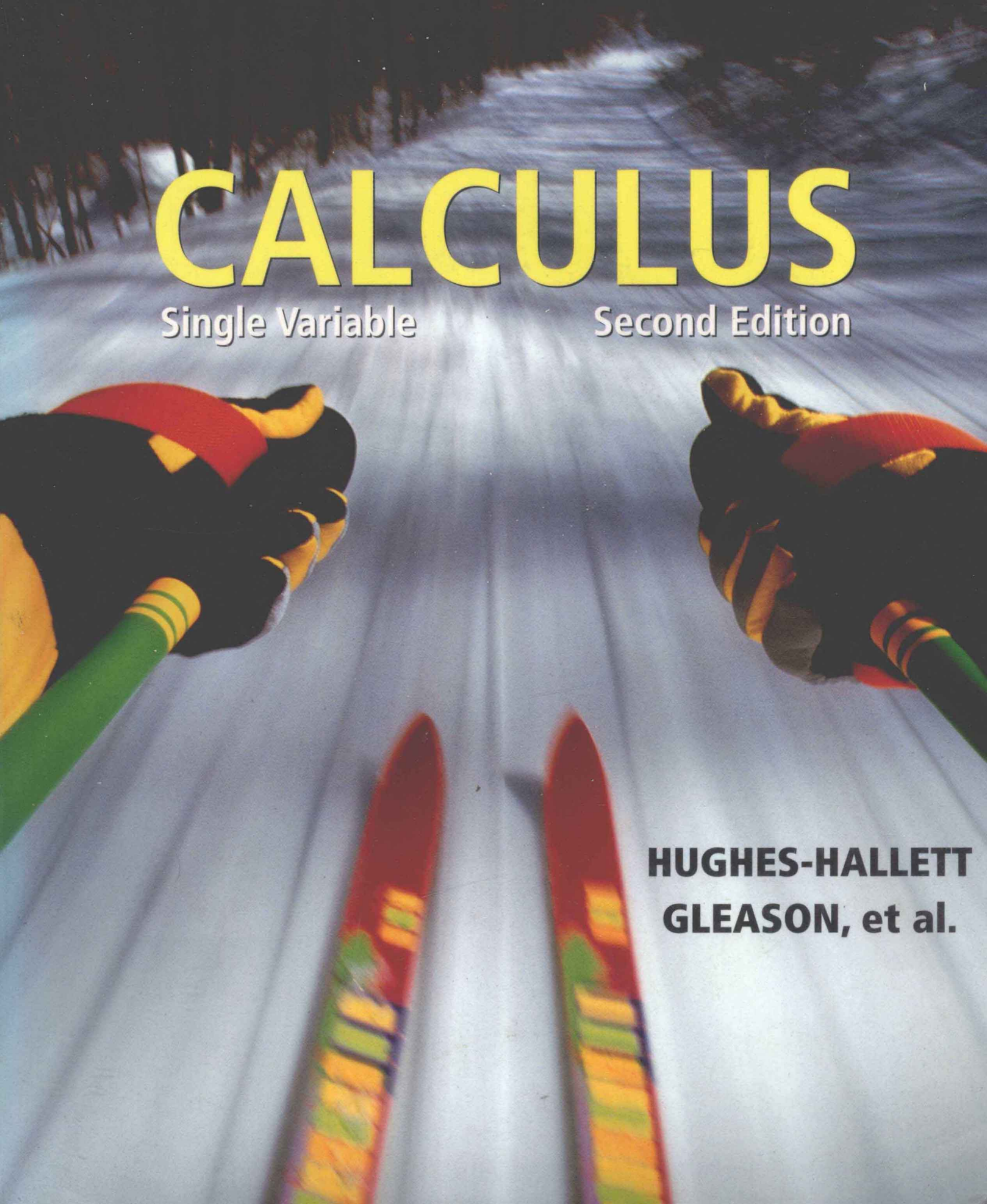
# CALCULUS

Single Variable

Second Edition



HUGHES-HALLETT
GLEASON, et al.

# CALCULUS

## Second Edition

Produced by the Consortium based at Harvard and funded by a National Science Foundation Grant.


