COLLEGE ALGEBRA
AND
PLANE TRIGONOMETRY

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PREFACE

There are many approaches to the problem of integrating the elementary collegiate mathematics course, ranging from a belief in the traditional separation to one of complete integration through the calculus with the introduction of the calculus coming very early. There is much to be said for each of these approaches. Without attempting to evaluate the relative merits of the various plans, the authors feel that there is a definite need for a book like the present one, one reason being the increasing popularity of integrated courses in analytic geometry and calculus, for which an integrated course in college algebra and trigonometry is a natural introduction.

The emphasis on the study of the elementary functions of mathematics forms a natural basis for the integration of algebra and trigonometry, and is the basis of the present treatment; thus, the mathematical concept of function has been used as the underlying theme of this book. The subject then becomes, in a very real sense, the study of the properties and applications of the elementary functions based on the principles of algebra, leaving the subsequent study of these functions based on the idea of limit as the province of the calculus.

In particular, in the treatment of the topics in trigonometry the emphasis is on the study of the trigonometric functions as functions. For example, the inverse trigonometric functions, in Chapter 10, are treated as an integral phase of the development of the elementary functions, instead of being relegated to an obscure role at the end of the book. In this sense, the solution of triangles becomes an application of the trigonometric functions. This emphasis, in addition to imparting a unity to the subject, also serves to remove early in the course the restricted concept of trigonometry that persists with many students who have studied the subject in high school.

Varied types of students must be served by a course in algebra and trigonometry. The student who pursues the course as a preliminary to the further study of mathematics, whether as a mathematics major or as a student of engineering or science, must, of course, obtain a firm foundation in the many skills required. On the other hand, he must also acquire the mode of thinking that considers mathematics as more than a mere collection of skills. The student for whom college algebra and trigonometry is the terminal mathematics course should attempt to achieve some of the feeling of mathematics as a cultural subject, and some knowledge of the concepts contained therein, that can persist when the special skills may be forgotten. It is the belief of the authors that both kinds of student are served to excellent effect by the emphasis on the function concept as applied to elementary mathematics.

In the first chapter, together with the usual development of the fundamental notions, there is included a brief discussion of number systems, starting with the positive integers or natural numbers and leading to the negative integers and zero, the rational numbers, the irrational numbers, and the complex numbers, with the need for each type indicated.

Except for the discussion of number systems, the material in the first four chapters represents essentially a review of the content of the usual one-year course in high-school algebra. To facilitate the review, especially for those students with more than this minimum preparation in algebra, an extensive list of review exercises is given at the end of the fourth chapter.

In Chapter 5 the function concept is introduced and applied to the trigonometric functions. In subsequent chapters this concept is related to many of the usual topics in algebra and trigonometry.

The use of the radian measure of an angle is introduced early, in conjunction with the degree measure, and continuous use is made of it in the following treatment. It is hoped thereby to avoid a common tendency of students to feel that radians may be useful only in considering lengths of arc on a circle, and should be ignored for every other purpose. The distinction between the idea of angle and the measure of an angle is clearly made.

With the abundance of material available for inclusion in the present book, the authors have deliberately omitted extended treatments of certain topics such as partial fractions, limits, and infinite series. However, some of these matters are not avoided where they arise naturally during a discussion. For example, binomial expansions for negative and fractional exponents are discussed in Section 7 of Chapter 12, and infinite geometric series are discussed in Section 3 of Chapter 17. For the topics which have been included the emphasis has been on as rigorous and logical a treatment as possible. To be sure, not every statement can be proved in an elementary course, but omissions of proof are stated. It is hoped that the student will thus know what has been proved and what has not been proved, and in each case will not have to "unlearn" later what has supposedly once been learned.

A feature of the presentation is an extended discussion of trigonometric reduction formulas. Proofs of these formulas are not readily available to the interested reader, and it is hoped that such students will appreciate the complete proof given. The study of the proof may be omitted without disturbing the remainder of the treatment.

The choice of 4-place tables on which to base the numerical aspects of the presentation was made with the idea that such tables involve the same principles as do more accurate tables and have an advantage in the economy of time involved in their use.

