# Advances in

# HEAT TRANSFER

# ACADEMIC PRESS, INC.

(Harcourt Brace Jovanovich, Publishers)

Orlando · San Diego · New York · London

Toronto • Montreal • Sydney • Tokyo

COPYRIGHT © 1984, BY ACADEMIC PRESS, INC.
ALL RIGHTS RESERVED.
NO PART OF THIS PUBLICATION MAY BE REPRODUCED OR
TRANSMITTED IN ANY FORM OR BY ANY MEANS, ELECTRONIC
OR MECHANICAL, INCLUDING PHOTOCOPY, RECORDING, OR ANY
INFORMATION STORAGE AND RETRIEVAL SYSTEM, WITHOUT
PERMISSION IN WRITING FROM THE PUBLISHER.

ACADEMIC PRESS, INC. Orlando, Florida 32887

United Kingdom Edition published by ACADEMIC PRESS, INC. (LONDON) LTD. 24/28 Oval Road, London NW1 7DX

Library of Congress Cataloging in Publication Data 63-22329
ISBN 0-12-020016-3

PRINTED IN THE UNITED STATES OF AMERICA

84 85 86 87 9 8 7 6 5 4 3 2 1

# Contributors to Volume 16

M. G. COOPER
BENJAMIN GEBHART
DAVID S. HILDER
W. M. KAYS
MATTHEW KELLEHER
R. J. MOFFAT
RICHARD A. W. SHOCK
JOHN R. THOME

#### CONTRIBUTORS

Numbers in parentheses indicate the pages on which the authors' contributions begin.

- M. G. COOPER (157), Department of Engineering Science, Oxford University, Oxford OX1 3PJ, England
- BENJAMIN GEBHART (1), Department of Mechanical Engineering and Applied Mechanics, University of Pennsylvania, Philadelphia, Pennsylvania 19104
- DAVID S. HILDER (1), Philadelphia Naval Shipyard, Philadelphia, Pennsylvania 19112
- W. M. KAYS (241), Department of Mechanical Engineering, Thermosciences Division, Stanford University, Stanford, California 94305
- MATTHEW KELLEHER (1), Department of Mechanical Engineering, Naval Postgraduate School, Monterey, California 93940
- R. J. MOFFAT (241), Department of Mechanical Engineering, Thermosciences Division, Stanford University, Stanford, California 94305
- RICHARD A. W. SHOCK (59), Heat Transfer and Fluid Flow Service, Atomic Energy Research Establishment Harwell, Didcot, Oxfordshire OX11 0RA, England
- JOHN R. THOME (59), Department of Mechanical Engineering, Michigan State University, East Lansing, Michigan 48824

#### PREFACE

The serial publication Advances in Heat Transfer is designed to fill the information gap between the regularly scheduled journals and university-level textbooks. The general purpose of this publication is to present review articles or monographs on special topics of current interest. Each article starts from widely understood principles and in a logical fashion brings the reader up to the forefront of the topic. The favorable response by the international scientific and engineering community to the volumes published to date is an indication of how successful our authors have been in fulfilling this purpose.

The Editors are pleased to announce the publication of Volume 16 and wish to express their appreciation to the current authors who have so effectively maintained the spirit of this serial.

## **CONTENTS**

Contr	ributors			٠			ix
Prefac	ce						xi
	The Diffusion of Turbulent Buoyant Jets						
BEN	NJAMIN GEBHART, DAVID S. HILDER, and MATTHE	EW	K	E	LL	EHI	ER
	Introduction						1
	Characteristics of Circular Discharges						3
	Review						6
	Properties and Ambient Stratification Modeling .						17
	General Formulation						20
VI.	Comparative Calculations, Jets in Unstratified						
	Quiescent Ambients						25
VII.	Comparative Calculations, Jets in Unstratified						
	Flowing Ambients						36
VIII.	Effects of Ambient Stratifications						42
IX.	Summary and Conclusions				·		50
X.	Review of Other Studies						53
	Appendix A						53
	Appendix B						54
	Nomenclature						55
	References						56
	Boiling of Multicomponent Liquid Mixture	28					
	JOHN R. THOME and RICHARD A. W. SHOO	CK					
Ι.	Introduction						60
	Fundamentals of Vapor-Liquid Phase Equilibria						61
	Inception of Boiling						64
IV	Bubble Growth						80
	Bubble Departure				•		95