Deborah Hughes-Hallett
*Harvard University*

Andrew M. Gleason
*Harvard University*

Daniel E. Flath
*University of South Alabama*

Patti Frazer Lock
*St. Lawrence University*

Sheldon P. Gordon
*Suffolk County Community College*

David O. Lomen
*University of Arizona*

David Lovelock
*University of Arizona*

William G. McCallum
*University of Arizona*

Douglas Quinney
*University of Keele*

Brad G. Osgood
*Stanford University*

Andrew Pasquale
*Chelmsford High School*

Jeff Tecosky-Feldman
*Haverford College*

Joe B. Thrash
*University of Southern Mississippi*

Karen R. Thrash
*University of Southern Mississippi*

Thomas W. Tucker
*Colgate University*

with the assistance of
Otto K. Bretscher
*Harvard University*

WILEY

John Wiley & Sons, Inc.
New York Chichester Weinheim Brisbane Singapore Toronto

*Cover Photo: Dennis O'Clair/Tony Stone Images/New York, Inc.*

Problems from *Calculus: The Analysis of Functions,* by Peter D. Taylor (Toronto: Wall & Emerson, Inc., 1992). Reprinted with permission of the publisher.

EXECUTIVE EDITOR Ruth Baruth
DEVELOPMENTAL EDITOR Nancy Perry
MARKETING MANAGER Leslie Hines
SENIOR PRODUCTION MANAGER Lucille Buonocore
SENIOR PRODUCTION EDITOR Monique Calello
SENIOR DESIGNER Laura Nicholls Boucher
ILLUSTRATION DIRECTOR Ishaya Monokoff
MANUFACTURING MANAGER Monique Calello

This book was set in Times Roman by the Consortium based at Harvard using LaTeX, Mathematica, and the package AsTeX, which was written by Alex Kasman for this project.

This book was printed and bound by Courier Westford, Inc. This cover was printed by Phoenix Color Corporation.

This book is printed on acid-free paper. ∞

The paper in this book was manufactured by a mill whose forest management programs include sustained yield harvesting of its timberlands. Sustained yield harvesting principles ensure that the number of trees cut each year does not exceed the amount of new growth.

Copyright © 1998 John Wiley & Sons, Inc. All rights reserved.

No part of this publication may be reproduced, stored in a retrieval system or transmitted in any form or by any means, electronic, mechanical, photocopying, recording, scanning or otherwise, except as permitted under Sections 107 or 108 of the 1976 United States Copyright Act, without either the prior written permission of the Publisher, or authorization through payment of the appropriate per-copy fee to the Copyright Clearance Center, 222 Rosewood Drive, Danvers, MA 01923, (508) 750-8400, fax (508) 750-4470. Requests to the Publisher for permission should be addressed to the Permissions Department, John Wiley & Sons, Inc., 605 Third Avenue, New York, NY 10158-0012, (212) 850-6011, fax (212) 850-6008, E-Mail: PERMREQ@WILEY.COM.

ISBN 0-471-16442-9 (paperback)

Printed in the United States of America.

10 9 8 7 6 5 4 3

This project was supported, in part, by the
**National Science Foundation**
Opinions expressed are those of the authors and not necessarily those of the Foundation

# CALCULUS

Second Edition

# PREFACE

Calculus is one of the greatest achievements of the human intellect. Inspired by problems in astronomy, Newton and Leibniz developed the ideas of calculus 300 years ago. Since then, each century has demonstrated the power of calculus to illuminate questions in mathematics, the physical sciences, engineering, and the social and biological sciences.

Calculus has been so successful because of its extraordinary power to reduce complicated problems to simple rules and procedures. Therein lies the danger in teaching calculus: it is possible to teach the subject as nothing but the rules and procedures—thereby losing sight of both the mathematics and of its practical value. This edition of *Calculus* continues our effort to refocus the teaching of calculus on concepts as well as procedures.

## A Focused Vision: Conceptual Understanding

Our goal is to provide students with a clear understanding of the ideas of calculus as a solid foundation for subsequent courses in mathematics and other disciplines. When we designed this curriculum we started with a clean slate. We included some new topics, such as differential equations, and omitted some traditional topics whose inclusion we could not justify after discussions with mathematicians, engineers, physicists, chemists, biologists, and economists. We focused on a small number of key concepts, emphasizing depth of understanding rather than breadth of coverage.

## The First Edition: A New Curriculum

The first edition of this book was the work of faculty at a consortium of eleven institutions, generously supported by the National Science Foundation. It represents the first consensus between such a diverse group of research mathematicians and instructors to have shaped a mainstream calculus text.

## The Second Edition: Expanded Choices

The second edition has the same vision as the first edition and provides instructors with additional choices through the *Focus On* sections. Instructors can select a focus for their course which reflects their interests and the needs of their students. In particular:

- All *Focus On* sections are optional.
- Chapter 3 (the definite integral) may be covered immediately before Chapter 6 (the antiderivative).
- In Chapters 5 and 8, instructors may select the sections relevant to their students.
- Chapters 9 and 10 may be covered in either order.

