \times 5 = 240$.
3	3, 4, 15	
	1, 4, 5	

At the commencement of the work, 8 is deleted because it is a factor of 16. All prime factors common to two or more numbers are then divided out, beginning with the smallest. When no factors remain which are common to two or more numbers, the divisors and the remaining factors are all multiplied together. The principle of the method is that when a factor is common to several numbers, it is used only once in their L.C.M.

EXAMPLES III

1. Write down all the multiples of 9 between 10 and 80.
2. Write down all the multiples of 12 between 10 and 80.
3. Write down all the common multiples of 9 and 12 between 10 and 80.

4. Write down the least common multiple of 9 and 12.

Find the L.C.M. of the numbers given in examples 5-16.

5. 2, 3, 4, 8.
6. 2, 3, 4, 5, 6.
7. 2, 4, 5, 6, 8, 10.
8. 5, 11, 55, 121.
9. 3, 7, 9, 14, 21.
10. 14, 16, 18, 24.
11. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.
12. 7, 14, 21, 28.
13. 9, 11, 18, 27, 44.
14. 5, 7, 15, 21, 35.
15. 13, 39, 65, 143.
16. 14, 17, 21, 68.
17. Find the smallest number that is exactly divisible by 108 and 144.
18. The product of any two numbers is equal to the product of their H.C.F. and L.C.M. Test the truth of this statement, using (a) 16 and 24, (b) 72 and 96.

19. Two gear-wheels which mesh with one another have 18 and 30 teeth respectively. How many revolutions of each wheel will bring the same pair of teeth again into contact ?

20. Two cyclists ride bicycles with different gears, so that A travels 22 ft. for each turn of his pedals, while B goes 18 ft. 4 ins. for each turn of his pedals. If they start together, with the right-hand pedals at the top of a stroke, and proceed at the same speed, how far will they travel before both pedals are again in the same position ?

CHAPTER II

VULGAR FRACTIONS

5. An integer consists of a whole number of units; thus 16 means sixteen units. Parts of a unit are called **fractions**. One fourth-part of a unit is written as $\frac{1}{4}$, three fourth-parts of a unit is written $\frac{3}{4}$, and so on. The number above the horizontal line is referred to as the **numerator**, and the number below the line as the **denominator**. Thus *the denominator indicates into how many equal parts a unit has been divided, while the numerator shows how many of these have been taken.*

Three fourth-parts of one unit is evidently equivalent to one fourth-part of three units; therefore the horizontal line may be looked upon as a sign of division, and $\frac{3}{4}$ may be considered to mean 3 divided by 4.

A **proper fraction** is a fraction in which the numerator is less than the denominator.

An **improper fraction** is a fraction in which the numerator is greater than the denominator; e.g. $\frac{13}{5}$ is an improper fraction.

If the numerator of a fraction is equal to the denominator, the value of the fraction is unity; e.g. $\frac{5}{5}=1$.

A **mixed number** is the sum of an integer and a proper fraction. The sign of addition between the integer and the fraction is omitted; thus $5\frac{3}{8}=5+\frac{3}{8}$.

An improper fraction may be reduced to a mixed number by dividing the numerator by the denominator; thus $\frac{13}{5}=2\frac{3}{5}$. The reason for this is evident on remembering that $\frac{5}{5}=1$.

A mixed number is reduced to an improper fraction by multiplying the integer by the denominator and adding the numerator to the result; thus $5\frac{3}{8}=\frac{40}{8}+\frac{3}{8}=\frac{43}{8}$.

6. The denominator of a fraction may be increased or dimin-

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ished provided that a corresponding change is made in the numerator; thus

$$\begin{aligned}\frac{3}{8} &= \text{three eighth-parts} \\ &= \text{six sixteenth-parts (since one eight-part is evidently equal} \\ &\quad \text{to two sixteenth-parts)} \\ &= \frac{6}{16}.\end{aligned}$$

From this and similar examples we see that the denominator of a fraction may be increased by multiplying the numerator and denominator by the same number.

Thus $\frac{5}{6} = \frac{5 \times 4}{6 \times 4} = \frac{20}{24}.$

Similarly, the denominator of a fraction may be diminished by dividing the numerator and denominator by any common factor.

Thus $\frac{45}{60} = \frac{9}{12} = \frac{3}{4}.$

When all the common factors have been divided out of numerator and denominator, the fraction is said to have been reduced to its lowest terms.

Ex. Reduce the fraction $\frac{219}{803}$ to its lowest terms.

