the 12th annual

SIMULATION Symposium

Edited by
R. DYSART CONINE
EPHRAIM D. KATZ
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RECORD OF PROCEEDINGS

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ANNUAL SIMULATION SYMPOSIUM

PREFACE

The Annual Simulation Symposium is a non-profit corporation formed to provide a forum for the interchange of information related to digital computer simulation. Its objectives are:

- to provide a continuation of the forum for the exchange of working experiences in the field of digital computer simulation, to
- permit an opportunity to survey the state-of-the-art across a broad range of applications, to
- demonstrate the widest possible range of simulation languages, with their strengths for individual problems, and to
- furnish an opportunity for comprehensive understanding of techniques through organized question and answer periods and personal contact. It also aims to
- provide potential users of simulation with first-hand exposure to methods, to
- display, for library type perusal, the range of literature available in the field, to
- maintain objectivity to the art of simulation, through a non-commercial meeting without obligation to any specific language or hardware, and to,
- underwrite, through grants, the advancement of the art of simulation.

Membership is provided as a result of registration at the Annual Symposium. A Board of Directors is elected by the membership, one Director per year for a three year term.

The Symposium is indebted to those corporations and universities whose support, through their representatives, make this totally independent organization capable of serving the Art of Simulation. This year particular recognition is afforded to those organizations whose members served in offices and on committees as shown.

The Annual Simulation Symposium is sponsored by the IEEE Computer Society, the Association for Computing Machinery, and the Society for Computer Simulation.

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ANALYTIC AND DISCRETE SIMULATION RESULTS FOR A STOCHASTIC MODEL OF LAW ENFORCEMENT AND SPEED LIMIT COMPLIANCE

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Abstract. An aggregate stochastic model of driver behavior is postulated and analyzed to determine the expected steady-state level of driver compliance with the speed limit as well as the duration of the transient period. The expected steady-state proportion of complying motorists is related to the probability of police surveillance and to two behavioral parameters of the driver population: mean compliance level following operation of a radar trap and mean percentage decrease in compliance following non-operation. The model's utility in cost/benefit analysis of police operations is elaborated, including other effects of speeding such as accident mortality, morbidity, and rate of energy usage. Applications of the model to more serious law-enforcement issues as well as entirely different contexts such as marketing are briefly explored.

INTRODUCTION

Recognizing the significant drain on resources and the threats to community stability which our national crime problem has posed, many researchers have focused their efforts on the development of improved planning and resource allocation aids. During the past decade, some of the most fruitful advances have come from the application of operations research methodologies to police patrol [1-17] , with Larson's seminal work on response and preventive patrol strategies representing a significant milestone [18] . While these models have varied in their treatment of the spatial and temporal aspects of the patrol problem, they all acknowledge explicitly the need to incorporate a random component in their patrol strategies. Underlying this feature of their models is the assumption that space-time coincidence between a criminal act and patrol unit implies detection and possible arrest. The unpredictable element in patrol thus serves to heighten the actual risk of apprehension and perhaps the criminal's perception of it as well.

The point of departure in this paper is two-fold. First, we address a specific, non-felonious crime, breaking the speed limit, and postulate a specific behavioral model for a cohort of potential violators. Second, as a preliminary step towards more comprehensive cost-effectiveness analyses of such problems, our model focuses on the question of the expected level of driver compliance as a function of radar surveillance frequency, rather than the spatial distribution of radar sites, or the effect of other forms of patrol surveillance. The impetus for this apparently innocuous and narrow concern stems not only from the recurring, significant levels of public expenditure in this law enforcement area, but, more recently, from energy conservation considerations reinforced by suggested measures such as federally imposed penalties to states with poor compliance. As we shall see, moreover, the model that we develop may have utility in other enforcement situations as well as in completely different contexts.

PROBLEM STATEMENT AND AGGREGATE STOCHASTIC MODEL

A key objective of highway patrol is to deter speeding and presumably, thereby, to reduce the mortality and morbidity of accidents as well as the rate of energy consumption. Knowledge on how drivers behave when traversing stretches of road where radar sensors are known to be irregularly deployed is still speculative. Moreover, the prognosis for empirically validating models of the deterrent effect of trap operation does not appear good. This is largely due to the legal difficulties and likely reporting artifacts that would be entailed in soliciting information both on the effect when a motorist violates the law and is or is not caught, and the effect when the driver complies with the traffic code and observes that it is or is not enforced.