## Guiding Principles

Since students usually learn most when they are active, we feel that the exercises in a text are of central importance. In addition, we have found that multiple representations encourage students to reflect on the meaning of the material. Consequently, we have been guided by the following principles.

- Our problems are varied and some are challenging. Most cannot be done by following a template in the text.
- The Rule of Four: Where appropriate, topics should be presented geometrically, numerically, analytically, and verbally.



此为试读,需要完整PDF请访问: www.ertongbook.com

## The Development of Mathematical Thinking

The first stage in the development of mathematical thinking is the acquisition of a clear intuitive picture of the central ideas. In the next stage, the student learns to reason with the intuitive ideas and explain the reasoning clearly in plain English. After this foundation has been laid, there is a choice of direction. Some students, for example mathematics majors, may benefit from a more theoretical approach, while others, for example science and engineering majors, may benefit from a further exploration of modeling. New supplementary sections, *Focus on Theory* and *Focus on Modeling*, provide material to support either choice.

*Focus on Theory*

Calculus as a logical structure of theorem and proof is a masterpiece of mathematics. Its beauty, if not glossed over in a broad but superficial treatment, attracts mathematically inclined students to their vocation. In the *Focus on Theory* sections we have chosen a few topics to cover in depth. We show how axioms, definitions, and theorems are formulated, and how proofs are constructed. Since we believe that to understand and appreciate this material students must work with it themselves, we have included challenging exercises that guide students to construct definitions and proofs on their own.

*Focus on Modeling*

Calculus is a powerful tool for analyzing the real world. Students gain an understanding of the power of calculus by focusing on its use in an extended problem. The *Focus on Modeling* sections take the time to explore selected applications of calculus in depth.

## The Development of Mathematical Skills

To use calculus effectively, students need skill in both symbolic manipulation and the use of technology. The exact proportions of each may vary widely, depending on the preparation of the student and the wishes of the instructor. The book is adaptable to many different combinations.

*Focus on Practice*

These new sections increase students' skills in the mechanics of differentiation and integration.

*Technology*

The book does not require any specific software or technology. It has been used with graphing calculators, graphing software, and computer algebra systems. Any technology with the ability to graph functions and perform numerical integration will suffice. Students are expected to use their own judgement to determine where technology is useful.

## Content

This content represents our vision of how calculus can be taught. It is flexible enough to accommodate individual course needs and requirements. Topics can easily be added or deleted, or the order changed.

### Chapter 1: A Library of Functions

This chapter introduces all the elementary functions to be used in the book. Although the functions are probably familiar, the graphical, numerical, verbal, and modeling approach to them is likely to be new. We introduce exponential functions at the earliest possible stage, since they are fundamental to the understanding of real-world processes.

We encourage you to cover this chapter thoroughly, as the time spent on it will pay off when you get to the calculus.

*Focus on Theory*

These sections discuss the theoretical underpinnings of calculus, illustrate the process of formulating and proving a theorem using the example of the binomial theorem, and show how axioms are developed using the example of the completeness axiom.

## Chapter 2: Key Concept---The Derivative

The purpose of this chapter is to give the student a practical understanding of the definition of the derivative and its interpretation as an instantaneous rate of change. The power rule is introduced; other rules are introduced in Chapter 4.

*Focus on Theory*

These sections present the definition of limit and continuity, and analyze the connection between differentiability and local linearity. Exercises lead students to a proof of the Intermediate Value Theorem.

## Chapter 3: Key Concept---The Definite Integral

The purpose of this chapter is to give the student a practical understanding of the definite integral as a limit of Riemann sums and to bring out the connection between the derivative and the definite integral in the Fundamental Theorem of Calculus. Some instructors using the first edition of the book have delayed covering Chapter 3 until after Chapter 5 without any difficulty.

*Focus on Theory*

This section explores the formal definition of the definite integral using upper and lower sums.

## Chapter 4: Short-Cuts to Differentiation

The derivatives of all the functions in Chapter 1 are introduced, as well as the rules for differentiating products, quotients, and composite functions.

*Focus on Practice*

This section provides a battery of drill problems on differentiation for students to practice their skills.

## Chapter 5: Using the Derivative

The aim of this chapter is to enable the student to use the derivative in solving problems. It is not meant to be comprehensive and you do not need to cover all the sections.

*Focus on Theory*

This section pulls together many of the strands of the earlier theory sections, showing how the completeness axiom and the definition of limit and continuity can be used to prove the Extreme Value Theorem, the Increasing Function Theorem, and the Mean Value Theorem. Some of the proofs are given in the text, others are developed through the exercises.

