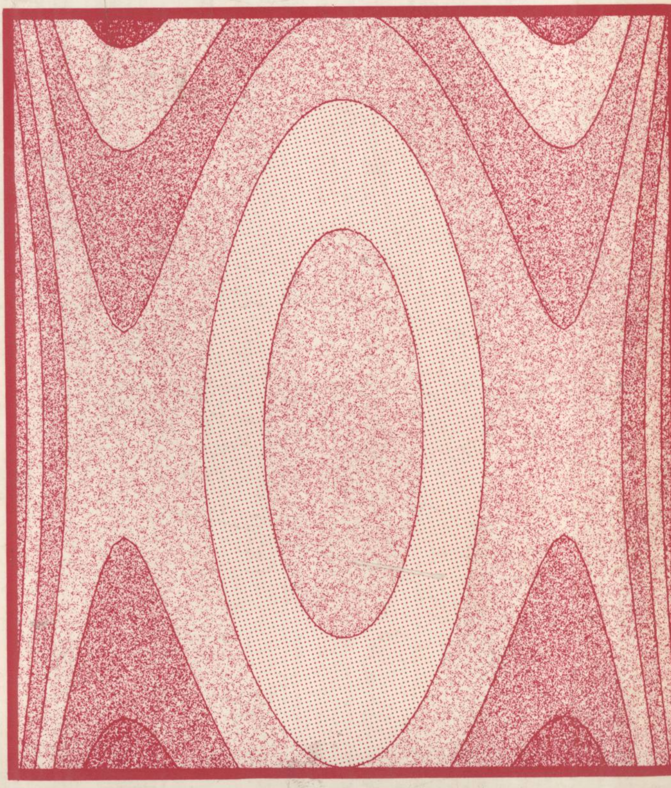
IEE CONTROL ENGINEERING SERIES 16

SYSTEMS MODELLING AND OPTIMIZATION

Edited by PETER NASH



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SYSTEMS MODELLING AND OPTIMIZATION

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Preface

This book is drawn from material presented at a vacation school for research students in control engineering, held in Cambridge in July, 1980, under the sponsorship of the (then) Control Engineering Committee of the Science Research Council. The school is one of a programme of five, soon to be six, vacation schools, whose aim is to broaden the training of control engineering research students. This particular school aims to explore some of the areas where control engineering and operational research overlap.

That this overlap is extensive can readily be seen, just by skimming through the many journals devoted, under one name or another, to "systems modelling". Indeed, it seems to me that there are a very large number of problems and theoretical areas which defy categorization as specifically operational research or management science or control engineering: the monitoring and control of environmental pollution, the efficient utilization of scarce natural resources, planning and control of large systems, control of integrated manufacturing operations are all obvious examples. Moreover, the common ground in these two disciplines must surely grow, as using the ever greater power of ever smaller computers becomes an essential feature of both.

A complete survey of the common ground of control engineering and operational research is obviously impossible in either a one-week school or in book of the length of this one. The vacation school was designed to concentrate largely on problems to which optimization techniques could sensibly be applied, and this book is about such problems and related thoeretical areas almost exclusively. Chapters 1 and 2 are surveys, of static optimization (mathematical programming) and dynamic optimization respectively. Chapter 1 in particular examines the problems of efficiently solving a mathematical optimization problem, once posed, and includes extensive references to up-to-date research in the field of mathematical programming algorithms. Some of these methods re-appear in chapter 2, where the use of function-space analogues of finite-dimensional programming algorithms is one of the methods discussed for solving problems in optimal control.

Chapter 3 surveys the theory of optimization under constraints as it applies to large-scale, and particularly decentralized or hierarchical, systems. The theoretical advantages of different methods of coordination in such systems are discussed, and it is shown how the theory provides a useful insight into the operation of systems subject to control at different levels. A particular type of problem decomposition, that of Dantzig and Wolfe for linear programs, is discussed in more detail in chapter 5.

Linear programming itself is almost certainly the most widely used and successful optimization technique. Chapter 4 is devoted to an introduction to the subject, and includes, with simple examples, an explanation of how the central technique of linear programming - the simplex method - works. Unusually in an introductory treatment, this chapter includes a brief discussion of the operational considerations involved in implementing a linear programming solution.

Chapters 6,8 and 9 provide specific case studies of problems of management and control approached by optimization techniques. Chapter 6 is concerned with the optimal operation of parts of a petrochemical plant; chapter eight is concerned with the optimal operation of reservoirs on the U. K. canal system; Chapter 9 is about the use of linear programming in a model used for optimizing the marketing strategy of an area of the National Coal Board. Chapter 7 is also case study material, but of a more general nature, and examines the problems of applying the techniques of control theory, and in particular optimal control, to models of the economy.