Students entering a course for which this text is designed will vary in their mathematical preparation. For the student with only one year of high-school algebra, the material in the first four chapters contains, in addition to a discussion of number systems, a detailed review of the usual material of that course and enough of high-school algebra to prepare him for Chapter 5, in which the idea of function is introduced. For the student with one and one-half years of high-school algebra, a very brief review of these first four chapters will suffice. For the better prepared student there is ample material for a five-semester hour course based on Chapters 5 through 18, with some leeway permitted the instructor in the selection of material. For the less well prepared student a course of six- to eight-semester hours can be given by varying the time spent on the first four chapters.

Since there are places where any attempt at integration would seem forced, such artificial attempts have been avoided. In such instances the most fruitful direction appears to be that in which skills acquired and principles learned are applied to any of the functions to which application can be made. The exercises to be worked by the student have been designed in many cases to carry out this idea. For the reader, answers are provided at the back of the book for odd-numbered exercises. For the teacher, all of the answers are available in a separate booklet.

We are indebted to Rinehart & Company, publishers, for permission to use Table 4, which was taken from *Fundamentals of College Mathematics* by Johnson, McCoy, and O'Neill, and to D. C. Heath & Company for permission to use Table 2, which was taken from *A Brief Course in Trigonometry* by Curtiss and Moulton.

A. S.
R. H. B.

February, 1955

USEFUL ITEMS FROM PLANE GEOMETRY

1. Two angles are complementary if their sum is 90°.
2. Two angles are supplementary if their sum is 180°.
3. The sum of the angles of any triangle is 180°.
4. *Theorem of Pythagoras.* In a right triangle the square of the hypotenuse is equal to the sum of the squares of the legs.
5. If a right triangle has a 30° angle, the side opposite that angle is equal to one-half of the hypotenuse.
6. If two angles of a triangle are equal, the opposite sides are equal, and conversely.
7. Two triangles are similar if the angles of one are equal, respectively, to the angles of the other.
8. If two triangles are similar, corresponding sides of the triangles are proportional.
9. If the sides of an angle are perpendicular, respectively, to the sides of another angle, the angles are either equal or supplementary.
10. The area of any triangle is equal to one-half the product of any side and the altitude drawn to that side.
11. The angle bisectors of a triangle intersect in a point, which is the center of the inscribed circle.
12. The perpendicular bisectors of the sides of a triangle meet in a point, which is the center of the circumscribed circle.

Greek Alphabet

α alpha	β beta	γ gamma	δ delta	ϵ epsilon	ζ zeta
η eta	θ theta	ι iota	κ kappa	λ lambda	μ mu
ν nu	ξ xi	o omicron	π pi	ρ rho	σ sigma
τ tau	υ upsilon	ϕ phi	χ chi	ψ psi	ω omega

CONTENTS

CHAPTER 1

NUMBER SYSTEMS AND FUNDAMENTAL OPERATIONS

The development of the number system has been a long and tedious process in the history of mankind. From its beginning in the simple counting of objects to its present highly developed stage the advances have often been slow and difficult. Among the greatest advances has been the introduction from time to time of various aspects of the simplified notation as presently used. For example, the use of a symbol to designate zero constituted an exceptionally notable achievement which did not come about until long after the Greek era.

In advanced treatments the construction of the number system is made to depend on a minimum number of assumptions, together with logical deductions from them. It is not our intent to present here a complete and rigorous exposition, nor would it be possible to do so. However, an outline of various phases of the development will prove interesting and helpful.

1–1 The integers. The most elementary experience with the number system occurs in learning to count. This use of numbers involves only the *positive integers*, 1, 2, 3, 4, etc., or the *natural numbers*, as they are also called. The next step consists in the combination of the positive integers in various ways, by addition and multiplication. Each operation of addition and multiplication with positive integers results again in a positive integer; this fact is expressed in the statement that *the system of positive integers is closed under the operations of addition and multiplication.*

DEFINITION 1–1. *If b and c are positive integers, the product of b and c means the sum $c + c + \cdots + c$, where there are b numbers c in the sum. The product of b and c is designated by bc, or $b \cdot c$, or $b \times c$.*

It is frequently desired to refer to the sum or product of two or more numbers as a single quantity. For this purpose various grouping symbols are used, such as *parentheses*, ( ), *brackets*, [ ], *braces*, { }, and the *vinculum*, ——. For example, $(a + b) + c$ means that the sum $a + b$ is first computed, and to that sum c is then added, and $a \cdot bc$ is obtained by computing the product bc and then multiplying that product by a.