## Chapter 6: Constructing Antiderivatives

This chapter focuses on going backward from a derivative to the original function, first graphically and numerically, then analytically. It introduces the Second Fundamental Theorem of Calculus and the concept of a differential equation.

*Focus on Modeling*

This section discusses why the equations of motion are based on acceleration rather than velocity.

## Chapter 7: Integration

This chapter includes several techniques of integration; others are included in the table of integrals. There is a discussion of numerical methods and of improper integrals.

*Focus on Practice*

This section presents a battery of integration problems for skill building.

### Chapter 8: Using the Definite Integral

This chapter emphasizes the idea of subdividing a quantity to produce Riemann sums which, in the limit, yield a definite integral. It aims to show how the integral is used in geometry, physics, and economics. You do not need to cover all the sections.

*Focus on Modeling*

These sections study applications of integration to probability distributions.

### Chapter 9: Approximations and Series

This chapter introduces Taylor Series and Fourier series via the idea of approximating functions with simpler functions. Geometric series, the ratio test, the harmonic series, and alternating series are discussed.

*Focus on Theory*

These sections present convergence tests in more detail, sketch a proof of the ratio test, and study bounds on the error in the Taylor approximation.

### Chapter 10: Differential Equations

This chapter introduces differential equations. The emphasis is on qualitative solutions, modeling, and interpretation.

*Focus on Modeling*

These sections show applications of the logistic equation and systems of differential equations to population models, the spread of disease, predator-prey interactions, and competitive exclusion.

### Appendices

The appendices contain material on roots and accuracy, compound interest, polar coordinates, complex numbers, Newton's method, and parametric equations.

## Our Experiences

In the process of developing the ideas incorporated in this book, we were conscious of the need to test the materials thoroughly in a wide variety of institutions serving many different types of students. Before the first edition was produced, consortium members and colleagues at over one hundred schools around the country class-tested preliminary versions of the book at large and small public universities, liberal arts colleges, two-year institutions, and high schools. The first edition was used by a very large and diverse group of schools in semester and quarter systems, in large lectures and small classes, in computer labs, small groups, and traditional settings, and with a number of different technologies. In preparing the second edition, we solicited comments from a large number of mathematicians who had used the text. We continued to engage our colleagues in client disciplines to determine the mathematical needs of their students. This included careful reviewing by a group of engineering faculty from highly regarded engineering programs around the country. We were offered many valuable suggestions, which we have tried to incorporate into this second edition of the text, while maintaining our original commitment to a focused treatment of a limited number of topics.

## Changes from the First Edition

We have streamlined some topics and added new sections on theory and on skill-building; we have moved some material into separate sections on modeling. The new arrangement divides the curriculum into a basic core and a set of supplementary sections from which the instructor can make choices to support a wide variety of courses.

*General Changes*

- There are more easy and medium level problems in each section, and a battery of drill problems in the *Focus on Practice* sections at the end of Chapters 4 (differentiation) and 7 (integration).

- There are short answers in the back of the book to those odd-numbered problems that have them.
- Each chapter concludes with a review of the main points, and there is a list of useful formulas on the endpapers.
- One or more projects are included at the end of each chapter; a wider selection is in the appendix.

*Summary of New Material*

- New *Focus on Theory* sections discuss the concepts of axiom, definition, and mathematical proof, and give a more theoretical treatment of selected topics, such as limits, differentiability, and the definite integral. The first edition's emphasis on concepts and on geometric and numerical reasoning serves as a natural introduction to this thread. These sections are optional and may be selected by instructors who prefer a more theoretical course.
- Chapter 4 contains a new section on local linearity and limits, which includes l'Hopital's rule.
- Chapter 5 contains a new section on the hyperbolic functions.
- Chapter 7 contains an expanded treatment of partial fractions.
- Chapter 8 contains a new subsection on center of mass.
- Chapter 9 contains new material on using the ratio test to find the radius of convergence of a power series, as well as new material on the harmonic and alternating series.
- There are two new appendices, one on parametric equations, and one containing additional projects.

