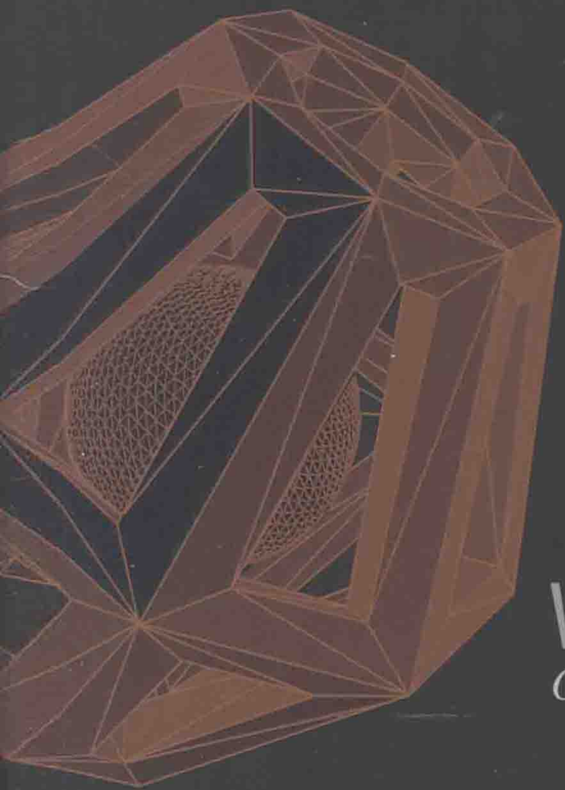

Vol. 3

Optimization Methods

Computer Aided and Integrated Manufacturing Systems

A 5-Volume Set

Cornelius T Leondes

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Cornelius T Leondes
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Preface

Computer Technology

This 5 volume MRW (Major Reference Work) is entitled "Computer Aided and Integrated Manufacturing Systems". A brief summary description of each of the 5 volumes will be noted in their respective PREFACES. An MRW is normally on a broad subject of major importance on the international scene. Because of the breadth of a major subject area, an MRW will normally consist of an integrated set of distinctly titled and well-integrated volumes each of which occupies a major role in the broad subject of the MRW. MRWs are normally required when a given major subject cannot be adequately treated in a single volume or, for that matter, by a single author or coauthors.

Normally, the individual chapter authors for the respective volumes of an MRW will be among the leading contributors on the international scene in the subject area of their chapter. The great breadth and significance of the subject of this MRW evidently calls for treatment by means of an MRW.

As will be noted later in this preface, the technology and techniques utilized in the methods of computer aided and integrated manufacturing systems have produced and will, no doubt, continue to produce significant annual improvement in productivity — the goods and services produced from each hour of work. In addition, as will be noted later in this preface, the positive economic implications of constant annual improvements in productivity have very positive implications for national economies as, in fact, might be expected.

Before getting into these matters, it is perhaps interesting to briefly touch on Moore's Law for integrated circuits because, while Moore's Law is in an entirely different area, some significant and somewhat interesting parallels can be seen. In 1965, Gordon Moore, cofounder of INTEL made the observation that the number of transistors per square inch on integrated circuits could be expected to double every year for the foreseeable future. In subsequent years, the pace slowed down a bit, but density has doubled approximately every 18 months, and this is the current definition of Moore's Law. Currently, experts, including Moore himself, expect Moore's Law to hold for at least another decade and a half. This is impressive with many significant implications in technology and economies on the international scene. With these observations in mind, we now turn our attention to the greatly significant and broad subject area of this MRW.

"The Magic Elixir of Productivity" is the title of a significant editorial which appeared in the *Wall Street Journal*. While the focus in this editorial was on productivity trends in the United States and the significant positive implications for the economy in the United States, the issues addressed apply, in general, to developed economies on the international scene.

