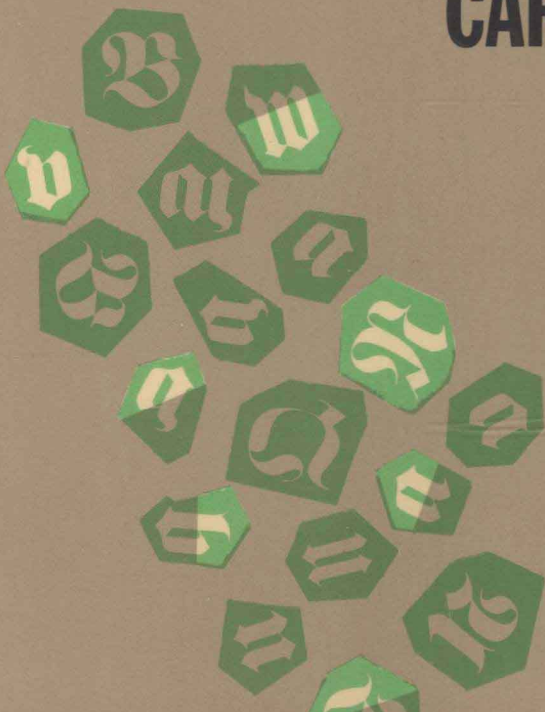
INTRODUCTION TO SYMBOLIC LOGIC AND ITS APPLICATIONS

BY RUDOLF CARNAP



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in deep gratitude

PREFACE TO THE ENGLISH EDITION

I wish to express my gratitude to my two translators, Professor William H. Meyer of the University of Chicago and Professor John Wilkinson of Wesleyan University, who between them provided the basic translation, revised it, made many improvements in wording and arrangement, and supplied additional explanations. The translation owes its existence to their generous devotion of time and interest. Translating a technical book requires a good knowledge of the subject matter in addition to linguistic abilities and sensitivities. In my opinion, the translators happily combined these abilities and performed an excellent job.

Except for numerous minor corrections and changes made either by me or by the translators, the translation follows in general the German original. In the following places, however, I made major changes or additions. In **20** ff., the explanations of the terms 'language', 'syntactical system', and 'semantical system' have been changed and made more exact. A new section, **26b**, has been added on the formalization of syntax and semantics. To the first explication of linear order in **31**, represented by Russell's concept of a series (D5), I have now added a second explication, represented by the concept of a simple order (D8, based on D6 and D7). This second concept has certain advantages and has recently seen increased use. The concept of a simple order is employed in some of the definitions of **38**. In **42a**, the distinction between the basic language L and the axiomatic language L′ is new. In **42b**, the distinction between interpretations and models has been made sharper. There are several changes in the axiom system of set theory (**43**). In **43a**, the axiom of regularity (A9) has been added. The original **43b** is omitted (it gave a second version of the system, with eight primitives, among them seven functors). The new **43b** is an expansion of a part of the original **43a**, with an altered form of the axiom of restriction (A10). Also, **43c** is newly added; here another version of the axiom system is described, which uses only individual variables. In the axiom system of neighborhoods (**46**), **46b** contains a new second version; and the definitions in **46c** are now based on this simpler version.

The bibliography (**56**) has been brought up to date. In chapters A, B, and C, many new exercises have been added; I wish to thank my student, David B. Kaplan, for his efficient help in this connection.

For the most part, the terminology in this English edition is based on terms used by me in classes and in recent publications. Suggestions for some other terms I owe to the translators and other colleagues. I went over the whole translation carefully and bear the sole responsibility for the accuracy of the content.

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May 1957

PREFACE TO THE GERMAN EDITION

During the past century logic has assumed an entirely new form, that of symbolic logic (or mathematical logic, or logistic). The use of symbols is, of course, the most striking feature of the new logic. Nevertheless, its essential characteristics lie in other directions: precision of formulation, greatly extended scope (especially in the theory of relations and of high-level concepts), manifold applications of its new methods. In consequence the last decades have seen an ever-increasing interest in symbolic logic, notably among mathematicians and philosophers, but also among those working in quite specialized fields who give attention to the analysis of the concepts of their disciplines.

Today, and particularly in the United States, symbolic logic is a recognized subject for teaching and research. The majority of American scholars who write on epistemology, analysis of language, scientific method, foundations of mathematics, axiomatic method, and the like, regard symbolic logic as an indispensable tool.

It is my hope that this book will reinforce, among German-speaking peoples, the general interest in symbolic logic.