*Chapter-by-chapter description of changes*

- Chapter 1. Notes on Compound Interest has been moved to an appendix. The introduction to continuity, formerly in an appendix, has been moved to this chapter. There are three new *Focus on Theory* sections: Underpinnings of Calculus, The Binomial Theorem, and Completeness of the Real Numbers.
- Chapter 2. The material on derivatives of constant and linear functions, and the statement of the power rule have been moved to Section 2.3. Approximations and Local Linearity, Notes on the Limit, and Notes on Differentiability have been replaced by two *Focus on Theory* sections: Limits and Continuity, and Differentiability and Linear Approximation.
- Chapter 3. The discussion in Section 3.2 of the relation between the value of $n$ and the convergence of Riemann sums has been shortened and the material on visualizing the integral as area has been moved to this section. The applications of the definite integral formerly in Section 6.1 have been moved to Section 3.3. Section 3.4 has been expanded to include both the Fundamental Theorem and the properties of definite integrals formerly in Section 6.2. Further Notes on the Limit has been replaced by a *Focus on Theory* section on the Definite Integral.
- Chapter 4. Notes on the Tangent Line Approximation has been incorporated into a new section on Local Linearity and Limits, which contains a treatment of l'Hopital's rule. More problems on the applications of rates have been included. A new *Focus on Practice* section provides drill problems in differentiation.
- Chapter 5. The material on local extrema has been shortened and put into Section 5.1. The material on families of functions has been simplified and is in Section 5.2. Section 5.3 is on global extrema and includes the first section on optimization (formerly Section 5.5). The section on Newton's Method has been moved to an appendix, and a new section on Hyperbolic Functions has been added. There is a new *Focus on Theory* section on Theorems about Continuous and Differentiable Functions.
- Chapter 6. This chapter has been refocused on antiderivatives; the material about definite integrals has been moved to Chapter 3. The chapter starts with a section on finding antiderivatives graphically and numerically. The material on finding antiderivatives analytically (formerly Section 7.1) has been moved to Section 6.2. The material on constructing antiderivatives using the definite integral has been moved here to form a new section on The Second Fundamental Theorem of Calculus. The section on equations of motion has become a *Focus on Modeling* section.
- Chapter 7 now starts with substitutions.
- Chapter 8. Applications to geometry have been put first, and new material on center of mass has been added to the material on density in Section 8.2. The sections on distributions and probability have been

moved to *Focus on Modeling* sections.

- Chapter 9. This chapter now covers Approximations and Series. There is new material on the ratio test for determining the radius of convergence of a power series, as well as on harmonic and alternating series. A new *Focus on Theory* section deals with the convergence of series. The Error in Taylor Approximations is now a *Focus on Theory* section.
- Chapter 10. This chapter now covers Differential Equations. The sections on Models of Population Growth, Systems of Differential Equations, and Analyzing the Phase Plane have become *Focus on Modeling* sections.
- Appendices. The material on continuity has been moved to Chapter 1. The sections on Compound Interest and Newton's Method have been moved to appendices. A new section on Parametric Equations and a new section containing projects have been added.

## Supplementary Materials

The following supplementary materials are available for the Second Edition.

- **Instructor's Manual with Sample Exam Questions** containing teaching tips, calculator programs, some overhead transparency masters and test questions arranged according to section.
- **Instructor's Solution Manual** with complete solutions to all problems.
- **Student's Solution Manual** with complete solutions to half the odd-numbered problems.
- **Instructor's Resource CD-ROM** which contains the Instructor's Manual, Instructor's Solutions Manual, additional projects, as well as other valuable resources.

### Acknowledgements

First and foremost, we want to express our appreciation to the National Science Foundation for their faith in our ability to produce a revitalized calculus curriculum and, in particular, to Louise Raphael, John Kenelly, John Bradley, and James Lightbourne. We also want to thank the members of our Advisory Board, Benita Albert, Lida Barrett, Simon Bernau, Robert Davis, M. Lavinia DeConge-Watson, John Dossey, Ron Douglas, Eli Fromm, William Haver, Seymour Parter and Stephen Rodi for their ongoing guidance and advice.