VII. Prediction Transfer C VIII. Peak Nucl IX. Film Boili X. Convectiv XI. Recomme Nomencla Reference	Pool Boiling Heat Tr of Nucleate Boiling Coefficients leate Heat Flux ng e Boiling ture s in Saturated Nuclei	Heat									105 117 127 133 142 150 151 153
	Examination Using 1						V IL	16-	.11	anş	ging
	M. G. C	COOPER									
II. Reformula III. Consequer IV. Data Anal V. Data Avai VI. Examinati VII. Data Anal VIII. Resulting IX. Conclusion X. Appendix XI. Appendix XII. Appendix Nomencla	tion of Correlations nces	nods	orodu orrela   	ncib	ility						158 161 164 166 169 175 185 203 206 210 229 232 237 237
A Review of	Furbulent-Boundary- Stanford, 1			Frai	nsfe	er l	Re	sea	arc	ch a	at
	R. J. Moffat ar	nd W. M	[. KA	YS							
II. Equilibrium III. The Appar IV. Factors Af	on	 s Used y Layer									242 249 254 272
Energy Eq	luations						÷	٠			341

							(	Со	N'	ГЕ	NT	S							vii
VI. Some Nome Refere	nclatu	ire		•															361
Author Inde	х												,				·		367
Subject Inde	х												,						375
Contents of	Previo	ous	3 \	/o	luı	ne	es												385

# The Diffusion of Turbulent Buoyant Jets

## **BENJAMIN GEBHART**

Department of Mechanical Engineering and Applied Mechanics, University of Pennsylvania, Philadelphia, Pennsylvania

#### DAVID S. HILDER

Philadelphia Naval Shipyard, Philadelphia, Pennsylvania

## **MATTHEW KELLEHER**

Department of Mechanical Engineering, Naval Postgraduate School, Monterey, California

I.	Introduction	
II.	Characteristics of Circular Discharges.	1
III.	Review	3
	A. Modeling Schemes.	6
	B. Experimental Studies	6
IV.	Properties and Ambient Stratification Modeling .	13
V.	General Formulation	17
VI.	Comparative Calculations Lets in Unatratification:	20
	A The Simplest Entrainment Madel	25
	A. The Simplest Entrainment Models	26
VII	2. Eater Entrainment Models, Detailed Comparisons	30
V 11.	Comparative Calculations, lets in Unstratified Flowing A 1:	36
III.	Effects of Ambient Stratifications	42
	and Conclusions	-
X.		50
XI.	Appendix A	53
	Appendix A .	53
	Difficulties variables	53
XXX.	Appendix B	54
	- contenedature	55
	References	
		6

#### I. Introduction

The cooling water discharge from a power plant into a large body of water, the thermally loaded condenser discharge from the condenser of a moving ship, and the high-temperature gas issuing from a stack or gas

turbine exhaust are all buoyant momentum jets. The trajectory and decay of such jets after discharge are influenced by such factors as initial jet velocity and buoyancy, ambient motion and stratification, and downstream mixing rate. Questions such as whether or not the jet will rise to a certain level, what the jet velocity and temperature will be at any point along its trajectory, or what effect ambient fluid stratification will have on behavior all require detailed and accurate analysis. There have been many contributions in the past few decades to the understanding of the mechanics of buoyant jet mixing and trajectory. The ultimate objective is to develop accurate general models that predict both trajectory and decay.