Economists split productivity growth into two components: Capital Deepening which refers to expenditures in capital equipment, particularly IT (Information Technology) equipment: and what is called Multifactor Productivity Growth, in which existing resources of capital and labor are utilized more effectively. It is observed by economists that Multifactor Productivity Growth is a better gauge of true productivity. In fact, computer aided and integrated manufacturing systems are, in essence, Multifactor Productivity Growth in the hugely important manufacturing sector of global economies. Finally, in the United States, although there are various estimates by economists on what the annual growth in productivity might be, Chairman of the Federal Reserve Board, Alan Greenspan — the one economist whose opinions actually count, remains an optimist that actual annual productivity gains can be expected to be close to 3% for the next 5 to 10 years. Further, the Treasure Secretary in the President's Cabinet is of the view that the potential for productivity gains in the US economy is higher than we realize. He observes that the penetration of good ideas suggests that we are still at the 20 to 30% level of what is possible.

The economic implications of significant annual growth in productivity are huge. A half-percentage point rise in annual productivity adds $1.2 trillion to the federal budget revenues over a period of ten years. This means, of course, that an annual growth rate of 2.5 to 3% in productivity over 10 years would generate anywhere from $6 to $7 trillion in federal budget revenues over that time period and, of course, that is hugely significant. Further, the faster productivity rises, the faster wages climb. That is obviously good for workers, but it also means more taxes flowing into social security. This, of course, strengthens the social security program. Further, the annual productivity growth rate is a significant factor in controlling the growth rate of inflation. This continuing annual growth in productivity can be compared with Moore's Law, both with huge implications for the economy.

The respective volumes of this MRW "Computer Aided and Integrated Manufacturing Systems" are entitled:

Volume 1: Computer Techniques
Volume 2: Intelligent Systems Technology
Volume 3: Optimization Methods
Volume 4: Computer Aided Design/Computer Aided Manufacturing (CAD/CAM)
Volume 5: Manufacturing Process

A description of the contents of each of the volumes is included in the PREFACE for that respective volume.

Optimization methods will become an increasingly important factor in manufacturing systems as they have proven to enhance productivity. The design of cellular manufacturing systems will also be optimized, again with significant implications for productivity. Computer Aided Design (CAD) methods for manufacturing processes, such as the ubliquitous injection molding process, will increasingly utilize optimization methods. Rapid prototyping has become a way of life in manufacturing systems and this will benefit greatly from the optimization methods. The CAD/CAM (Computer Aided Design/Computer Aided Manufacturing) process will necessarily require visual assessment techniques of free-form surfaces in order to assure an optimal product. These and numerous other significant topics are treated rather comprehensively in Volume 3.

As noted earlier, this MRW (Major Reference Work) on "Computer Aided and Integrated Manufacturing Systems" consists of 5 distinctly titled and well integrated volumes. It is appropriate to mention that each of the volumes can be utilized individually. The great significance and the potential pervasiveness of the very broad subject of this MRW certainly suggests the clear requirement of an MRW for an adequately comprehensive treatment. All of the contributors to this MRW are to be highly commended for their splendid contributions that will provide a significant and unique reference source for students, research workers, practitioners, computer scientists and others, as well as institutional libraries on the international scene for years to come.

Contents

CHAPTER 1

OPTIMAL DYNAMIC FACILITY DESIGN OF MANUFACTURING SYSTEMS

TIMOTHY L. URBAN

Operations Management,
The University of Tulsa,
600 South College Avenue,
Tulsa, Oklahoma, USA
E-mail: unbantl@utulsa.edu

The physical layout of manufacturing systems is a major determinant of a firm's efficiency. With the rapidly-changing environment facing most firms today as well as the shortened life cycles of many products and process technologies, facility rearrangement and redesign become critical in sustaining productivity and competitiveness. Consequently, operations managers and researchers have recently focused on the dynamic aspects of facility design. This paper investigates various approaches of analyzing and solving the dynamic facility layout problem. Optimization, bounding, and heuristic methodologies are presented, and issues concerning the application and implementation of these dynamic models are presented.

Keywords: Facility design; dynamic models; optimization.

