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ABSTRACT

Let f be a stable map from a compact, oriented, three manifold M into $\mathbb{R}^2$. The main object of these notes, treated in Chapter III, is the classification of immersions (f,h) of M into $\mathbb{R}^2 \times \mathbb{R}^2$. (Here, immersions (f,h_0) and (f,h_1) are equivalent if they are connected by a regular homotopy, (f,h_t).) A descriptive device used for this study is the space W_f obtained by identifying points of M that belong to the same component of the f-fibre. In Chapter I, the local geometry of W_f is studied and all local descriptions of W_f are given. In Chapter II, semi-local canonical coordinate expressions for f near the singular set of f are derived.

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Introduction

Our primary objective in this work is to study stable maps of compact three-dimensional manifolds into the plane. Stable maps generally are those whose character is unchanged by small perturbations, i.e. any small perturbation of a stable map can be obtained from it by composition with diffeomorphisms of the source and target manifolds. These maps have been characterized by John Mather [Mather, G^2].

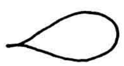
Let f be a smooth map of a compact three-dimensional manifolds, M, into the plane. The regular points of f are those at which the Jacobian of f has rank 2 and the singular points, $S(f)$, are the non-regular points. The stability of f is equivalent to the following:

At each $P \in S(f)$, there are coordinates (u,x,y) centered at P and (V,X) centered at $f(P)$ such that $(V(f(u,x,y)),X(f(u,x,y)))$ has one of the following expressions:

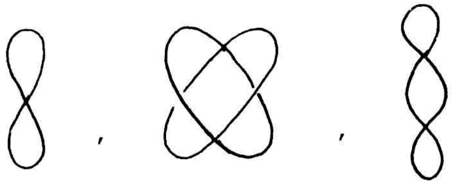
$$\begin{cases} (u,x^2+y^2), & P \text{ a definite fold point} \\ (u,x^2-y^2), & P \text{ an indefinite fold point} \\ (u,y^2+ux-\frac{x^3}{3}), & P \text{ a cusp point.} \end{cases}$$

In addition no cusp point is a double point of $f \mid S(f)$ and on $S(f)-\{\text{cusps}\}$, f is an immersion with normal crossings.

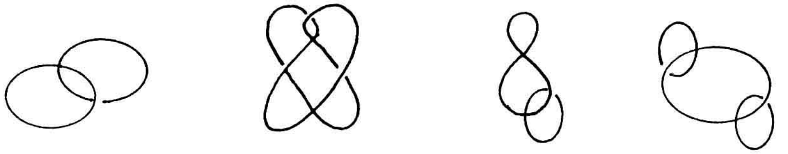
In attempting to understand the map f, we consider first the possible fibres of the map. Obviously, above every point in $\mathbb{R}^2 - f(S(f))$, the f-fibre is a set of disjoint embedded circles. However for $P \in S(f)$, the part of the f-fibre over $f(P)$ in the three above mentioned neighborhoods are an isolated point, two intersecting line segments, and a cusp (i.e. $\{u = 0,\ y^2 = x^3/3\}$). Thus the connected component of the f-fibre through a definite fold point is just the point itself and through a cusp point is:



However, on the connected component of the f-fibre through an indefinite point there may be one or two indefinite points. Thus those connected components of the f-fibres are connected graphs with one or two 'X-nodes'. The distinct possibilities are:

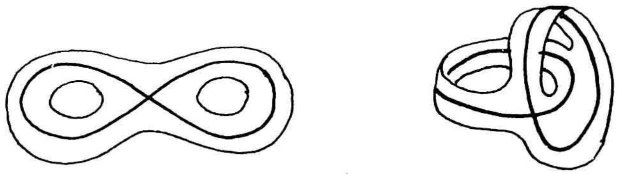


if M is orientable, and in addition if M is not orientable:



An indefinite point is called simple if it is the only point of $S(f)$ on the f-fibre component through it.

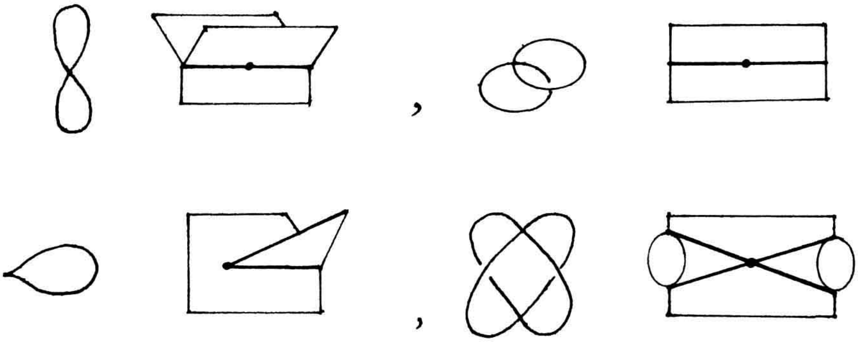
Call a subset of M, saturated if it is the union of connected components of f-fibres. A saturated neighborhood of an arc of definite points is a product of a disc with the arc, the f-fibre components of which are the concentric circles as well as the center of the disc at each point of the arc. A saturated neighborhood of an arc of simple indefinite points is a product of the arc with either a disc with two holes or a Möbius band with one hole (see §1.3).