In the absence of such information, we postulate the following aggregate stochastic model. At a particular site, or on a stretch of highway where drivers have been warned that a trap may be operating, we assume that the police actually operate the trap on each occasion (day, morning, afternoon, etc.) with fixed probability p. The trap need not be stationary, but may be attached to a car patroling the designated area. Whether fixed or mobile, the police unit's decision to operate the trap is independent from period to period, irrespective of operation on previous occasions, their associated outcomes vis-à-vis the proportion of speeding drivers, etc. Following the operation of the trap, the proportion of the driver population that comply with the speed limit is assumed to return to a level π. On each occasion following non-operation, however, the proportion of law-abiding drivers falls by the percentage δ. For this population, then, we wish to determine the steady-state expected proportion of drivers that cooperate with the law as a function of the surveillance likelihood p and return level π, as well as the number of epochs necessary to achieve a steady-state. The first case considered will be for deterministic π and δ, where the steady-state solution can be derived analytically. This will be followed by the case in which π and δ are stochastic, where the results are obtained through simulation.

STEADY-STATE AND TRANSIENT ANALYSIS: DETERMINISTIC π AND δ

The possible states (i.e., levels of driver cooperation expressed as a population percentage) and allowable transitions between levels from period to period permit the computation of the expected level of driver cooperation in a straightforward manner. Letting t = 0, 1, ... ,n denote the individual occasions on which the police trap may be operated and assuming that the trap was operated at t = 0, the expected level of driver cooperation $\bar{\pi}_t$ at each stage t can be expressed as follows

$$\bar{\pi}_n = \pi - \delta \sum_{t=1}^{n} q^t = \bar{\pi}_{n-1} - q^n \delta \qquad n \leq [\pi / \delta] = [\gamma] \tag{1}$$

where q = (1 - p), the likelihood of non-surveillance, and where $[\gamma]$ denotes the largest integer not greater than γ . The non-recursive formulation of in the first part of (1) can be further simplified by expressing the familiar power series in (1) as follows:

$$\bar{\pi}_n = \begin{cases} \pi - \delta q (q^n - 1) / (q - 1) & q \neq 1 \quad t = n \leq [\pi / \delta] \\ 0 & q = 1 \end{cases} \tag{2}$$

Since the level of driver compliance cannot fall below 0, for $t > [\gamma]$ we obtain the steady-state result for the expected level of cooperation $\bar{\pi}_{ss}$ as a function of q

$$\bar{\pi}_{ss}(q ; \pi, \delta) = \begin{cases} \bar{\pi}_{[\gamma]} - q^{[\gamma]+1} \left[\pi - [\gamma] \delta \right] & q \neq 1 \quad t > [\gamma] \\ 0 & q = 1 \end{cases} \tag{3}$$

where $\bar{\pi}_{[\gamma]}$ is given by (2).

As this last result indicates, the steady-state value of the expected proportion of complying drivers depends only on the ratio of the compliance level π following operation of the trap and the decrement δ following non-operation, for a given likelihood of operation p. Smaller values for the decay factor and higher population compliance percentages produce a more protracted transient period $0 \leq t \leq [\gamma]$ for $\bar{\pi}_t$. As (2) and (3) show, in the limit as q approaches 1, or p approaches 0, $\bar{\pi}_{ss}$ becomes 0 for any $\delta > 0$. Similarly, $\bar{\pi}_{ss}$ becomes π as q and p attain the respective values of 0, 1. These and other key analytic properties of (3), as well as some policy implications, are clarified in Figures 1-3 which show the variation of $\bar{\pi}_{ss}$ with p for the different driver cooperation return levels π = .5, .75, and 1. Each family of curves corresponds to the selected decrements in driver cooperation, δ = .1, .2, .3, .4, .5, and π . The graphs show that for $\delta = \pi$, the variation of $\bar{\pi}_{ss}$ with p is linear, while for $\delta \neq \pi$, the variation of $\bar{\pi}_{ss}$ with p is nonlinear. This property is a consequence of (3) which for $\delta = \pi$ yields