1. Introduction

It is a well-established fact that products and processes exhibit life cycles evolving through initial development, growth, maturity, and decline stages. The understanding of these life cycles can be quite beneficial in determining appropriate marketing and manufacturing strategies for an organization. Schmenner[1] proposed that facilities also progress through life cycles and that the knowledge of this concept can be exploited to plan the use and change of the facility, leading to improved productivity and prolonged facility life. Nandkeolyar *et al.*[2] further developed a conceptual model for facility life cycles, identifying the characteristics that describe each stage. The throughput, the number of products, the capacity utilization, and the process technology — all of which have an effect on the design of the facility at any particular stage of the life cycle — change throughout the life of a facility which, in turn, necessitates the redesign of the facility. In general, manufacturing facilities tend to be quite capital intensive and have long-range implications for the organization, which underscores the importance of dynamic facility design in response to the changing demands placed on the facility.

Nicol and Hollier[3] reported on the results of a field study of 33 manufacturing companies in the United Kingdom. One of the aspects investigated was the stability of the firms' facility layout. They found that layouts were frequently designed for a predetermined fixed level of production which could only be marginally exceeded, yet many companies experienced or anticipated volume changes by a factor of two or more. Nearly half of the companies had an average layout stability of two years or less; the mean of all firms was just over three years. The authors concluded "that radical layout changes occur frequently and that management should therefore take this into account in their forward planning".

More recently, Hales[4] reported on a survey of facility management organizations (199 respondents, two-thirds of which were manufacturing firms) designed to measure current business practices and concerns. One of the facilities management practices that was identified as being in serious need of improvement was that of "planning horizon" for major buildings and facilities; over 80 percent of the respondents categorized their organization with poor or inadequate performance on this dimension. Furthermore, three of the eight key findings that were identified are:

(i) Rearranging for cells and continuous flow: This was the greatest management concern, cited by 60% of respondents, including a number in government, insurance, health care, and other non-manufacturing facilities.
(ii) Many facilities organizations lack readiness: They are playing 'one-move chess' with no plan beyond their next major project.
(iii) Planning horizons are still too short among manufacturing organizations: The most common practice is still calendar-based, typically three to five years, rather than being tied to industry cycles or the life cycles of key products and process technologies.

Obviously, facilities managers see the rearrangement and redesign of facilities as an important part of their organizational efficiency and competitiveness, yet this is one aspect of their planning efforts that is apparently underemphasized.

The purpose of this paper is to investigate various approaches of analyzing and solving the dynamic facility layout problem. Current formulations of the problem and techniques for determining the optimal solution are presented. We present a linearization of the problem, such that a commercially-available, linear-programming computer package can be utilized, either in conjunction with a CAD system or as a stand-alone package for a facility manager solving relatively small problems. Bounding techniques are demonstrated to make the problem more tractable. Finally, implementation issues concerning the understanding and application of the dynamic facility layout problem are presented.

2. The Dynamic Facility Layout Problem

Despite the obvious practical relevance concerning the dynamics of facility design, the vast majority of relevant research has focused on the static problem, in which the layout design is determined with no consideration of future requirements. Literally hundreds of papers have been written on the static facility (plant) layout problem since the development of operations sequence analysis[5] and systematic layout planning.[6] Several reviews of the facility layout problem have been published, including Levary and Kalchik,[7] Kusiak and Heragu,[8] and Meller and Gau.[9] Recently, a survey paper on the dynamic facility layout problem was published by Balakrishnan and Cheng.[10]

Hitchings[11] was one of the first authors to recognize the importance of planned changes in the layout of facilities. Based on the observation that the material handling cost for all feasible layouts tends to be approximately normally distributed, Hitchings argued that controlling a layout could be conducted similar to the control of a production process. Thus, the use of control charts was suggested to identify at which point in time a layout change is warranted; that is, when the cost of effecting the change is less than the savings that result from the change. The use of statistical quality control techniques is common in manufacturing systems and could then easily be applied to determine the timing of the facility redesign.