The need for such predictive models has grown. Since nuclear- and fossil-fueled power plants have thermal efficiencies on the order of 30–40%, the immense discharge of heat into either the atmosphere or a body of water has a very large effect. Sewage is often discharged as treated effluent into rivers, lakes, and oceans. The proper evaluation of the ecological impact of such discharges requires that their subsequent behavior be predictable. More stringent environmental regulations and heightened public awareness require increasing accuracy in such prediction.

The need for the prediction of jet behavior is not limited to environmental issues. Rapid advancement of the ability to detect small temperature and concentration differences, and other anomalies, may make it increasingly easy to detect many physical effects, changes, and motions in the environment. The implications for increasing knowledge of environmental, geophysical, and technological processes are enormous.

Given the wide range of applications in which jet behavior is to be analyzed, the range of possible jet and/or ambient characteristics that may be of interest is equally wide. The variables include initial jet geometry, discharge momentum, thermal and concentration loading, turbulence characteristics, as well as ambient flow conditions, turbulence, and stratification. An extremely large number of appreciably different combinations arise.

The summary and calculations here concern a single, fully turbulent, circular buoyant jet, discharged into a surrounding ambient of the same fluid. Two-dimensional trajectories are included, wherein any ambient flow is taken as parallel to the horizontal component of jet velocity. Jet encounter with an abrupt ambient discontinuity, such as a two-phase interface, is not treated here. That is, the ambient is considered infinite in extent.

Among the variables are:

(1) buoyancy effects, arising from density differences between the jet and the ambient (differences may arise from temperature and/or concentration variations);

(2) ambient density stratification, arising from vertical nonuniformity of temperature and/or concentration in the ambient;

(3) ambient flow conditions, with respect to the jet, of differing magni-

tude and orientation relative to the jet;

(4) initial jet discharge characteristics, including direction of momentum.

## II. Characteristics of Circular Discharges

The terms *jet*, *momentum jet*, *forced plume*, and *plume* are often used to describe qualitatively the differing characteristics of a discharge penetrating an ambient medium. In general usage, *jet*, *momentum jet*, and *forced plume* refer to the downstream region wherein the momentum of the initial discharge is still sufficient to influence jet mixing and trajectory. A discharge in which the discharge momentum is everywhere negligible, relative to the eventual total momentum produced by buoyancy, is called a plume. In this account these will be called buoyant jets and plumes.

The jet-ambient interaction mechanisms are classified according to the

following characteristics:

(1) Jet buoyancy

(a) neutrally buoyant

(b) buoyant (positively or negatively)

(2) Orientation of discharge

(a) horizontal (perpendicular to the gravity field)

(b) inclined

(3) Ambient motion

(a) quiescent

(b) flowing

(4) Ambient stratification

(a) unstratified

(b) linearly stratified

(c) other stratifications

Independent of the jet-ambient mechanisms, each jet passes through several flow regimes along its trajectory. They are shown for an inclined submerged buoyant jet in Fig. 1. The regimes are as follows:

(1) The zone of flow establishment. In this region, flow characteristics are dominated by the discharge conditions. Velocity and scalar quantity profile (temperature, salinity, etc.) undergo transition from their initial discharge configurations, through a turbulent shear layer formed around

WATER SURFACE

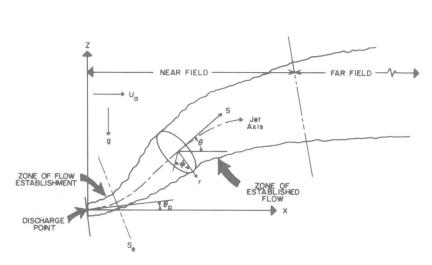


FIG. 1. Flow regimes of a buoyant momentum jet and the coordinate system and dimensions in which its trajectory and growth are described.

the jet periphery. As mixing with the ambient progresses, the turbulent shear layer grows inward and the extent of the core of undisturbed profiles becomes smaller. The zone of flow establishment ends at the point where turbulent mixing reaches the jet centerline. The jet behavior in this region is strongly influenced by initial momentum and discharge conditions and is only slightly influenced by the ambient.