As no factors common to numerator and denominator can readily be seen, we employ the division method to find the H.C.F., and obtain the result 73. Dividing numerator and denominator by 73 we find that

$$\frac{219}{803} = \frac{3}{11}.$$

EXAMPLES IV

1. Express the following mixed numbers as improper fractions:

$$1\frac{1}{2}, 2\frac{3}{7}, 3\frac{1}{4}, 1\frac{15}{16}, 7\frac{5}{8}, 10\frac{1}{8}, 33\frac{1}{3}, 111\frac{1}{9}.$$

2. Reduce to mixed numbers: $\frac{7}{2}, \frac{10}{3}, \frac{16}{5}, \frac{9}{5}, \frac{100}{9}, \frac{80}{11}, \frac{60}{13}, \frac{27}{8}, \frac{16}{4}, \frac{20}{5}.$

3. Express as fractions having 12 as denominator: $\frac{1}{2}, \frac{2}{3}, \frac{5}{6}, 2\frac{1}{4}, 1\frac{1}{6}, 3\frac{1}{2}.$

4. Reduce the following fractions to fractions having 20 as denominator, and hence find which is the greatest and which is the least: $\frac{1}{2}, \frac{3}{5}, \frac{11}{20}, \frac{3}{4}, \frac{7}{10}.$

Reduce to their lowest terms the fractions given in examples 5-14.

$$\begin{array}{lllll} 5. \frac{6}{8}. & 6. \frac{5}{10}. & 7. \frac{18}{27}. & 8. \frac{16}{24}. & 9. \frac{121}{22}. \\ 10. \frac{75}{100}. & 11. \frac{48}{44}. & 12. \frac{67}{134}. & 13. \frac{825}{4400}. & 14. \frac{221}{238}. \end{array}$$

7. Addition of fractions. It is clear that we cannot *directly* add fractions that have different denominators, as, for instance, $\frac{2}{3}$ and $\frac{1}{2}$. But if we remember that the numerator and denomina-

tor of a fraction may be changed by multiplying both by the same number, we have

$$\frac{2}{3} = \frac{4}{6}, \text{ and } \frac{1}{2} = \frac{3}{6};$$

$$\therefore \frac{2}{3} + \frac{1}{2} = \frac{4}{6} + \frac{3}{6} = \frac{7}{6} = 1\frac{1}{6}.$$

The denominator 6 is selected because it is a common multiple of the other denominators 3 and 2, and the addition is performed with the least possible labour if the denominator selected in this way is the least common multiple of the given denominators.

Ex. Add together $2\frac{5}{6}$, $1\frac{7}{8}$, $\frac{5}{12}$.

The L.C.M. of the denominators is 24.

$$\begin{aligned} \therefore 2\frac{5}{6} + 1\frac{7}{8} + \frac{5}{12} &= 2 + 1 + \frac{20 + 21 + 10}{24} \\ &= 3 + \frac{51}{24} \\ &= 3 + \frac{17}{8} \\ &= 3 + 2\frac{1}{8} \\ &= 5\frac{1}{8}. \end{aligned}$$

8. The subtraction of one fraction from another is accomplished in a similar manner.

Ex. 1. Subtract $\frac{5}{12}$ from $\frac{9}{16}$.

The L.C.M. of the denominators is 48.

$$\therefore \frac{9}{16} - \frac{5}{12} = \frac{27 - 20}{48} = \frac{7}{48}.$$

Ex. 2. Subtract $5\frac{7}{8}$ from $9\frac{1}{4}$.

$\frac{7}{8}$ cannot be subtracted from $\frac{1}{4}$. The mixed numbers may be converted into improper fractions, but it is better to proceed thus:

$$\begin{aligned} 9\frac{1}{4} - 5\frac{7}{8} &= 8 - 5 + 1\frac{1}{4} - \frac{7}{8} \quad (\text{since } 9\frac{1}{4} = 8 + 1\frac{1}{4}) \\ &= 3 + \frac{10 - 7}{8} \\ &= 3\frac{3}{8}. \end{aligned}$$

9. It is often necessary to simplify combinations of fractions in which both *plus* and *minus* signs occur.

Ex. 1. Simplify $2\frac{1}{8} + 3\frac{7}{8} - 5\frac{1}{4} + 1\frac{1}{2}$.

Taking the integers separately, we have

$$2 + 3 + 1 = 6, \text{ and } 6 - 5 = 1.$$

Simplifying the fractions, we have

$$\frac{1}{8} + \frac{7}{8} - \frac{1}{4} + \frac{1}{2} = \frac{4 + 21 - 6 + 12}{24} = \frac{31}{24} = 1\frac{7}{24}.$$

Finally,

$$1 + 1\frac{7}{24} = 2\frac{7}{24}.$$

Ex. 2. The construction of a line of railway was commenced at both ends. When $\frac{11}{36}$ of the whole line had been laid from one end, $\frac{2}{9}$ had been laid from

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the other. If the distance remaining to be laid was 34 miles, what was the length of the line?