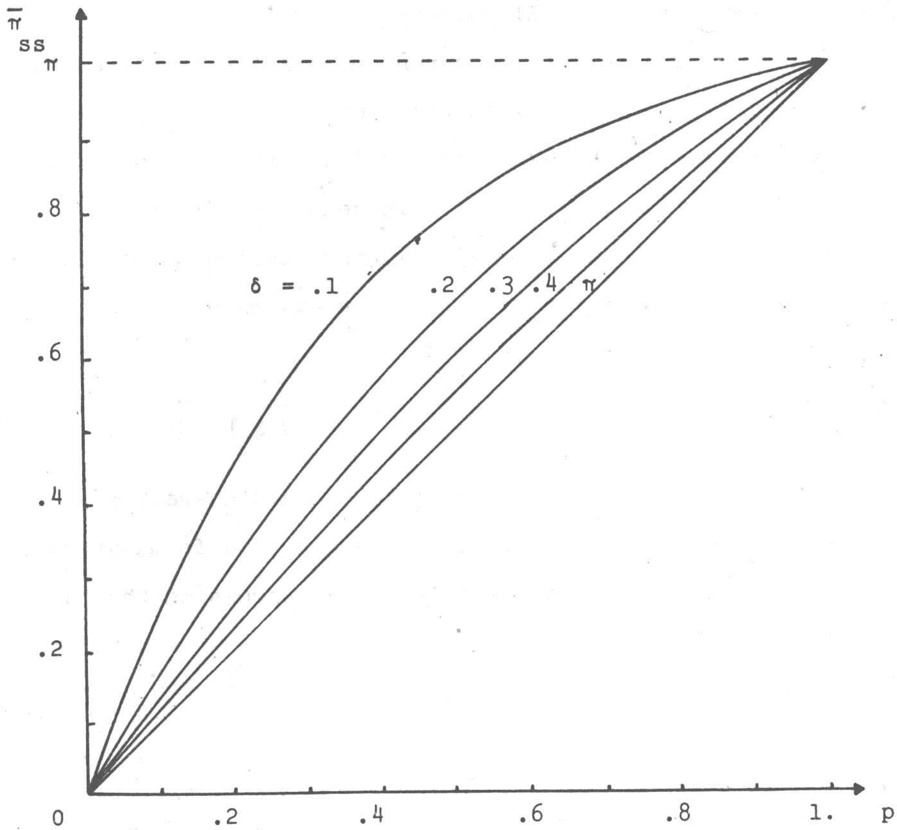

Figure 1. $\bar{\pi}_{ss}$ vs. p for $\pi = .5$ and $\delta = .1, .2, .3, .4, \pi$

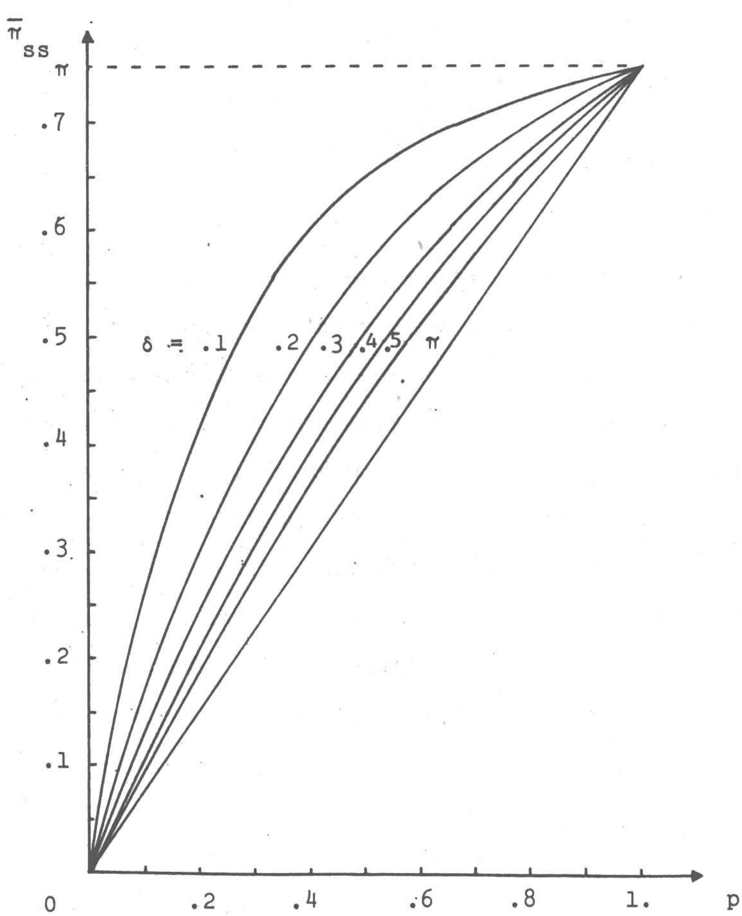

Figure 2. $\bar{\pi}_{ss}$ vs. p for $\pi = .75$ and $\delta = .1, .2, .3, .4, .5, \pi$