(2) The zone of established flow. This region begins when turbulent mixing has reached the jet centerline. The motion of the jet and its physical characteristics are governed by its initial and acquired momentum, by its buoyancy, and by ambient stratification and flow conditions. Initial discharge conditions play a progressively smaller role. The flow progresses from jetlike to plumelike behavior.

(3) The far field. In this region the jet's initial momentum has a negligible effect, and the jet may be passively convected by ambient motions. The jet fluid may be further diffused by ambient turbulence, and the distribution of the jet as a separate entity gradually disappears.

The regions of flow establishment and established flow regimes are the near field. They are the regions of the most vigorous mechanisms. These are the concern of this account and of the calculations of trajectory and decay.

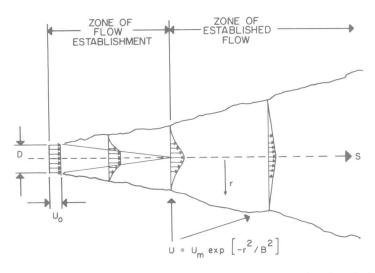


Fig. 2. Development of Gaussian velocity profiles in a momentum jet after discharge.

Experimental work, begun by Albertson *et al.* [1] and continued and expanded by many others, has shown that within the zone of established flow, mean velocity profiles are nearly Gaussian:

$$U = U_{\rm m} \exp[-r^2/B^2] \tag{1}$$

where  $U_{\rm m}$  is the local centerline velocity, r is the radial jet coordinate, and B is a characteristic jet width. It is the radial distance at which U is equal to 1/e times the mean centerline value,  $U_{\rm m}$ .

Profiles of jet scalar quantities, such as temperature and concentration, have also been found to be Gaussian in the zone of established flow by investigators such as Fan [2], Hoult *et al.* [3], and others. The profiles may be expressed as:

$$\Delta t = \Delta t_{\rm m} \exp[-r^2/\lambda_t^2 B^2] \tag{2}$$

$$\Delta c = \Delta c_{\rm m} \exp[-r^2/\lambda_c^2 B^2] \tag{3}$$

where  $\Delta t = (t - t_a)$ ,  $\Delta t_m = (t_m - t_a)$ ,  $\Delta c = (c - c_a)$ , and  $\Delta c_m = (c_m - c_a)$ . The values  $\lambda_t$  and  $\lambda_c$  are the relative radial spreading ratio between velocity and the scalar properties t and c. These quantities are related to the turbulent entrainment Prandtl and Schmidt number effects. Figure 2 illustrates a profile within the jet.

A coordinate system to describe the trajectory and physical dimensions of a jet system is shown in Fig. 1. The X coordinate is horizontal. The Z positive direction is vertical, opposite to the gravity vector. The

streamwise coordinate S is measured along the direction of the mean centerline of the jet. The local angle between S and X, the inclination of the jet from the horizontal, is  $\theta$ . The polar coordinates  $\phi$  and r, defining the jet cross section, are normal to S. Herein any ambient medium motion is assumed to be horizontal, that is, in the direction of X.

A principal quantitative measure of relative momentum and buoyancy is the densimetric Froude number F given by

$$F = \frac{U_0}{[gD(\rho_a - \rho_0)/\rho_0]^{1/2}}$$
 (4)

The contribution of momentum is reflected in the numerator by the discharge velocity  $U_0$ . The buoyancy effect is included in the denominator by the density difference, or units of buoyancy  $(\rho_a - \rho_0)/\rho_0$ . This quantity is the measure of the velocity level generated by the buoyancy force. Thus the value of the densimetric Froude number ranges from near zero for plumes to infinity for pure, nonbuoyant momentum jets. Hereafter, the term *Froude number* will be used to mean the densimetric form in Eq. (4).