The fraction of the line already laid $= (\frac{1}{3} + \frac{2}{9}) = \frac{1}{3} + \frac{2}{9}$;

\therefore the fraction remaining $= 1 - \frac{1}{3} + \frac{2}{9} = \frac{3}{3} - \frac{1}{3} + \frac{2}{9} = \frac{1}{3}$;

$\therefore \frac{1}{3}$ of the whole line $= 34$ miles;

$\therefore \frac{1}{3}$ of the whole line $= 2$ miles;

\therefore the length of the line $= 72$ miles.

Check: $\frac{1}{3}$ of 72 mls. $= 22$ mls., and $\frac{2}{9}$ of 72 mls. $= 16$ mls.;

\therefore the length of track laid $= 38$ mls.;

\therefore the distance remaining $= 72$ mls. $- 38$ mls. $= 34$ mls.;

which agrees with the statement of the problem.

Answers to problems should always be checked in some such manner as this.

EXAMPLES V

1. $\frac{2}{3} + \frac{1}{2}$.
2. $\frac{4}{7} + \frac{7}{8}$.
3. $\frac{5}{11} + \frac{11}{12}$.
4. $\frac{6}{19} + \frac{29}{57}$.
5. $\frac{5}{6} + \frac{9}{14}$.
6. $\frac{13}{15} + \frac{15}{17}$.
7. $\frac{3}{4} + \frac{2}{3} + \frac{1}{6}$.
8. $\frac{3}{5} + \frac{1}{3} + \frac{7}{15}$.
9. $\frac{3}{8} + \frac{1}{4} + \frac{5}{6}$.
10. $\frac{4}{7} + \frac{2}{3} + \frac{20}{21}$.
11. $\frac{1}{2} + \frac{2}{3} + \frac{5}{6} + \frac{7}{9}$.
12. $\frac{5}{8} + \frac{2}{3} + \frac{5}{4} + \frac{5}{6}$.
13. $\frac{1}{3} + \frac{3}{4} + \frac{8}{9} + \frac{7}{36}$.
14. $2\frac{1}{2} + 1\frac{1}{3}$.
15. $4\frac{3}{8} + 3\frac{5}{12}$.
16. $5\frac{2}{7} + 3\frac{11}{14}$.
17. $2\frac{9}{10} + 5\frac{7}{8}$.
18. $4\frac{5}{9} + 5\frac{13}{18} + 2\frac{10}{27}$.
19. $\frac{3}{8} + \frac{2}{5} + \frac{31}{16}$.
20. $\frac{25}{9} + \frac{17}{3} + \frac{25}{27}$.
21. $\frac{39}{5} + \frac{27}{8} + \frac{91}{40}$.
22. $1\frac{10}{17} + 2\frac{33}{34}$.
23. $3\frac{1}{19} + 1\frac{53}{57} + \frac{5}{8}$.
24. $2\frac{7}{30} + 5\frac{5}{9} + 1\frac{44}{45}$.
25. $\frac{3}{4} - \frac{1}{2}$.
26. $\frac{5}{12} - \frac{1}{6}$.
27. $\frac{7}{8} - \frac{5}{6}$.
28. $\frac{9}{11} - \frac{3}{7}$.
29. $\frac{4}{5} - \frac{3}{15}$.
30. $\frac{7}{9} - \frac{7}{12}$.
31. $6\frac{5}{12} - 3\frac{1}{4}$.
32. $4\frac{6}{7} - 1\frac{1}{6}$.
33. $2\frac{10}{10} - 1\frac{3}{5}$.
34. $8\frac{5}{6} - 3\frac{9}{16}$.
35. $5\frac{1}{2} - 1\frac{13}{16}$.
36. $9\frac{2}{10} - 4\frac{8}{25}$.
37. $\frac{1}{6} + \frac{4}{9} + \frac{2}{3} - \frac{3}{4}$.
38. $2\frac{3}{5} + \frac{2}{3} - \frac{1}{2} + \frac{5}{6}$.
39. $\frac{7}{9} - \frac{3}{5} - \frac{1}{3} + \frac{13}{16}$.
40. $\frac{9}{14} + \frac{15}{16} - \frac{7}{7} - \frac{1}{3}$.
41. $1\frac{3}{4} - \frac{7}{12} + 2\frac{5}{24} - 1\frac{1}{6}$.
42. $2\frac{1}{3} - \frac{1}{2} + 1\frac{1}{4} - 1\frac{5}{6}$.
43. Find the greatest and the least of the following fractions: $\frac{7}{10}$, $\frac{3}{8}$, $\frac{2}{5}$, $\frac{9}{16}$, $\frac{1}{24}$.

44. Gunpowder consists of saltpetre, charcoal, and sulphur. If $\frac{3}{4}$ of the weight is saltpetre and $\frac{3}{20}$ is charcoal, how much is sulphur?

45. When $\frac{5}{8}$ of a chimney stalk had been built, the stalk was 200 ft. high. How many more feet had to be built in order to finish the stalk?

46. Brass contains $\frac{9}{25}$ of its weight of zinc, the remainder being copper. How much brass would 48 lbs. of copper make?