In addition, a host of other people around the country and abroad deserve our thanks for all that they did to help our project succeed. They include: Leonid Andreev, Huriye Arikan, Ruth Baruth, Martin Batey, Maria Betkowski, Laura Boucher, Melkana Brakalova, John Bravman, Stephen Boyd, Jackie Boyd-DeMarzio, Otto Bretscher, Mark Bridger, Morton Brown, Lucille Buonocore, Monique Calello, Ray Cannon, Marilyn Carlson, Phil Cheifetz, Bob Condon, Jurie Conradie, Sterling G. Crossley, Ehud de Shalit, Bob Decker, Persi Diaconis, Tom Dick, Srdjan Divac, Wade Ellis, Paul Feehan, Hermann Flaschka, Leonid Fridman, Lynn Garner, Tom Gearhart, James F. Gibbons, Allan Gleason, Florence Gordon, Danny Goroff, Robin Gottlieb, Biddy Greene, Dudley Herschbach, JoEllen Hillyer, Leslie Hines, Luke Hunsberger, Richard Iltis, Rob Indik, Adrian Iovita, Pete Janzow, Mary Johenk, Jerry Johnson, Mille Johnson, Matthias Kawski, David Kazhdan, Mike Klucznik, Donna Krawczyk, Mark Kunka, Carl Leinbach, David Levermore, Don Lewis, Jim Lewis, John Lucas, Reginald Luke, Tom McMahon, Dan Madden, Brad Mann, Barry Mazur, Eric Mazur, Georgia Mederer, Ansie Meiring, Dave Meredith, Andrew Metrick, Ishaya Monokoff, Lang Moore, David Mumford, Don Myers, Bridget Neale, Alan Newell, Steve Olson, John Orr, Arnie Ostebee, Jose Padro, Mike Pavloff, Nancy Perry, Tony Phillips, John Prados, Nancy Prinz, Ted Pyne, Amy Radunskaya, Wayne Raskind, Gabriella Ratay, Janet Ray, George Rublein, Anneliese Schauerte, Wilfried Schmid, Walter Seaman, Marilyn Semrau, Alida Setford, Pat Shure, Esther Silberstein, Lynne Small, David Smith, Sharon Smith, Don Snow, Bob Speiser, Howard Stone, Steve Strogatz, "Suds" Sudholz, Cliff Taubes, Peter Taylor, Tom Timchek, Alan Tucker, Jerry Uhl, Bill Vélez, Charles Walter, Mary Jean Winter, Chris Yetman, Debbie Yoklic, Lee Zia, Paul Zorn, and all the people in the Harvard mathematics department who shared their computers and their space with us.

Reports from the following reviewers were most helpful in shaping the second edition. Mathematics faculty and students: Frank Beatrous, Dennis De Turck, G. Martin Forrest, John Hagood, Kenneth Hannsgen, Kerry Johnson, Lisa A. Mantini, Joe A. Marlin, Grace Orzech, Howard Penn, Anthony L. Peressini, David R. Pitts, Lynne Small, Michell Anne Stillman, Brian Tankersley.

Engineering faculty: Howie Choset, Michael R. Foster, Terry L. Kohutek, Dorian Liepmann, Howard Littman, Vijay Modi, J. W. Rogers, Jr., John Sanchez.

Most of all, to the remarkable team that worked day and night to get the text into the computer (and out again), to get the solutions written and the pictures labeled: we greatly appreciate your ingenuity, energy and dedication. Thanks to: Ebo Bentil, Duff Campbell, Kenny Ching, John Cho, Dave Chua, Eric Connally, Jie Cui, Mike Esposito, Greg Fung, David Grenda, David Harris, Alex Kasman, Misha Kazhdan, Alex Mallozzi, Elliot Marks, Jean Morris, Kyle Niedzwiecki, Ed Park, Dave Richards, Natalka Roshak, Ann Ryu, Adrian Vajiac, Mihaela Vajiac, Alice Wang, Eric Wepsic, Gang Zhang, and the team from Arizona, Josh Cowley, Noah Syroid, and Xianbao Xu.

| | | |
|---|---|---|
| Deborah Hughes-Hallett | David O. Lomen | Douglas Quinney |
| Andrew M. Gleason | David Lovelock | Jeff Tecosky-Feldman |
| Daniel E. Flath | William G. McCallum | Joe B. Thrash |
| Patti Frazer Lock | Brad G. Osgood | Karen R. Thrash |
| Sheldon P. Gordon | Andrew Pasquale | Thomas W. Tucker |

此为试读,需要完整PDF请访问: www.ertongbook.com

## To Students: How to Learn from this Book

- This book may be different from other math textbooks that you have used, so it may be helpful to know about some of the differences in advance. At every stage, this book emphasizes the *meaning* (in practical, graphical or numerical terms) of the symbols you are using. There is much less emphasis on "plug-and-chug" and using formulas, and much more emphasis on the interpretation of these formulas than you may expect. You will often be asked to explain your ideas in words or to explain an answer using graphs.
- The book contains the main ideas of calculus in plain English. Success in using this book will depend on reading, questioning, and thinking hard about the ideas presented. It will be helpful to read the text in detail, not just the worked examples.
- There are few examples in the text that are exactly like the homework problems, so homework problems can't be done by searching for similar–looking "worked out" examples. Success with the homework will come by grappling with the ideas of calculus.
- Many of the problems in the book are open-ended. This means that there is more than one correct approach and more than one correct solution. Sometimes, solving a problem relies on common sense ideas that are not stated in the problem explicitly but which you know from everyday life.
- This book assumes that you have access to a calculator or computer that can graph functions, find (approximate) roots of equations, and compute integrals numerically. There are many situations where you may not be able to find an exact solution to a problem, but can use a calculator or computer to get a reasonable approximation. An answer obtained this way is usually just as useful as an exact one. However, the problem does not always state that a calculator is required, so use your own judgement.