For saturated neighborhoods of the non-simple points, see §1.4.

Essentially as a device for describing how the connected components of the f-fibres cohere to form the manifold, M, we introduce the auxiliary space W_f, defined by identifying points of M which belong to the same component of the f-fibre. We let $q : M \longrightarrow W_f$ be the identification map and $\overline{f} : W_f \longrightarrow \mathbb{R}^2$ be the map defined by $f = \overline{f} \circ q$. This factorization of f into the composition with a map with connected fibres, q, and one with finite fibres, $\overline{f}$, is known in algebraic geometry as the Stein factorization [Hart, p. 280]. This identification space was also used by Burlet and de Rham [B-de R] in the special case of stable maps of 3-manifolds into the plane having only definite fold singularities. In that case, W_f can be given the structure of a smooth surface with boundary, the boundary being the q-image of the definite fold curves.

For a general stable f, the local descriptions of W_f in the neighborhood of the q-image of points of S(f) are detailed in §1.4. We give three examples: the drawings show a q-fibre and a neighborhood in W_f of the q-image of that q-fibre



In all of the drawings of neighborhoods in W_f, the dark lines are the q-images of the arcs of S(f) in the neighborhood and in each case the heavy point is the q-image of the pictured q-fibre.

Chapter I is devoted to the description of stable maps, their fibres, the auxiliary space W_f and a decomposition of M following the pattern given by the obvious stratification of W_f.

In Chapters II and III we restrict ourselves to the M-orientable case. In Chapter II we construct coordinate systems that yield simple canonical forms for the coordinate expressions of f in tubular neighborhoods of arcs of S(f) and in neighborhoods of cusps and double points of $f \mid S(f)$. The proof of the principal technical result of this chapter from which everything else follows is rather long and is therefore deferred to an appendix. The final chapter makes extensive use of the results of this chapter.

In the final chapter we address the problem: Given a stable map $f : M \longrightarrow \mathbb{R}^2$ for M a compact orientable 3-manifold, classify the f-regular homotopy classes of immersions $(f,h) : M \longrightarrow \mathbb{R}^2 \times \mathbb{R}^2$, where (f,h) and (f,h') are f-regular homotopic if there is a homotopy, H, joining h to h' such that (f,H_t) is an immersion for each t.

In case N is a compact surface, the necessary and sufficient condition for lifting a stable map $g : N \longrightarrow \mathbb{R}^2$ to an immersion in

$\mathbb{R}^3$ is that each component of $S(g)$ with an even (odd) number of cusps has an orientable (non-orientable) neighborhood. This result of Haefliger [Haef] has been generalized by Blank and Curley [B-C] to lift stable maps (admitting only folds and cusps) between manifolds of the same dimension to an immersion into a line bundle over the target. The problem of classifying all immersions that lift a given map has, as far as I know, been studied only in the case that both the source and target manifolds are 2-dimensional [B, F-T].

The general idea of our classification involves the following: If (f,h) is an immersion of M into $\mathbb{R}^2 \times \mathbb{R}^2$, h must immerse the fibres of $f \mid M - S(f)$ in $\mathbb{R}^2$ and the total turning of the Th-image of the tangent to such fibre arcs or circles can be computed. Furthermore Th at $S(f)$ has rank two in the kernel of Tf. Thus the total turning of the Th-images of those kernel planes along the arcs of $S(f)$ can also be computed. Although neither of these functions is invariant under f-regular homotopy, there is a geometrically-defined equivalence relation on the pairs of such functions whose equivalence classes are invariants of f-regular homotopy. Furthermore the kernel bundle of Tf along $S(f)$ can be oriented and another f-regular homotopy invariant is the orientation preserving or reversing of Th on this bundle.

In addition to the above mentioned invariants which give information about the germ of (f,h) on $q^{-1}(q(S(f))) = \hat{\Sigma}$, the f-regular homotopy class of $(f,h) \mid M - \hat{\Sigma}$ is determined by the homotopy class of the map which assigns to each point $P \in M - \hat{\Sigma}$, the unit vector in $\mathbb{R}^2$ in the direction of the Th-image of any vector at P orienting the q-fibre.

We then give conditions that guarantee that the information so far recorded about the germ of (f,h) at $\hat{\Sigma}$ and on $(M-\hat{\Sigma})$, can arise from an immersion (f,h) on all of M. Finally we complete the classification with an invariant that distinguishes among those (f,h) which agree on a neighborhood of $\hat{\Sigma}$ and whose restrictions to the complement of the neighborhood of $\hat{\Sigma}$ are f-regularly homotopic.

I have begun Chapters II and III with introductions which will, I hope give accessible resumés of their contents. Some long and technical proofs have been deferred to the Appendix. A Reader's Guide to Notation and a Subject Index are included after the Appendix.

Although this work is the continuation of that begun with León Kushner and Paulo Porto [K, K-L-P], I have tried to make it self contained. I wish to acknowledge here with gratitude the precious contributions to the early development of this work that were made by León Kushner and Paulo Porto.