Hicks and Cowan[12] developed an extension of the well-known Craft heuristic,[13] a pairwise-exchange procedure in which the cost of moving a department and the resulting process improvements (resulting from the redesign) are incorporated in the layout decision logic. One shortcoming of their approach, however, is that it only provides one opportunity for facility redesign, changing the layout when the resulting cost improvement exceeds the rearrangement cost; no future rearrangements are considered. Slepicka and Rajchel[14] proposed a dynamic programming procedure to select from a set of feasible layout arrangements over a finite time period. They formulated the problem with two conflicting goals — minimizing the cost of the rearrangement and maximizing the operational efficiency of the resulting layouts. It was assumed the time "when a major expansion takes place" is defined, and a feasible set of layouts within each time frame can be determined.

Rosenblatt[15] was the first to present a comprehensive treatment of the dynamic facility layout problem; he provided an explicit formulation of the problem, developed an optimal solution methodology, identified bounding procedures, and established heuristic techniques. He proposed a model analogous to the quadratic assignment problem (QAP) in which the objective is to assign each of the N departments to one of the N specified locations (dummy departments or locations can be used when an unequal number of departments/locations are available). In the dynamic layout problem, this assignment must be made for each of the T periods in the planning horizon; thus, there are $(N!)^T$ possible solutions to the overall problem. Even symmetric layouts must be considered in the dynamic situation, as different rearrangement costs will be incurred for different layouts. Since the publication of

Rosenblatt's paper, a great deal of research activity has focused on dealing with various aspects of the dynamic facility layout problem.

3. Mathematical Formulation

The dynamic facility layout problem (DFLP) is one in which the layout arrangement of a facility — that is, the relative location of departments, machines, cells, workstations, etc. — is determined for each period of a finite planning horizon. The principal costs associated with this problem are the material handling costs for each period as well as any rearrangement costs involved in changing the layout between periods. On the one hand, we would like to change the layout arrangement over time to minimize the material handling effort; on the other hand, we would like to maintain the same layout from one period to the next to avoid costs associated with the rearrangement of the facility.

3.1. *Extension of the quadratic assignment problem*

The most common formulation of the DFLP is an extension of the quadratic assignment problem; reviews of the QAP can be found in Burkard,[16,17] Finke *et al.*,[18] and Pardalos *et al.*[19] Under this formulation, the performance (efficiency) of a particular layout arrangement is measured as the sum of the workflows (material handling cost per unit distance) times the distance traveled. This measure must be expressed as a cost in the DFLP, as opposed to a distance measure, so it can directly correspond to the rearrangement costs. This is the approach that was presented by Rosenblatt[15] and has been the focus of the majority of research conducted on the dynamic problem.

Let $N = \{1, 2, \ldots, i, \ldots, |N|\}$ represent the number of departments and locations and $T = \{1, 2, \ldots t, \ldots, |T|\}$ represent the time periods in the planning horizon. The problem can then be formulated as a quadratic binary programming problem (see, e.g. Kaku and Mazzola[20]) as follows:

DFLP–1

Minimize

$$C = \sum_{t \in T} \left[\sum_{i \in N} \sum_{j \in N} \sum_{k \in N} \sum_{l \in N} f_{ikt} d_{jl} x_{ijt} x_{klt} + \sum_{i \in N} \sum_{j \in N} c_{ijt} x_{ijt} + \sum_{i \in N} s_{it} y_{it} + r_t z_t \right]. \tag{1}$$

Subject to:

$$\sum_{j \in N} x_{ijt} = 1 \qquad i \in N,\ t \in T, \tag{2}$$

$$\sum_{i \in N} x_{ijt} = 1 \qquad j \in N,\ t \in T, \tag{3}$$

$$x_{ijt} \in \{0, 1\} \qquad i \in N,\ j \in N,\ t \in T, \tag{4}$$

$$y_{it} = \sum_{j \in N} x_{ijt-1} \left(\sum_{l \in N \setminus \{j\}} x_{ilt} \right) \qquad i \in N,\ t \in T \setminus \{1\}, \tag{5}$$