  If you mistrust technology, listen to this student, who started out the same way:

  > Using computers is strange, but surprisingly beneficial, and in my opinion is what leads to success in this class. I have difficulty visualizing graphs in my head, and this has always led to my downfall in calculus. With the assistance of the computers, that stress was no longer a factor, and I was able to concentrate on the concepts behind the shapes of the graphs, and since these became gradually more clear, I got increasingly better at picturing what the graphs should look like. It's the old story of not being able to get a job without previous experience, but not being able to get experience without a job. Relying on the computer to help me avoid graphing, I was tricked into focusing on what the graphs meant instead of how to make them look right, and what graphs symbolize is the fundamental basis of this class. By being able to see what I was trying to describe and learn from, I could understand a lot more about the concepts, because I could change the conditions and see the results. For the first time, I was able to see how everything works together . . . .

  That was a student at the University of Arizona who took calculus in Fall 1990, the first time we used the text. She was terrified of calculus, got a C on her first test, but finished with an A for the course.
- This book attempts to give equal weight to three methods for describing functions: graphical (a picture), numerical (a table of values) and algebraic (a formula). Sometimes it's easier to translate a problem given in one form into another. For example, you might replace the graph of a parabola with its equation, or plot a table of values to see its behavior. It is important to be flexible about your approach: if one way of looking at a problem doesn't work, try another.
- Students using this book have found discussing these problems in small groups helpful. There are a great many problems which are not cut-and-dried; it can help to attack them with the other perspectives your colleagues can provide. If group work is not feasible, see if your instructor can organize a discussion session in which additional problems can be worked on.
- You are probably wondering what you'll get from the book. The answer is, if you put in a solid effort, you will get a real understanding of one of the most important accomplishments of the millennium – calculus – as well as a real sense of how mathematics is used in the age of technology.

# Table of Contents


## 1 A LIBRARY OF FUNCTIONS 1

1.1 WHAT'S A FUNCTION? 2
1.2 LINEAR FUNCTIONS 7
1.3 EXPONENTIAL FUNCTIONS 13
1.4 POWER FUNCTIONS 24
1.5 NOTES ON INVERSE FUNCTIONS 31
1.6 LOGARITHMS 35
1.7 THE NUMBER $e$ AND NATURAL LOGARITHMS 40
1.8 NEW FUNCTIONS FROM OLD 46
1.9 THE TRIGONOMETRIC FUNCTIONS 52
1.10 POLYNOMIALS AND RATIONAL FUNCTIONS 60
1.11 INTRODUCTION TO CONTINUITY 67
REVIEW PROBLEMS 70
PROJECTS 76

**FOCUS ON THEORY 77**
UNDERPINNINGS OF CALCULUS 77
THE BINOMIAL THEOREM 78
COMPLETENESS OF THE REAL NUMBERS 82

## 2 KEY CONCEPT: THE DERIVATIVE 87

2.1 HOW DO WE MEASURE SPEED? 88
2.2 THE DERIVATIVE AT A POINT 95
2.3 THE DERIVATIVE FUNCTION 105
2.4 INTERPRETATIONS OF THE DERIVATIVE 112
2.5 THE SECOND DERIVATIVE 116
REVIEW PROBLEMS 122
PROJECTS 126

**FOCUS ON THEORY 127**
LIMITS AND CONTINUITY 127
DIFFERENTIABILITY AND LINEAR APPROXIMATION 136

## 3 KEY CONCEPT: THE DEFINITE INTEGRAL 145

3.1 HOW DO WE MEASURE DISTANCE TRAVELED? 146
3.2 THE DEFINITE INTEGRAL 153
3.3 INTERPRETATIONS OF THE DEFINITE INTEGRAL 160
3.4 THEOREMS ABOUT DEFINITE INTEGRALS 167
REVIEW PROBLEMS 175
PROJECTS 180

**FOCUS ON THEORY 181**
THE DEFINITE INTEGRAL 181

## 4 SHORT-CUTS TO DIFFERENTIATION 189

4.1 POWERS AND POLYNOMIALS 190
4.2 THE EXPONENTIAL FUNCTION 197
4.3 THE PRODUCT AND QUOTIENT RULES 202
4.4 THE CHAIN RULE 208
4.5 THE TRIGONOMETRIC FUNCTIONS 213
4.6 APPLICATIONS OF THE CHAIN RULE 219
4.7 IMPLICIT FUNCTIONS 224
4.8 LINEAR APPROXIMATIONS AND LIMITS 227
REVIEW PROBLEMS 233
PROJECTS 236