Note: If X and Y are sets, (X,Y) will denote the set of functions from X to Y. In general, functions between manifolds will be assumed to be smooth, i.e. C^{∞}.

CHAPTER I

The Singularities of Stable Maps of Three Manifolds into the Plane

1.1 Stable Maps

In this paragraph we describe the stable maps from three dimensional manifolds into the plane. Throughout, we will denote by M, a compact orientable 3-manifold without boundary. Let $C(M,\mathbb{R}^2$ be the smooth maps of M into $\mathbb{R}^2$. For $f \in C(M,\mathbb{R}^2)$, the singular set of f, $S(f) = \{x \in M \mid \text{rank } Tf(x) < 2\}$. The stable maps $\mathcal{S}(M,\mathbb{R}^2) \subseteq C(M,\mathbb{R}^2)$ are those whose multijet extensions satisfy the usual transversality conditions [Mather, G^2]. We give an equivalent description of $\mathcal{S}(M,\mathbb{R}^2)$.

Definition. $f \in \mathcal{S}(M,\mathbb{R}^2)$, if near each point $P \in S(f)$, and in some local coordinates centered at P and $f(P)$, f is one of the following:

L_0) $(u,x,y) \longrightarrow (u,x^2+y^2)$, definite fold point or fold point.

L_1) $(u,x,y) \longrightarrow (u,x^2-y^2)$, indefinite fold point or saddle point.

L_2) $(u,x,y) \longrightarrow (u,y^2+ux-x^3/3)$, cusp point.

In addition the following global conditions are satisfied:

G_1) If P is a cusp point, then $\{P\} = f^{-1}(f(P)) \cap S(f)$.

G_2) $f \mid S(f) - \{\text{cusps}\}$ is an immersion with normal crossings. (In particular $f \mid S(f)$ has no triple points.)

We let $S_0 = \{\text{definite fold points}\}$, $S_1 = \{\text{indef. fold points}\}$, $C = \{\text{cusps}\}$.

As an immediate consequence of these conditions, we see that for $f \in \mathcal{S}(M,\mathbb{R}^2)$, $S(f)$ is a finite disjoint union of embedded circles in M and C is a finite set of points. On any component of $S(f)$, the points of C separate arcs belonging to S_0 from those of S_1. Hence on every component of $S(f)$ there is an even number of cusps.

A component of $S(f) - C$ is in either S_0 or S_1 and is called *definite* or *indefinite*, accordingly.

From now on we will write $\mathcal{S}$ for $\mathcal{S}(M,\mathbb{R}^2)$.

1.2 *Coordinatized Product Neighborhoods (CPN)*

Let $f \in \mathcal{S}$. Here we show that each point, p, of M is contained in a neighborhood which is diffeomorphic to the product of an open interval, I, and a surface with boundary, H. The f-image of this neighborhood is diffeomorphic to a rectangle, $I \times J$ where J is a closed interval. If $\phi : I \times H \longrightarrow M$ and $\psi : I \times J \longrightarrow \mathbb{R}^2$ are the diffeomorphisms, then $\psi^{-1} \circ f \circ \phi = 1 \times g : I \times H \longrightarrow I \times J$. We take I and J centered at 0 and assume $\psi(0,0) = f(p)$. For all $u \in I$, $g(u,\cdot)(\partial H) \subseteq \partial J$ and $g(u,\cdot)$ is a Morse function except for $g(0,\cdot)$ in case p is a cusp.

Definition. We say that smooth mappings $f : M \longrightarrow P \longleftarrow Q : g$ *meet transversally* or f *is transverse to* g, $f \pitchfork g$ at $z \in P$ if either $z \notin f(M) \cap g(Q)$ or $z \in f(M) \cap g(Q)$, and $Tf(T_xM) + Tg(T_yQ) = T_zP$ for all $x \in f^{-1}(z)$ and $y \in g^{-1}(z)$. We say f and g *meet transversally* or f *is transverse to* g if the condition is satisfied at each $z \in P$. If f, g or both are inclusions we say $M \pitchfork g$, $f \pitchfork Q$ or $M \pitchfork Q$ respectively.

Proposition 1. *Let* $f \in \mathcal{S}$, $I = (-1,1)$, $J = [-1,1]$. *For each* $y \in \mathbb{R}^2$ *there is a diffeomorphism* $\psi : I \times J \longrightarrow \mathbb{R}^2$ *such that* $y = \psi(0,0)$ *and the composition,*

$$f^{-1}(\psi(I\times J)) \xrightarrow{f} \psi(I\times J) \xrightarrow{\psi^{-1}} I \times J \xrightarrow{proj} I, \qquad \underbrace{\qquad\qquad\qquad}_{h}$$

is a trivial bundle.

Proof. This is an easy application of Ehresmann's Theorem [E,T] which states:

Let X *and* Y *be connected manifolds without boundary. If* f *is a proper submersion onto* Y, *then* $f : X \longrightarrow Y$ *is a locally trivial fibration. If* $\partial X \neq \emptyset$ *and* f *and* $f \mid \partial X$ *are submersions onto*