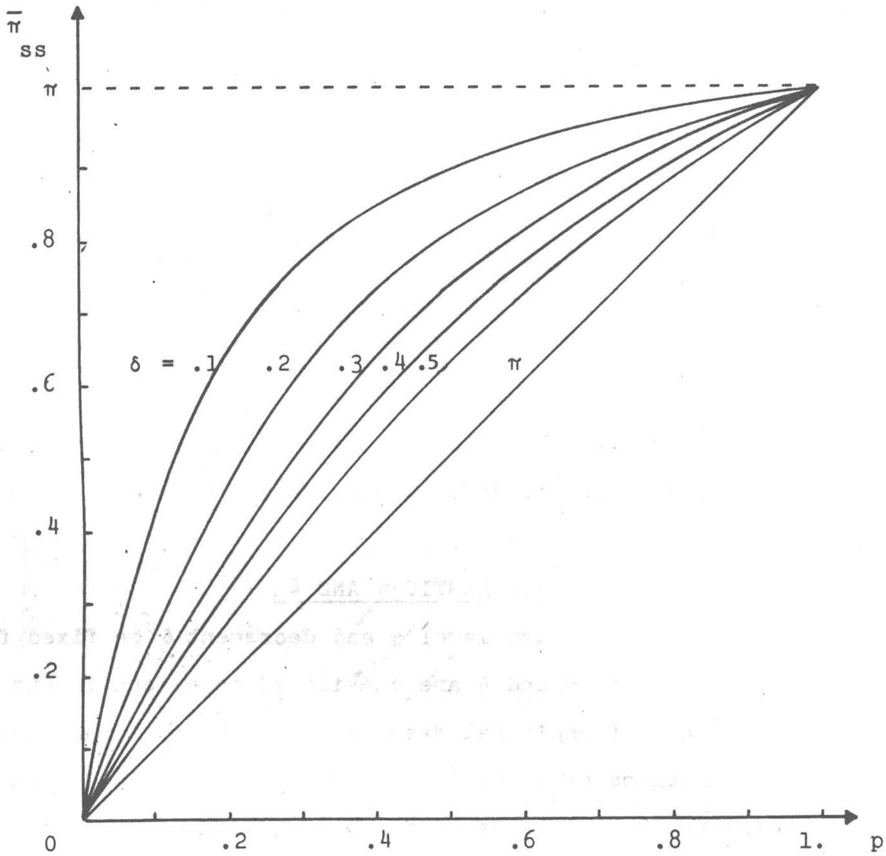

Figure 3. $\bar{\pi}_{ss}$ vs. p for π=1 and δ = .1, .2, .3, .4, .5, π

$$\bar{\pi}_{ss} = \begin{cases} \bar{\pi}_1 - q^2(\pi - \delta) = \pi p & q \neq 1 \quad \delta = \pi \\ 0 & q = 1 \end{cases} \tag{4}$$

where $\bar{\pi}_1$ is given by (1) and p= (1-q).

From an operational viewpoint, the result contained in (3) for this simple aggregate model suggests that for small δ (on the order of 20% or less) and for all π, significant increases can accrue to the steady-state level of expected driver compliance if the police are willing to increase a low value of p (in the 0-to-5% range, say) to a value in the 5-to-25% range. For all δ except δ = π, it is in this region that the rate of change of $\bar{\pi}_{ss}$ with p is clearly greatest. By contrast, we see that except for δ = π (for which $d\bar{\pi}_{ss}/dp = \pi$, a constant), the gains in $\bar{\pi}_{ss}$ from increases in p are least for larger values of p (in the region p > .7), especially for small δ. More precisely, these diminishing returns in $\bar{\pi}_{ss}$ with increasing p are due to the monotonically non-increasing nature of the derivative of $\bar{\pi}_{ss}$, namely

$$\frac{d\bar{\pi}_{ss}}{dp} = \begin{cases} \{1 + [\gamma]\}\{\pi - [\gamma]\}(1-p)^{[\gamma]} & \delta \neq \pi \\ \pi & \delta = \pi \end{cases} \qquad 0 < p < 1 \tag{5}$$