$$z_t \geq y_{it} \qquad i \in N,\ t \in T \setminus \{1\}, \tag{6}$$

$$y_{it}, z_t \geq 0 \qquad i \in N,\ t \in T \setminus \{1\}, \tag{7}$$

where f_{ikt} is the workflow cost from department i to department k in time period t (likely measured as the product of the volume of material flow, assumed to be deterministic, and the cost per unit to move the material), d_{jl} is the distance from location j to location l (assumed to be time invariant), c_{ijt} is the cost of assigning department i to location j in period t, s_{it} is the variable rearrangement cost of moving department i at the beginning of the time period t, and r_t is the fixed rearrangement cost associated with making any layout changes at the beginning of period t. The decision variables are:

$$x_{ijt} = \begin{cases} 1 & \text{if department } i \text{ is placed at location } j \text{ in period } t \\ 0 & \text{otherwise} \end{cases}$$

$$y_{it} = \begin{cases} 1 & \text{if department } i \text{ is moved at the beginning of period } t \\ 0 & \text{otherwise} \end{cases}$$

$$z_t = \begin{cases} 1 & \text{if any rearrangement is made at the beginning of period } t \\ 0 & \text{otherwise} \end{cases}$$

Constraint sets (2) and (3) are the typical constraints associated with an assignment problem and they ensure that each department is assigned to one location and each location contains exactly one department for each time period. Constraint sets (5) and (6) are definitional constraints ensuring that the rearrangement variables (y_{it} and z_t) take on a value of one if a rearrangement takes place. Note that these constraints also allow the rearrangement variables to be expressed as continuous variables. As shown, these definitional constraints and variables assume that there is no existing layout at the beginning of the planning horizon, implying that this is a new facility; hence, they are not required for the first period (any layout arrangement can be used with no rearrangement cost at $t = 1$). If there is an existing layout involving a rearrangement at the current time, this can be represented by incorporating the appropriate assignment variables (i.e. we would have x_{ij0} to take on appropriate values) and extending Constraint sets (5) and (6) over the entire planning horizon ($t \in T$).

This formulation of the dynamic facility layout problem can easily be generalized. For example, Balakrishnan *et al.*[21] formulated the variable rearrangement costs to reflect the origin and destination of a department's move, $\sum_i \sum_j \sum_l s_{ijlt} y_{ijlt}$. This could take into account, for instance, situations in which the rearrangement cost is dependent on the distance the department is moved. Discounting considerations can also be easily incorporated into the formulation, if desired, by simply including a discount factor for each of the cost coefficients.

3.2. *Strategic interpolative design*

An alternative approach to the DFLP was presented by Montreuil and Venkatadri[22] in which the facility designer has developed a target layout arrangement for the final (mature) phase of a facility expansion. The intent is then to identify the intermediary layout over several phases. It is assumed that each department will remain within the boundaries it is assigned in the final arrangement. Thus, the size of the department must remain less than or equal to its final size during each phase, although the input/output stations of the departments are not required to be stationary and can be moved at no cost. The location of each department relative to the other departments, however, must remain the same.

Since the size of the departments can change over time, the quadratic assignment formulation (in which departments are allocated to specific locations) is not appropriate. Instead, a formulation is used in which the layout is defined on a planar coordinate system (see, e.g. Montreuil[23] and Banerjee *et al.*[24]). The departments are rectangular, and their locations are characterized by the coordinates of the input/output stations and the length and width of the department.