#### III. Review

#### A. Modeling Schemes

Several kinds of predictive models have been developed for the motion of circular buoyant momentum jets. Although specific calculations have considered different circumstances (e.g., in origin of buoyancy, for stratified or uniform ambients and quiescent or coflowing ambients, etc.), all models are one of two kinds.

- (1) Algebraic models are algebraic equations based on either empirical data or simplification of differential models. These most typically predict only trajectory and jet width. Some, such as the model of Shirazi et al. [4], also predict velocity, concentration, and temperature residuals. Databased algebraic models tend to become unreliable when the basic conditions on which they were based, such as general temperature and salinity range of the jet and ambient, are significantly changed.
- (2) Differential models are based on the relevant conservation equations of mass, momentum, energy, and chemical species. This modeling technique allows prediction of jet trajectory and width, as well as velocity, temperature, and concentration decay downstream in the jet. Stratification and motion of the ambient may also be accommodated. The promi-

nent differential models are entrainment, mixing length, and  $k-\varepsilon$  and eddy diffusivity turbulence modeling.

Because of their limited applicability, algebraic models are not treated here. They are sometimes useful for prediction when the jet-ambient system involved is simple and only information such as trajectory is required. However, by far the greatest effort in recent years has involved the more general and inclusive differential approach to jet modeling. In a majority of these differential models the entrainment mixing concept has been used, rather than models utilizing mixing length  $k-\varepsilon$ , or turbulent diffusion hypotheses.

Morton et al. [5] were the first to use the entrainment concept to develop a buoyant jet model, as previously suggested by Taylor [6]. The concept supposes that the downstream induction of quiescent ambient fluid into the moving jet is proportional to the local jet centerline velocity  $U_{\rm m}$  and a characteristic jet periphery  $2\pi B$ . Thus

$$E \propto 2\pi B U_{\rm m}$$

where E represents volumetric rate of entrainment, or ambient inflow per unit of jet length, into the jet. The definition of  $\alpha$  is completed by

$$dQ/dS = \rho E \tag{5}$$

where Q is the total mass flow in the jet at any downstream location S. The constant of proportionality for E,  $\alpha$ , is called the entrainment constant, or coefficient. The rate of entrainment is then written as:

$$E = 2\pi B\alpha U_{\rm m} \tag{6}$$

Solutions of the governing equations for differential modeling have, in the past, been based on the following assumptions.

- (1) The turbulent jet flow is steady.
- (2) Since the jet flow is fully turbulent, radial molecular diffusion is neglected, compared to radial turbulent transport.
- (3) Streamwise turbulent transport is a negligible downstream transport mode, compared with streamwise convective transport.
- (4) The variation of fluid density throughout the flow field remains small compared to a chosen reference density. Density variation is included only in buoyancy terms. This is a Boussinesq approximation.
  - (5) Other fluid properties are taken constant throughout the field.
  - (6) Pressure is hydrostatic throughout the flow field.
  - (7) The jet remains axisymmetric throughout the near field. That is,

velocity, temperature, density, and salinity profiles are assumed not to develop circumferential variations.

The governing conservation equations, in the forms used in differential modeling, are presented in Table I, in the physical variables.

With the exception of Hoult *et al.* [3], all studies cited in the following discussion have assumed that the velocity, temperature, salinity, and density profiles are all Gaussian. This assumption, therefore, limits the applicability of such models to the zone of established flow.

Hoult  $et\ al.$  instead used a "top hat" velocity profile throughout, rather than a Gaussian distribution. This assumption certainly applies very near the jet origin but not downstream in the rest of the near field. The reduced form of the conservation equations for this model require the cross-stream integration, with the Gaussian assumption. Since Hoult assumed top hat profiles for both the zone of flow establishment and the zone of established flow, initial conditions are those at the jet outlet. In this method the values of  $\rho$ , t, c, and U ascribed to the jet at various points along the path are taken to be mean values for the entire jet cross section. This is a more limiting result than that with models using Gaussian profiles, where maximum values of jet properties result, and the entire cross-