Thus, for fixed π and δ we obtain the limits in slope

$$\lim_{p \to 1^-} \bar{\pi}'_{ss} = \begin{cases} 0 & \delta \neq \pi \\ \pi & \delta = \pi \end{cases} \tag{6}$$

and

$$\lim_{p \to 0^+} \bar{\pi}'_{ss} = \begin{cases} \{1 + [\gamma]\}\{\pi - [\gamma]\delta\} & \delta \neq \pi \\ \pi & \delta = \pi \end{cases} \tag{7}$$

with the +/- signs in these last equations indicating respectively left- and right-hand limits, since $\bar{\pi}'_{ss}$ is not defined outside $0 < p < 1$.

STEADY-STATE SIMULATION RESULTS: STOCHASTIC π AND δ

The constraint that the return level π and decrement δ be fixed from epoch to epoch is relaxed here. Both π and δ are considered to be continuous random variables. In the absence of empirical data, a simple model is assumed for the probability density functions of π and δ, namely, that each is identically distributed according to the triangular pdf. Thus, for arbitrary breakpoint β, and $x = \pi$ or δ, the corresponding density function for π or δ is given by

$$f(x) = \begin{cases} (2/\beta)\, x & 0 \leq x \leq \beta \quad x = \delta, \pi \\ [2/(1-\beta)]\,(-x + 1) & \beta \leq x \leq 1 \quad 0 < \beta < 1 \end{cases} \tag{8}$$

The distribution function for the random variables π and δ is thus

$$F(x) = \begin{cases} (1/\beta)\, x^2 & 0 \leq x \leq \beta \\ [1/(1-\beta)]\{-x^2 + 2x + [-2\beta + \beta^2 + (1-\beta)/\beta]\} & \beta \leq x \leq 1 \end{cases} \tag{9}$$

For a given choice of parameter β, this relation is used to generate successive values for π, δ by analytic inversion in the simulation program. The expected value of either π or δ is given by

$$E(x) = \frac{2}{3}\beta^2 + \left[\frac{2}{\beta - 1}\right]\left[\frac{-1}{3}\beta^3 + \frac{1}{2}\beta^2 - \frac{1}{6}\right] \qquad \begin{matrix} 0 < \beta < 1 \\ x = \pi, \delta \end{matrix} \tag{10}$$

This result can be related to the maximum steady-state response $\bar{\pi}_{ss}$, as explained next.

Figure 4 shows the discrete simulation results corresponding to the choices β = .5, .75, and 1. As can be seen, the expected steady-state level in driver compliance, $\bar{\pi}_{ss}$, is a moderately nonlinear function of p. The extremal values shown in Figure 4 at the end points p = 0, 1 are readily verified analytically to