The order in which the positive integers are combined, whether in addition or multiplication, is immaterial, i.e., the following *properties* hold:

A. $b + c = c + b$
B. $bc = cb$
C. $b + (c + d) = (b + c) + d = b + c + d$
D. $b(cd) = (bc)d = bcd.$

A further property, involving addition and multiplication, is

E. $b(c + d) = bc + bd.$

Properties A and B are known as the *commutative laws* of addition and multiplication, respectively, and C and D are the *associative properties* of addition and multiplication, respectively. Finally, E is the *distributive property* of multiplication with respect to addition. It should be noted that the associative properties in C and D make the use of parentheses unnecessary for a sum or product of integers.

The next operation with the integers would naturally be to find the difference of two integers. The *difference* between two positive integers a and b is defined as the number which must be added to b to give a.

ILLUSTRATION. The difference between 5 and 2 is 3, since $2 + 3 = 5$.

It is very convenient at this point to introduce the minus sign (−) and to indicate the difference between b and c as $b - c$. Thus we would have $5 - 2 = 3$. The process of finding the difference between two integers is called *subtraction*. We note that it is not always possible to find the difference between two positive integers as a positive integer (for example, $3 - 5$ is not a positive integer), so that if subtraction is to be always possible it is necessary to expand the system of positive integers to a new system which contains *zero* and the *negative* integers in addition to the positive integers. *Zero* is defined as the difference between any positive integer and itself, $0 = a - a$. A negative integer $(-a)$, where a is a positive integer, is defined as the number such that $a + (-a) = 0$. In the new system, which contains the positive and negative integers and zero, it is always possible to find the difference of two numbers as a number of the system. The new system of numbers is *closed* under addition, multiplication, and subtraction. Moreover, *all the properties previously developed or stated apply to the new system.*

1–2 Algebraic sums. The expression $ax + by - c$ is an example of an *algebraic sum.* In an algebraic sum each part, together with the sign which is present or implied before it, is called a *term* of the expression. The name algebraic sum is used even though some of the signs of the terms may be minus; thus $ax - by$ is a sum of the two terms ax and $(-by)$. The process of determining the sum (or difference) of two algebraic quantities involves combining similar terms, as illustrated below.

ILLUSTRATION 1. The sum of $(3x + 2y - 5)$ and $(2x - 3y + 7)$ is

$$(3x + 2y - 5) + (2x - 3y + 7) = 3x + 2y - 5 + 2x - 3y + 7$$
$$= 3x + 2x + 2y - 3y - 5 + 7 = 5x - y + 2.$$

ILLUSTRATION 2. The difference between $(3x + 2y - 5)$ and $(2x - 3y + 7)$ is

$$(3x + 2y - 5) - (2x - 3y + 7) = 3x + 2y - 5 - 2x + 3y - 7$$
$$= 3x - 2x + 2y + 3y - 5 - 7 = x + 5y - 12.$$

ILLUSTRATION 3.

$$3x - [2x + (x - 5) - \overline{3x - 1}] = 3x - [2x + x - 5 - 3x + 1]$$
$$= 3x - [-4] = 3x + 4.$$

Note that the removal of parentheses preceded by a minus sign necessitates a change in sign of each term within the parentheses.

ORAL EXERCISES

Simplify each of the following expressions by removing parentheses and combining similar terms.

1. $3 + (-2)$ 2. $-3 - 5$ 3. $-7 + 2$ 4. $5 - 7$ 5. $8 - 21$
6. $3x - 8x$ 7. $-10x + 16x$ 8. $-8a + 7a$
9. $5y - 12y$ 10. $5 - 6y - (2 - 3y)$ 11. $2a - 3b + 4a$
12. $(a + b) - 2a$ 13. $2a - (a - b)$ 14. $a - (3a - b)$
15. $2a + (b - a)$ 16. $-(a - b) + (b - 2a)$ 17. $-(a + b) - (b - a)$
18. $2a - (3b + a)$ 19. $(b - 2a) - (3b + 4a)$
20. $(2a - 3b) - (3a - 2b)$ 21. $a + x - (y + x) - a + y$
22. $2x - u + v - 2x$ 23. $2a - (3b - a) + (3a - b)$
24. $a - (2b + 3a) - (4a + 5b)$ 25. $3a - (b + 2a) + (5b - a)$

EXERCISE GROUP 1–1

In the following six exercises, (a) add the numbers, (b) subtract the lower number from the upper one.

1.	2.	3.	4.	5.	6.
35	−43	59	−71	0	2798
17	19	−23	−36	75	−3462