TABLE I  $\begin{tabular}{ll} The General Equations, in Dimensional Form, for Entrainment Modeling of Buoyant Momentum Jets \\ \end{tabular}$ 

Equation	Form						
Continuity	$\frac{d}{dS} \left\{ \int_0^{2\pi} \int_0^\infty Ur  dr  d\phi \right\} = 2\pi \alpha U_{\rm m} B = E$						
Horizontal momentum	$\frac{d}{dS} \left\{ \int_0^{2\pi} \int_0^{\infty} U^2 \cos \theta  r  dr  d\phi \right\} = U_a E$						
Vertical momentum	$\frac{d}{dS} \left\{ \int_0^{2\pi} \int_0^{\infty} \rho U^2 \sin \theta  r  dr  d\phi \right\} = \int_0^{2\pi} \int_0^{\infty} (\rho_a - \rho) gr  dr  d\phi$						
Energy	$\frac{d}{dS} \left\{ \int_0^{2\pi} \int_0^{\infty} U(t - t_a) r  dr  d\phi \right\} = -\frac{dt_a}{dS} \int_0^{2\pi} \int_0^{\infty} Ur  dr  d\phi$						
Concentration (or scalar species)	$\frac{d}{dS} \left\{ \int_0^{2\pi} \int_0^{\infty} U(c - c_a) r  dr  d\phi \right\} = -\frac{dc_a}{dS} \int_0^{2\pi} \int_0^{\infty} Ur  dr  d\phi$						
Horizontal component of trajectory	$dX = dS \cos \theta$						
Vertical component of trajectory	$dZ = dS \sin \theta$						

section profile may be deduced from the appropriate Gaussian distribution.

Abraham [7] initially used the vertical and horizontal momentum equations, as well as the energy equation, to model jets discharged to a quiescent ambient. The continuity equation was not needed or used in the calculations. The solution required a prespecification of the variation of B, in (1), as a function of S. Most other models have included the continuity equation in lieu of prespecifying the B variation.

The solution of the seven equations in Table I yields values of jet centerline velocity  $U_{\rm m}$  and temperature and concentration differences  $\Delta t_{\rm m}$  and  $\Delta c_{\rm m}$ , as well as jet width D(S) and trajectory (X,Z), all as functions of S. The solution of the equations, of course, also requires that the entrainment function E be specified, that  $\alpha$  be given. Therein lie the principal differences between entrainment models. These models fall into two general categories: those for a quiescent ambient and those for a flowing ambient.

#### 1. Quiescent Ambient Media

Albertson *et al.* [1] and others have verified through measurements that for nonbuoyant momentum jets, (i.e.,  $F = \infty$ ), an appropriate value of  $\alpha$  within the zone of established flow is 0.057. There seems to be little disagreement with this value, judged from numerous comparisons of differential modeling with this value and with experimental data.

Abraham [7] suggested, also on the basis of experimental evidence, that, for flows resulting largely from buoyancy, small F, the value is  $\alpha = 0.085$ . This is in good agreement with the suggestion of List and Imberger [8] of  $\alpha = 0.082$  for pure buoyant plumes (F = 0). Fan and Brooks [9] had suggested  $\alpha = 0.082$  for all flows except pure momentum jets. Fan and Brooks also recommended, on the basis of their experiments,  $\alpha = 0.057$  for pure momentum jets.

In applications, however, buoyant discharges are seldom either pure jets or plumes. Typically, their flow is some stage of transition away from jet behavior toward plume behavior. Morton *et al.* [5] proposed to model this the whole range by:

$$\alpha = 0.057 + a_2/F_L \tag{7}$$

where  $a_2$  is an empirically determined constant and  $F_L$  is a local Froude number, based on the local centerline velocity  $U_m$ . The same general form was derived by Fox [10] for a vertically discharged buoyant jet.

Hirst [11] postulated that, for a buoyant discharge into a quiescent ambient, the entrainment function should depend on