What chiefly differentiates the present book from other logic texts (mostly in English) may be summarized under the following heads. In addition to the elementary portions of the theory, whose treatment is customary in most books, there is also a detailed presentation of the more advanced topics (especially the logic of relations) required for the application of logic. Further, the entire second part of the present book is given over to the application of symbolic logic. In this second part we first explain the construction of various language forms that must be considered in the application of logic; thereafter, we give in symbolic form axiom systems from different fields. Finally, in accordance with modern views, the present book outlines the theories of formal language systems (logical syntax) and interpreted language systems (semantics).

It may be thought that these last theories transcend the natural limits of an introductory text. However, I consider it important for anyone who would make the new symbolic methods his own that he learn from the very beginning to think from the point of view of the construction of deductive systems: in so doing, he gains for himself the insight that symbolism is a language conforming to exact rules whose use can sharpen the forms of his own thinking. It is this deliberate consideration of logical syntax and semantics which—apart from essentially greater length—

mainly distinguishes the present book from my former *Abriss der Logistik* (Wien 1929, 114 p.), now out of print and in many respects out of date because of rapid developments in the field.

The present book can be used as the text of a two-semester course in symbolic logic. The first semester, the introductory part of the course, could e.g. be based on Chapter A together with several illustrative applications drawn from Part II (see my explanations in **42e**). The second semester of the course could center chiefly on Chapter C supplemented by other applications from Part II; and to these matters can be added (to a degree desired by the instructor) considerations of syntactical and semantical theory, based either on the sketch provided in Chapter B or on the fuller presentations found in other books. Of course, the whole field of modern logic—including the theory of formal and interpreted language systems—is so extensive that two one-year courses are far more appropriate to it.

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PART ONE

SYSTEM OF SYMBOLIC LOGIC

Chapter A

The simple language A

1. THE PROBLEM OF SYMBOLIC LOGIC

1a. The purpose of symbolic language. Symbolic logic (also called mathematical logic or logistic) is the modern form of logic developed in the last hundred years. This book presents a system of symbolic logic, together with illustrations of its use. Such a system is not a theory (i.e. a system of assertions about objects), but a *language* (i.e. a system of signs and of rules for their use). We will so construct this symbolic language that into it can be translated the sentences of any given theory about any objects whatever, provided only that some signs of the language have received determinate interpretations such that the signs serve to designate the basic concepts of the theory in question. So long as we remain in the domain of pure logic (i.e. so long as we are concerned with building this language, and not with its application and interpretation respecting a given theory), the signs of our language remain uninterpreted. Strictly speaking, what we construct is not a language but a schema or skeleton of a language: out of this schema we can produce at need a proper language (conceived as an instrument of communication) by interpretation of certain signs.

Part Two of this book sees a variety of such interpretations, and the symbolic formulation (axiomatically, for the most part) of theories from various domains of science. All this is *applied logic*. Part One of the book attends to *pure logic*: here we describe the structure of the symbolic language by specifying its rules. In the present Chapter A, the first of the three chapters comprising Part One, we describe a simple symbolic language A containing the following sorts of signs (to be explained later): sentential constants and variables, individual constants and variables, predicate constants and variables of various levels and types, functor constants and variables, sentential connectives, and quantifiers. The third chapter, Chapter C, presents a more comprehensive language C. In Chapter B a symbolic language B is represented both as a syntactical system and as a semantical system.

If certain scientific elements—concepts, theories, assertions, derivations, and the like—are to be analyzed logically, often the best procedure is to translate them into the symbolic language. In this language, in contrast to ordinary word-language, we have signs that are unambiguous and formulations that are exact: in this language, therefore, the purity and correctness of a derivation can be tested with greater ease and accuracy. A derivation is counted as pure when it utilizes no other presuppositions than those specifically enumerated. A derivation in a word-language often involves presuppositions which were not made explicitly, but which entered unnoticed. Numerous examples of this are afforded by the history of geometry, especially in connection with attempts to derive Euclid's axiom of parallels from his other axioms.

A further advantage of using artificial symbols in place of words lies in the brevity and perspicuity of the symbolic formulas. Frequently a sentence that requires many lines in a word-language (and whose perspicuity is consequently slight) can be represented symbolically in a line or less. Brevity and perspicuity facilitate manipulation and comparison and inference to an extraordinary degree. The twin advantages of exactness and brevity appear also in the usual mathematical notations. Had the mathematician been confined to words and denied the use of numerals and other special symbols, the development of mathematics to its present high level would have been not merely more difficult, but psychologically impossible. To appreciate this point, one need only attempt to translate into the word-language e.g. so elementary a formula as "$(x+y)^3=x^3+3x^2y+3xy^2+y^3$" ("The third power of the sum of two arbitrary numbers equals the sum of the following summands: ..."). The symbolic method gives mathematics an advantage in its investigation of numbers, numerical functions, etc.; symbolic logic seeks this same advantage in full generality for its treatment of concepts of any kind.