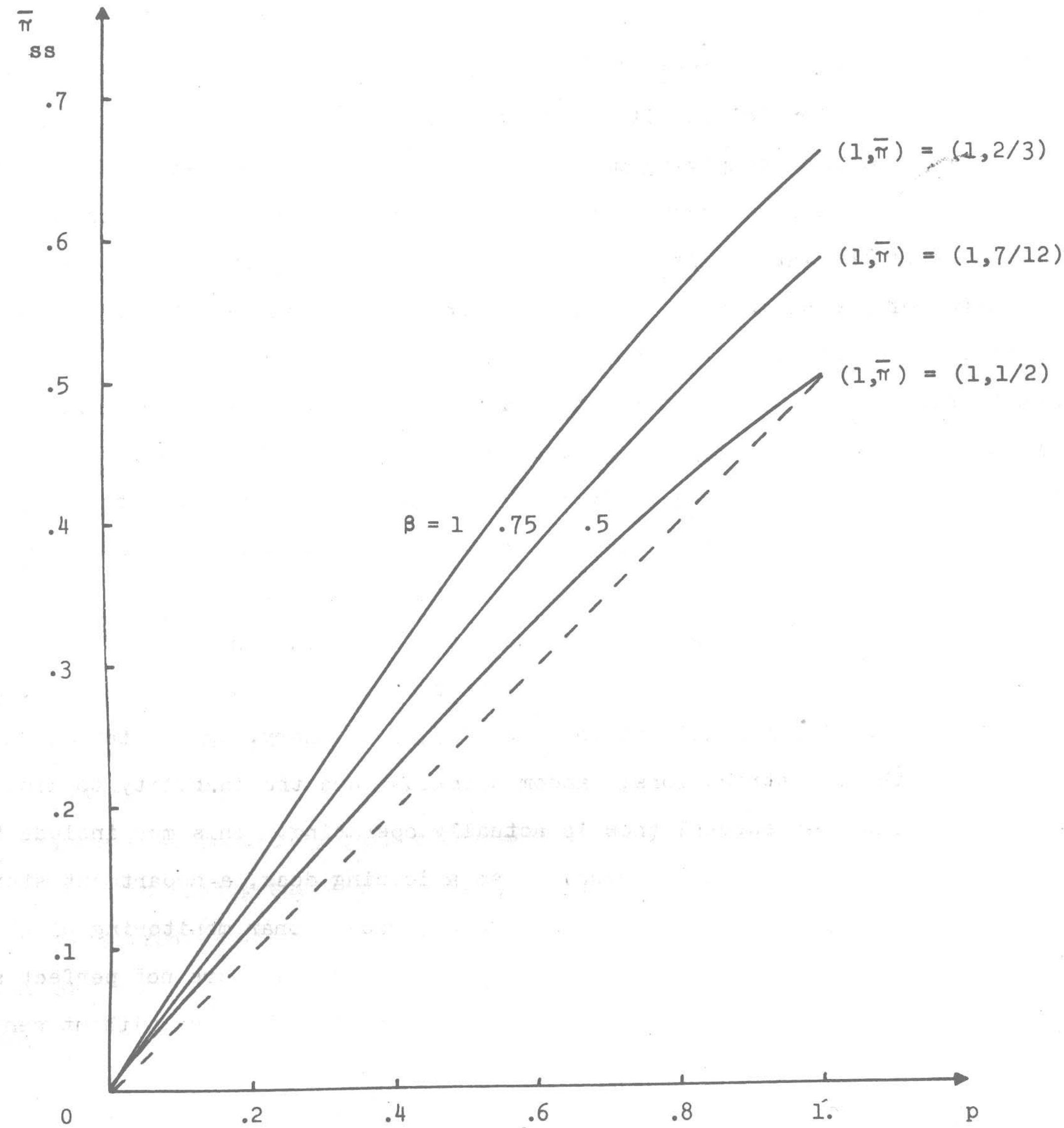

Figure 4. Stochastic Model Simulation Results for $\bar{\pi}_{ss}$ vs. p: Triangular PDF Breakpoints at β = 1, .75, .5

be

$$\bar{\pi}_{ss} = \begin{cases} 0 & p = 0 \\ \bar{\pi} & p = 1 \end{cases} \tag{11}$$

where $\bar{\pi}$, the mean return level following trap operation, is given by (10). Thus, the maximum for $\bar{\pi}_{ss}$ is $\bar{\pi}$ itself. Unlike the deterministic case, the rate of change of $\bar{\pi}_{ss}$ with p is nearly constant, since $\bar{\pi}_{ss}$ (p) is nearly linear. Although not evident from the graphs, the simulation experiments indicate that the resultant levels of $\bar{\pi}_{ss}$ for all values of p are insensitive to initial conditions in compliance level after approximately 50 iterations or epochs, the most pronounced changes in $\bar{\pi}_{ss}$ occurring in the first 10 iterations.

MODEL APPLICATIONS

The accompanying graphs of $\bar{\pi}_{ss}(p)$ can be viewed as cost-effectiveness curves. This interpretation follows if effectiveness is defined as the expected steady-state proportion of complying motorists and if cost is related to the surveillance probability, p. Use of accepted cost-accounting principles should permit good estimates of the latter. These incremental costs should vary significantly with the level of p contemplated at each site and will likely be different for those traps operated under municipal, rather than state police aegis. If levels of $\bar{\pi}_{ss}$ can be further related to mortality and morbidity, or some general index of dysfunction [19] due to speeding accidents, then the relationship $\bar{\pi}_{ss}(p; \pi, \delta)$ may permit more penetrating analysis of cost/benefit tradeoffs. Comprehensiveness could be further enhanced by relating $\bar{\pi}_{ss}$ to environmental impacts.