As well as illustrating the application of theory discussed in the first five chapters, this case study material is intended to shed light on the problems of modelling systems in the ways implied by approaches involving optimization. Indeed, it is evident in some of the studies that the most important part of the modelling effort comes in setting up the model. Once this is done, solution can be relatively simple, and it is perhaps worth saying that, while the development of efficient general algorithms is clearly necessary and important, a surprisingly large number of optimization problems that arise in practice are solved in an ad hoc way. Indeed, it seems sensible always to look carefully at any optimization problem in case the solution to it, or something close to it, can be found by a combination of mathematical technique, and insight born of knowledge of the system being modelled.

At the other end of the scale from these considerations, it is to be remembered that considerable help can be given with the solution of what may present itself as a problem in optimization without attempting any formal optimization at all. Just to be able to construct a model which can elucidate the consequences of different options may be sufficient to enable a satisfactory course of action to be chosen. This is to some extent the point made by chapter 10, which describes some continuing work on the modelling of the growth of telecommunications systems in less developed countries. This chapter also illustrates the very first steps of building a model to help people to make decisions, when there is no clear framework yet established and the goals of planning are known at best vaguely.

All of the models discussed in the case studies have in common the feature that their essential output is recommendations for action, whether by a computer controlling a plant or an engineer controlling a reservoir system or a treasury minister concerned with the country's economy. If these recommendations are put into effect, there is a clear implication that the people who decide that they should be implemented 'believe' in the models. It is interesting to ask where that belief comes from, and whether there are any criteria by which a model can be measured to see whether it is credible. These are the questions addressed by the final chapter. Here the 'optimizing model' is analysed into the three components of behavioural, cost and solution sub-models, and methods for evaluating these - in particular the first two - discussed.

Editing this book has been both interesting and rewarding, and has been made much easier by the help of a number of people. I should like to express my thanks to all the contributors, as well as to the students who made the original vacation school so interesting. I have had help, advice and patience from Tim Hills of Peter Peregrinus, and considerable encouragement from Josie Spring of the Science and Engineering Research Council. I am very grateful to Dr. G.W.T. White, who in addition to making a contribution in the form of a chapter, spent a considerable amount of time modifying the software that was used in producing the book.

Peter Nash

Contents

Chapter 1

Mathematical programming

P. Toint

1.1 INTRODUCTION

A mathematical programming problem is usually of the following type. Associated with some situation under study, there exists a real valued function that measures the performance or quality of the system that is considered, and it is desired to modify the system so that this performance index or objective function is as small as possible. Usually it is not possible to modify a part of the system arbitrarily without regard to the others: there are constraints that link the different components of the system. The only allowable modifications are those which satisfy these constraints. One wishes to find a state of the system that gives to the objective function the least possible value, while satisfying the constraints.

This can be mathematically expressed in the following way. Assume that the state of the system under consideration can be adequately described by a vector of n real numbers, x say. Let the objective function be f(x) and suppose that the constraints are expressed by some equations and bounds involving the components of x. Then the problem can be formally stated as

P1: minimize $f(x)$

subject to $e_i(x) = 0 \quad , \quad i \in I_e$ (1.1a)

$h_i(x) \geq 0 \quad , \quad i \in I_i$ (1.1b)

$x \in \mathbb{R}^n$

where the (1.1a) and (1.1b) represent equations and inequalities that describe the constraints , and where I_e and I_i are sets of indices of the constrained functions.

It is clear that P1 is fairly general, and that further assumptions will be needed in order to obtain practically solvable problems. These assumptions will generate several overlapping classes of optimization problems, depending, for example, on the mathematical description of the objective function, and on the effective presence of constraints on the x variable. At the same time, the formulation of P1 does not include a number of practically important problems. For example, problems where the performance of a system has to be measured by more than one criterion cannot be readily put into this framework.

The generality of problem P1 has led to mathematical programming becoming a very wide area of research, and the variety of specific problems that are addressed as well as the variety of the proposed solutions make a complete discussion impossible. Consequently, we will restrict ourselves in this chapter to an exploratory survey, with the main emphasis on algorithms and without much in the way of proofs. We begin with some general theoretical background.

1.2 THEORETICAL BACKGROUND

Consider again the general problem P1, where we now assume that all the functions involved are everywhere twice continuously differentiable. The following paragraphs will deal with conditions on the derivatives of these functions which are necessary or sufficient for a vector x^* to be a solution. First, let us define what we mean by a solution.

Definition: The vector $x \in \mathbb{R}^n$ is feasible for P1 if x satisfies (1.1a) and (1.1b). The vector $x^* \in \mathbb{R}^n$ is a local solution of P1 if x^* is feasible and there exists a neighbourhood of x^* such that for all feasible x in this neighbourhood, $f(x) > f(x^*)$.