**FOCUS ON PRACTICE 237**
DIFFERENTIATION 237

## 5 USING THE DERIVATIVE 239

5.1 USING FIRST AND SECOND DERIVATIVES 240
5.2 FAMILIES OF CURVES: A QUALITATIVE STUDY 250
5.3 OPTIMIZATION 254
5.4 APPLICATIONS TO MARGINALITY 262
5.5 MORE OPTIMIZATION: INTRODUCTION TO MODELING 270
5.6 HYPERBOLIC FUNCTIONS 277
REVIEW PROBLEMS 281
PROJECTS 284

**FOCUS ON THEORY 286**
THEOREMS ABOUT CONTINUOUS AND DIFFERENTIABLE FUNCTIONS 286

## 6 CONSTRUCTING ANTIDERIVATIVES 293

6.1 ANTIDERIVATIVES GRAPHICALLY AND NUMERICALLY 294
6.2 CONSTRUCTING ANTIDERIVATIVES ANALYTICALLY 298
6.3 DIFFERENTIAL EQUATIONS 304
6.4 SECOND FUNDAMENTAL THEOREM OF CALCULUS 309
REVIEW PROBLEMS 313
PROJECTS 316

**FOCUS ON MODELING 317**
THE EQUATIONS OF MOTION 317

## 7 INTEGRATION 321

7.1 INTEGRATION BY SUBSTITUTION: PART I 322
7.2 INTEGRATION BY SUBSTITUTION: PART II 328
7.3 INTEGRATION BY PARTS 333
7.4 TABLES OF INTEGRALS 339
7.5 APPROXIMATING DEFINITE INTEGRALS 344
7.6 APPROXIMATION ERRORS AND SIMPSON'S RULE 350
7.7 IMPROPER INTEGRALS 356
7.8 MORE ON IMPROPER INTEGRALS 364
REVIEW PROBLEMS 369
PROJECTS 371

**FOCUS ON PRACTICE 373**
INTEGRATION 373

## 8 USING THE DEFINITE INTEGRAL 375

8.1 APPLICATIONS TO GEOMETRY 376
8.2 DENSITY AND CENTER OF MASS 384
8.3 APPLICATIONS TO PHYSICS 390
8.4 APPLICATIONS TO ECONOMICS 398
REVIEW PROBLEMS 405
PROJECTS 408

**FOCUS ON MODELING 409**
DISTRIBUTION FUNCTIONS 409
PROBABILITY AND MORE ON DISTRIBUTIONS 416

## 9 APPROXIMATIONS AND SERIES 425

9.1 TAYLOR POLYNOMIALS AND SERIES 426
9.2 CONVERGENCE OF SERIES 435
9.3 FINDING AND USING TAYLOR SERIES 444
9.4 GEOMETRIC SERIES 451
9.5 FOURIER SERIES 458
REVIEW PROBLEMS 471
PROJECTS 474

**FOCUS ON THEORY 476**
CONVERGENCE THEOREMS 476
THE ERROR IN TAYLOR APPROXIMATIONS 481

## 10 DIFFERENTIAL EQUATIONS 489

10.1 WHAT IS A DIFFERENTIAL EQUATION? 490
10.2 SLOPE FIELDS 494
10.3 EULER'S METHOD 501
10.4 SEPARATION OF VARIABLES 505
10.5 GROWTH AND DECAY 510
10.6 APPLICATIONS AND MODELING 520
10.7 MODELS OF POPULATION GROWTH 530
10.8 SECOND-ORDER DIFFERENTIAL EQUATIONS: OSCILLATIONS 542
10.9 LINEAR SECOND-ORDER DIFFERENTIAL EQUATIONS 550
REVIEW PROBLEMS 558
PROJECTS 561

**FOCUS ON MODELING 563**
SYSTEMS OF DIFFERENTIAL EQUATIONS 563
ANALYZING THE PHASE PLANE 572

**APPENDIX** 579

A ROOTS, ACCURACY, AND BOUNDS 580
B COMPOUND INTEREST 589
C POLAR COORDINATES 593
D COMPLEX NUMBERS 595
E NEWTON'S METHOD 600
F PARAMETRIC EQUATIONS 604
G PARAMETRIC EQUATIONS AND CALCULUS 610
H PROJECTS 614

**ANSWERS TO ODD NUMBERED PROBLEMS** 623

**INDEX** 639

此为试读,需要完整PDF请访问: www.ertongbook.com