The model may have broader law-enforcement applicability than simply traffic control. Potential candidates are those situations which have the ingredients of a stable population; repeated exposures to some form of surveillance or other stimulus; significant penalties (or rewards); stationary, aggregate values for π and δ, or their distributions; random stimulus; and the inability to discern in advance whether the surveillance is actually operating. This may include the random monitoring of closed-circuit TV at a loading dock, a department store location, library exit, or building entrance; radar/sonar monitoring of a restricted geographical space; and so on. These analogies are not perfect since in any of these contexts it may be possible to test the system without penalty (e.g., by staging a fake theft to see if the system responds).

In an entirely different context (with penalties replaced by rewards), the model may have applicability to marketing. Here the central concern is the effect of repeated advertising exposure on buyer behavior, including actual purchase, purchase intention, recall, total attendant audience or "reach," etc. While much research has been devoted to such issues [20-24] , the effect of repetition frequency has received least attention, and the present model in which exposures are stochastic has not been postulated or analyzed. In this regard, it would appear that the flexibility required by the model in deciding whether to operate a campaign or not obviates its application to such media as TV, radio, or magazines. This would not be the case, however, for those able to purchase enough blocks of time or ad space in advance (as brokers do) and then simply substitute another ad of equal duration whenever the random choice mechanism yielded "no operation." The more significant problem here pertains to the need for a stable audience from exposure to exposure, although such program and station loyalties are not uncommon. A context which is far more amenable to this type of application -- and even testing -- of the model would be a supermarket, for example. Here the marketing manager would be relatively free to randomly expose steady customers to a private brand discount, say, or to mail out coupons to the neighborhood that the store serves.

MODEL ASSUMPTIONS AND POSSIBLE REFINEMENTS

As stated, the aggregate stochastic model assumes that a population of motorists traversing a given stretch of highway from period to period behave so as to return to a compliance level π following trap operation, or to deteriorate in compliance by δ following non-operation. The motorists themselves will have individual π_i and δ_i. What the model requires is that the central tendancies of these variables remain stationary with values π and δ. This is more likely to be the case for commuter populations, and therefore for operational periods within the rush hours, than for cohorts of predominantly transient (i.e., atypical, sporadic) motorists. Further, it is assumed that the random surveillance takes place along such segments where there is in fact an opportunity to speed, otherwise traffic conditions and road topology may also intervene in the driver's decision to speed.

As a step toward further refinement and possible model validation, it would be useful to disaggregate the driver population. The subpopulations of motorists conceptualized earlier provide one possible stratification, although attributes such as prior arrests and cumulative points towards loss of license, age, marital status, number of children, sex, vehicle type, etc., may be shown to be more salient vis-à-vis the propensity to speed. Accordingly, significantly different values of π and δ are likely to be at play among those speeders who have been caught when a trap is operating; those who are not caught during operation; those who do not speed even when they observe that there is no surveillance; and those who speed when their experience indicates that there is no enforcement. Thus, the level of π is likely to be high and δ small for the first group; π and δ may also be good for the second group (presumably counting themselves lucky to have escaped). The third group would be characterized by the extremal values $\pi = 1$ and $\delta = 0$, whereas the fourth stratum would have $\pi = 1$ and $\delta \approx 1$. In short, while the present model treats π and δ as stable averages for the population as a whole, a better approximation to the cohort's behavior might be obtained by modeling the separate, more homogeneous subpopulations with individual distributional parameters π_i and δ_i, $i = 1, 4$. On a more detailed level, allowing for further realism, a discrete simulation model could be developed in which each driver would be associated with a particular, perhaps nonstationary, distribution for π and δ.

Were experiments to determine these π_i and δ_i not so legally awkward, useful insights might be gained on the deterrent effects of such surveillance. Although extrapolations to serious criminal activity would be at obvious risk, at least some light could be shed on the deterrent aspect of law enforcement. The behavior of offenders who are not caught (when they believe there is no enforcement) is particularly interesting since its observation for felonies would be essentially impossible (i.e., officers would be legally and morally bound to make an arrest). However, it would probably be acceptable to contrive such a situation in a speeding context (i.e., where the motorists believe there is no surveillance and yet are actually observed by police over an extended period). Further research and empirical results along these lines would assist the remaining problem of model validation.

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