In the course of constructing our symbolic language systems, it frequently happens that a new precisely-defined concept is introduced in place of one which is familiar but insufficiently precise. Such a new concept is called an *explicatum* of the old one, and its introduction an *explication*. (The concept to be explicated is sometimes called the *explicandum*.) E.g. the concept of L-truth (to be defined technically later (**5b**) on the basis of exact rules) is an explicatum of the concept of logical or necessary truth, which is defined with insufficient exactness despite its frequent occurrence in philosophy and traditional logic. Again, the concept of the inductive cardinal numbers (**37c**) is an explicatum for the concept of finite number that has been widely used in mathematics, logic and philosophy, but never exactly defined prior to Frege. [For a more complete exposition of the methods of explication and the requirements an adequate explicatum must meet, see Carnap [Probability], Chapter I.]

1b. The development of symbolic logic. Symbolic logic was founded around the middle of the last century and carried on into the present more by mathematicians than philosophers (cf. references to the literature, **57**). The reason for this lies in the historical fact that during the past century mathematicians became increasingly more conscious of the need to reexamine and reconstruct the foundations of the whole edifice of mathematics. Finding the traditional (i.e. aristotelian-scholastic) logic a totally inadequate instrument for this purpose, the mathematicians set about to develop a system of logic that was at once more appropriate, more accurate and more comprehensive.

The resulting new symbolic logic (especially in the systems of Frege, Whitehead-Russell, and Hilbert) clearly evinced a suitability to the first task set it, viz. to provide a basis for *the reconstruction of mathematics* (arithmetic, analysis, function theory, and the infinitesimal calculus). Further, in its *logic of relations* the new symbolic logic developed an abstract theory of arbitrary order-forms, and thereby created the possibility of representing and logically analyzing theories in which relations play an essential role, e.g. the various geometries, physical theories (especially in reference to space and time), epistemology and, latterly, even certain branches of biology. This development was a particularly significant advance beyond traditional logic. For traditional logic had neglected relations almost completely and hence proved entirely useless in connection with the axiomatic method (e.g. in geometry) that has become so important in recent decades. Still another merit of symbolic logic—minor, but nonetheless valuable—is that it achieved the complete solution of certain contradictions, viz. the so-called logical *antinomies* (cf. **21c**), whose analysis and elimination were beyond the reach of the old logic.

For *literature* on matters treated here, see the references, **57**. In the text of this book, citations of the literature are phrased with the help of abbreviated titles in square brackets; cf. the bibliography, **56**. ('[P.M.]' is used without author names for: Whitehead and Russell, *Principia mathematica*; and similarly for several of my own works.)

Regarding *terminology*. In the domain of symbolic logic the expressions "algebraic logic", "algebra of logic", etc., were employed at an earlier date but are no longer customary today. In addition to "symbolic logic" and "mathematical logic", the designation "logistics" is often used, especially on the European continent; it is short and permits the formulation of the adjective "logistic". The word "logistics" originally signified the art of reckoning, and was proposed by Couturat, Itelson and Lalande independently in 1904 as a name for symbolic logic (according to the assertion of Ziehen, *Lehrbuch der Logik*, p. 173, note 1, and Meinong, *Die Stellung der Gegenstandstheorie*, p. 115).

Concerning results of the new symbolic logic in comparison with traditional logic, cf. Russell [World], Chap. II; Carnap [Neue Logik]; Menger [Logic]. On the special importance of the logic of relations, cf. Russell, *ibid.*

Concerning *the reconstruction of mathematics* on the basis of the new logic, cf. the basic older works: Frege [Grundlagen] and [Grundgesetze]; Peano [Formulaire]; as chief work, [P.M.]; and also Russell [Introduction]; a more recent work: Hilbert and Bernays [Grundlagen]; for an easy presentation of the basic ideas: Carnap, "Die Mathematik als Zweig der Logik", *Blätter f. dt. Philos.* 4, 1930; Carnap, "Die logizistische Grundlegung der Mathematik", *Erkenntnis* 2, 1931.