Since each department remains in the same location (although it may grow over time), there are no rearrangement costs associated with this formulation. Thus, the objective is simply to minimize the sum of the material handling costs for each phase over a finite planning horizon. Let $N = \{1, 2, \ldots, i, \ldots, |N|\}$ represent the number of departments, $S = \{1, 2, \ldots, s, \ldots, |S|\}$ represent the number of input/output stations for the departments, and $P = \{1, 2, \ldots, p, \ldots, |P|\}$ represent the phases of the facility expansion. The formulation of the problem is then:

DFLP–2

Minimize

$$C = \sum_{p \in P} \sum_{i \in N} \sum_{j \in N} \sum_{s \in S} \sum_{r \in S} w_p f_{ijsrp} \left(|x_{isp} - x_{jrp}| + |y_{isp} - y_{jrp}| \right). \tag{8}$$

Subject to:

$$\underline{X}_{ip} \leq x_{isp} \leq \overline{X}_{ip} \qquad i \in N,\ s \in S,\ p \in P, \tag{9}$$

$$\underline{Y}_{ip} \leq y_{isp} \leq \overline{Y}_{ip} \qquad i \in N,\ s \in S,\ p \in P, \tag{10}$$

$$0 \leq \underline{LX}_{ip} \leq \overline{X}_{ip} - \underline{X}_{ip} \leq \overline{LX}_{ip} \qquad i \in N,\ p \in P, \tag{11}$$

$$0 \leq \underline{LY}_{ip} \leq \overline{Y}_{ip} - \underline{Y}_{ip} \leq \overline{LY}_{ip} \qquad i \in N,\ p \in P, \tag{12}$$

$$0 \leq \underline{B}_{ip} \leq 2\left[\left(\overline{X}_{ip} - \underline{X}_{ip}\right) + \left(\overline{Y}_{ip} - \underline{Y}_{ip}\right)\right] \leq \overline{B}_{ip} \qquad i \in N,\ p \in P, \tag{13}$$

$$\underline{X}_{i(p-1)} \geq \underline{X}_{ip} \qquad i \in N,\ p \in P \backslash \{1\}, \tag{14}$$

$$\overline{X}_{ip} \geq \overline{X}_{i(p-1)} \qquad i \in N,\ p \in P\backslash\{1\}, \qquad (15)$$

$$\underline{Y}_{i(p-1)} \geq \underline{Y}_{ip} \qquad i \in N,\ p \in P\backslash\{1\}, \qquad (16)$$

$$\overline{Y}_{ip} \geq \overline{Y}_{i(p-1)} \qquad i \in N,\ p \in P\backslash\{1\}, \qquad (17)$$

where w_p is the weight associated with phase p. It is also a function of the length of the phase, a discounting factor, etc. f_{ijsrp} is the workflow between input/output station s of department i and station r of department j in phase p; (x_{isp}, y_{isp}) are the coordinates of input/output station s of department i in phase p; $(\underline{X}_{ip}, \underline{Y}_{ip})$ and $(\overline{X}_{ip}, \overline{Y}_{ip})$ are the coordinates of the lower and upper boundaries of department i in phase p, respectively; $(\underline{LX}_{ip}, \underline{LY}_{ip})$ and $(\overline{LX}_{ip}, \overline{LY}_{ip})$ are the lower and upper bounds on the length of the sides of department i in phase p, respectively; and $\underline{B}_{ip}$ and $\overline{B}_{ip}$ are the lower and upper bounds on the perimeter of department i in phase p. Linear programming can be utilized to solve this problem as the absolute values in the objective function can be easily linearized through the use of additional variables.

Constraint sets (9) and (10) ensure that the input/output station coordinates are within the departmental boundaries. Constraint sets (11) and (12) restrict the length of the sides of the department to be within bounds; the same is done for the perimeter of the department with Constraint set (13). Finally, Constraint sets (14)–(17) ensure that the department in any phase is located within the boundaries of the department in the next phase, allowing growth over time but maintaining its relative location. Montreuil and Venkatadri[22] also identified some variations of their model by considering aspects such as temporary reductions in the size of a department and phasing out a facility.

Montreuil and Laforge[25] introduced a dynamic facility layout model to take into consideration the probabilistic nature of future requirements. They proposed a set of possible future states, each with a probability of occurrence as well as workflow and spatial requirements. The designer is also required to propose a design skeleton for each future, so, as with the Montreuil and Venkatadri[22] model, the relative positions of the departments do not change. The authors argued, however, that an experienced layout designer could investigate multiple design skeletons to identify good layouts.