We choose to follow in this section the structure that is presented in the book by Fiacco and McCormick [13]. In order to proceed, we need another definition: that of the Lagrangian function associated with P1, namely

$$L(x,u,v) = f(x) - u^T e(x) - w^T h(x). \tag{1.2}$$

Here e(x) and h(x) are vector-valued functions whose components are $\{e_i(x): i \in I_e\}$ and $\{h_i(x): i \in I_i\}$ respectively. In expression (1.2), the vectors u and w are called Lagrange multipliers or Lagrange parameters associated with problem P1. We shall discover that the Lagrangian function and its derivatives are of paramount importance in deriving optimality conditions.

We now state, without proof, a classical result, due to Farkas [11].

Lemma 1.1

Let $\{a^k: k=0,1,\ldots,q\}$ be a set of vectors in $\mathbb{R}^n$. If

$$z^T a^0 \geq 0$$

for every $z \in \mathbb{R}^n$ such that

$$z^T a^k \geq 0 \quad , \quad (k=1,\ldots,q) \tag{1.3}$$

then there exist non-negative coefficients $\{\alpha_k: k=1,2,\ldots,q\}$ such that

$$a^0 = \sum_{i=1}^{q} \alpha_i \, a^i \tag{1.4}$$

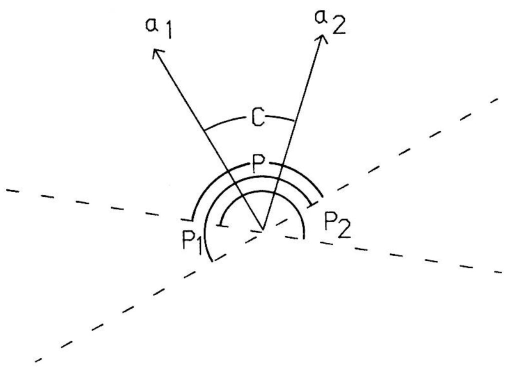

Fig 1.1 Farkas' Lemma.

This result is illustrated by the two-dimensional example in Fig 1.1. The sets of vectors satisfying (1.3) for $k=1,2$ are respectively P_1 and P_2. Hence the set of vectors satisfying both inequalities is P. It is easy to verify that the set of vectors that have non-negative projections on any vector of P is C, and hence that any such vector is a non-negative linear combination of a_1 and a_2.

Consider a feasible point x^* for P1 and define the following sets

$$I_i^* = \{i:\ i \in I_i,\ h_i(x^*) = 0\} \quad \text{(active set)} \quad , \tag{1.6}$$

$$Z_1^* = \{z:\ z^T \nabla h_i(x^*) \geq 0 \text{ for } i \in I_i^*,\ z^T \nabla e_i(x^*) = 0 \text{ for } i \in I_e,\ z^T \nabla f_i(x^*) \geq 0\}, \tag{1.7}$$

$$Z_2^* = \{z:\ z^T \nabla h_i(x^*) \geq 0 \text{ for } i \in I_i^*,\ z^T \nabla e_i(x^*) = 0 \text{ for } i \in I_e,\ z^T \nabla f_i(x^*) < 0\}, \tag{1.8}$$

$$Z_3^* = \{z:\ z \in \mathbb{R}^n,\ z \notin Z_1^*,\ z \notin Z_2^*\} \tag{1.9}$$

The point of these definitions is that Z_1^* (Z_2^*) is the set of perturbations of x^* which, to first order, maintain feasibility with respect to equality and saturated inequality constraints and produce an increase (decrease) in objective function value. Z_3^* is the set of infeasible perturbations.

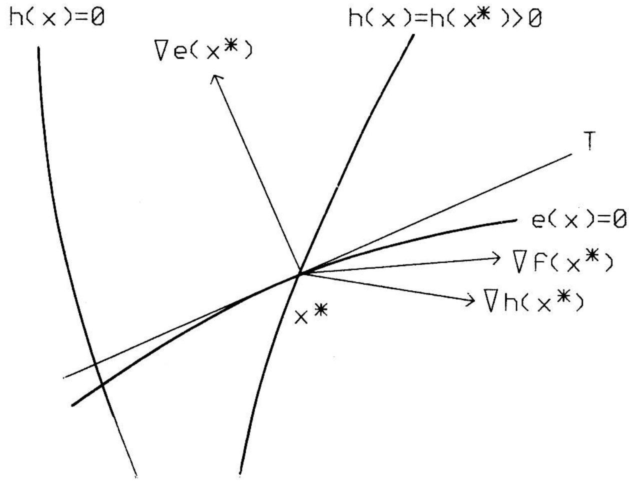

Fig 1.2 The sets Z_1^*, Z_2^*, Z_3^*.