Lacksonen[26] has since proposed an approach to address the limitation that pre-specifying the design skeleton has on the analysis of the trade-off between material handling costs and rearrangement costs. He proposed a two-stage formulation in which the first stage is a QAP-type of analysis that considers both workflow and department rearrangement, then he fixes the rearrangement costs and estimates the department arrangements. The second stage is a mixed integer program (an extension of the Montreuil formulations) which takes the results of the first stage to minimize the flow costs and define the specific department locations; preprocessing operations were identified to improve on the solution time.[27]

4. Optimal Solution Methodologies

In this section, we will investigate various existing optimal solution methodologies to the dynamic facility layout problem. We will also emphasize on the DFLP-1 formulation, as the majority of research conducted on the DFLP has focused on this formulation. A linearization of DFLP-1 is also developed that will allow the optimal solution of the DFLP to be found using commercially-available, linear-programming software.

4.1. *Dynamic programming*

The first technique developed to identify the optimal solution for the dynamic facility layout problem was presented by Rosenblatt[15] using a dynamic programming algorithm. Each of the states of the dynamic program (DP) corresponds to a particular layout arrangement, and each of the stages corresponds to a time period in the planning horizon, resulting in a problem with $N!$ states and T stages. As mentioned above, symmetric layouts must be included, since different rearrangement costs will result from the different layout arrangements.

A recursive relationship was established to identify the combination of layout arrangements with the minimum total cost as follows:

$$C_{tm}^* = \min_k \left\{C_{t-1,k}^* + R_{km}\right\} + Q_t^m, \tag{18}$$

where R_{km} is the rearrangement cost as a result of changing from layout arrangement A_k to layout arrangement A_m ($R_{kk} = 0$). This could easily be modified to provide a different cost for different time periods. Q_t^m is the material handling cost for layout arrangement A_m in period t; and C_{tm}^* is the minimum total cost for all periods up to t, in which layout arrangement A_m is being used in period t ($C_{01}^* = 0$, assuming an initial layout is given). To restrict the state space of the model, Rosenblatt noted that the Sweeney and Tatham[28] results for the dynamic location problem are applicable to the dynamic layout problem. In particular, a layout arrangement for a given period does not need to be included in the DP if the difference between the material handling cost of that arrangement, Q_t^m, and the material handling cost of the optimal static solution for that period, Q_t^*, is greater than the difference between the values of the upper bound, C^+, and the lower bound, C^-, of the dynamic problem; that is:

$$Q_t^m - Q_t^* \geq C^+ - C^-. \tag{19}$$

Therefore, it is necessary to include only the best static solutions for each period in the planning horizon. To identify the best ranked static solutions that must be included in the DP state space, a constrained quadratic assignment problem (precluding higher ranked solutions) can be solved. Balakrishnan[29] developed an alternative fathoming procedure in which the right-hand side of Constraint set (19) is replaced by twice the maximum rearrangement cost that could occur in a given period. This, in the general case, is the sum of the fixed and all variable rearrangement costs.

Due to the computational requirements necessary to optimally solve the quadratic assignment problem as well as that for dynamic programming algorithms (the infamous 'curse of dimensionality'), this approach is practical only for small problems. The development of strong bounds — to reduce the DP state space — and an efficient method of finding the best ranked static solutions have become consequential in finding the optimal solution to the DFLP.

4.2. *Mixed integer programming*

Another approach to solving the dynamic facility layout problem is to linearize the quadratic binary program in order to utilize integer linear programming algorithms, which are readily accessible using commercially-available software. As shown in the previous section, DFLP-1 contains a quadratic objective function and a set of quadratic constraints. To linearize the constraint set, we replace Constraint set (5) with the following constraints:

$$y_{it} \geq x_{ijt-1} - x_{ijt} \quad i \in N,\ j \in N,\ t \in T, \tag{5a}$$

$$y_{it} \geq x_{ijt} - x_{ijt-1} \quad i \in N,\ j \in N,\ t \in T. \tag{5b}$$

While this approach obviously requires additional constraints ($2N^2T$ versus NT), it results in an all-linear constraint set.

We can linearize the objective function in a manner similar to the approaches used in the linearization of the quadratic assignment problem. Various QAP linearizations have been proposed by Lawler,[30] Bazaraa and Sherali,[31] Frieze and Yadegar,[32] and Kettani and Oral[33,34]; we follow the approach of Kaufman and Broeckx[35] which, in turn, is based on the method of Glover.[36] To do this, first consider the quadratic component of the objective function (Eq. 1):

$$\sum_{t \in T} \sum_{i \in N} \sum_{j \in N} \sum_{k \in N} \sum_{l \in N} f_{ikt} d_{jl} x_{ijt} x_{klt}. \tag{20}$$

Now define a set of N^2T continuous variables, w_{ijt}, such that:

$$w_{ijt} = x_{ijt} \sum_{k \in N} \sum_{l \in N} f_{ikt} d_{jl} x_{klt}. \tag{21}$$

If x_{ijt} is equal to zero, then w_{ijt} will also equal zero; if x_{ijt} is equal to one, then w_{ijt} will equal the material handling cost for the corresponding layout arrangement, $\sum_k \sum_l f_{ikt} d_{jl} x_{klt}$. Therefore, this component of the objective function can be simply expressed as the sum of the w_{ijt} variables. For each new variable, we also add the constraints:

$$u_{ijt}x_{ijt} + \sum_{k\in N}\sum_{l\in N} f_{iklt}d_{jl}x_{klt} - w_{ijt} \leq u_{ijt}, \tag{22}$$

$$w_{ijt} \geq 0, \tag{23}$$

where $u_{ijt} = \max\{\sum_k \sum_l f_{ikt}d_{jl}, 0\}$. The DFLP can now be expressed as the following mixed integer program:

DFLP–1 (MIP)
Minimize

$$C = \sum_{t\in T}\left[\sum_{i\in N}\sum_{j\in N}(w_{ijt} + c_{ijt}x_{ijt}) + \sum_{i\in N} s_{it}y_{it} + r_t z_t\right]. \tag{24}$$

Subject to:

$$\sum_{j\in N} x_{ijt} = 1 \qquad i\in N,\ t\in T,$$

$$\sum_{i\in N} x_{ijt} = 1 \qquad j\in N,\ t\in T,$$

$$y_{it} \geq x_{ijt-1} - x_{ijt} \qquad i\in N,\ j\in N,\ t\in T,$$

$$y_{it} \geq x_{ijt} - x_{ijt-1} \qquad i\in N,\ j\in N,\ t\in T,$$

$$z_t \geq y_{it} \qquad i\in N,\ t\in T,$$

$$u_{ijt}x_{ijt} + \sum_{k\in N}\sum_{l\in N} f_{ikt}d_{jl}x_{klt} - w_{ijt} \leq u_{ijt} \qquad i\in N,\ j\in N,\ t\in T,$$

$$x_{ijt}\in\{0,1\}, w_{ijt}, y_{it}, z_t \geq 0 \qquad i\in N,\ j\in N,\ t\in T.$$

This formulation contains N^2T integer variables and $(N^2+N+1)T$ continuous variables. While readily available software can be used to solve this problem (the Ampl model, a modeling language for mathematical programming, and the data presented by Lacksonen and Enscore[37] are contained in the appendix for illustrative purposes), it will still only be practical for relatively small problems. The use of MIP solution strategies — strong bounds, depth-first versus breadth-first branching, special ordered sets, etc. — can be utilized to increase the size of the problem; Crowder *et al.*[38] discussed strategies for solving general, large-scale, zero-one linear programming problems.

4.3. *Special case: fixed rearrangement costs*

In many situations, the fixed cost of rearranging a facility far outweighs the variable (i.e. departmental) rearrangement costs. For example, if the entire facility must frequently be shut down when any rearrangement occurs, the cost of moving any particular department may be negligible as compared to the lost production time.