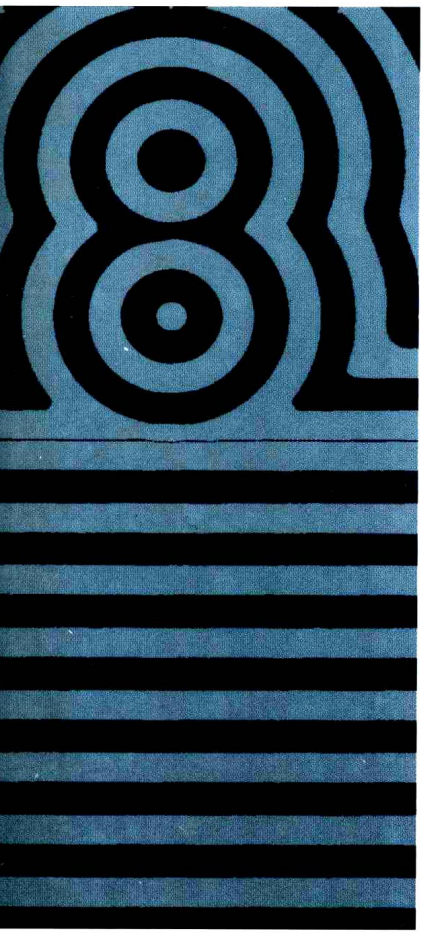
Innovative Methods for Numerical Solutions of Partial Differential Equations



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INNOVATIVE METHODS FOR NUMERICAL SOLUTION OF PARTIAL DIFFERENTIAL EQUATIONS

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Dedication

This volume consists of papers presented at a symposium honoring Phil Roe on the occasion of his 60th birthday and in recognition of his original contributions to Computational Fluid Dynamics (CFD) over the past twenty years.

The symposium entitled "Progress in Numerical Solutions of Partial Differential Equations" was held in Arcachon, France, on July 11-13, 1998. The authors from U.S., U.K., France, Italy, India and Japan, are internationally known researchers in this field.

The book covers many topics including theory and applications, algorithm developments and modern computational techniques for industry.

Phil Roe was born on May 4, 1938 in Derby, U.K. He received his B.A. in 1961 and Diploma in Aeronautics in 1962 from Cambridge University, Department of Engineering. He worked at the Royal Aircraft Establishment, Bedford, U.K. from 1962 to 1984. He joined the Cranfield Institute of Technology as a professor of aeronautics from 1984 to 1990 and he has been a professor in the Department of Aerospace Engineering, University of Michigan since 1990.

Prof. Roe became internationally known in the CFD community immediately after he published a paper on his flux differencing and averaging technique entitled "Approximate Riemann Solvers, Parameter Vectors, and Difference Schemes" in the *Journal of Computational Physics* in 1981. Since then this paper has been cited over 500 times and it has been selected for reprinting in the 25th Anniversary issue of *Journal of Computational Physics.* Prior to this publication, he had written several Royal Aircraft Establishment reports on high speed aerodynamics. He also wrote three reports on shock capturing and numerical algorithms for the linear wave equation.

Prof. Roe made many important contributions to CFD during the last two decades, covering many aspects of this field, including grids, schemes and solvers. In particular, one should mention his work on acceleration of Runge-Kutta integration algorithms, his optimal smoothing multistage schemes and soft walls and remote boundary condition for unsteady flows, his characteristic-based schemes and multidimensional upwinding, his limiters and high resolution schemes for structured as well as unstructured grids together with preconditioning techniques and his recent work on vorticity preserving schemes.

He worked with many researchers in U.S. and abroad and in many areas of aeronautics and beyond. His research interests include robust algorithms with applications to stiff flow problems, two phase flows and magnetohydrodynamics, where he has recently made fundamental contributions.

He has supervised, so far, 11 M.Sc. and 25 Ph.D. students and has been an external examiner for Ph.D. candidates in over 20 British, French and Swiss universities.

Prof. Roe has received many awards, including NASA Group Achievement Award "for work... which has formed the foundation of modern computational fluid dynamics" in 1993, and the University of Michigan College of Engineering Research Excellence Award in 2000–2001. He was awarded, jointly with B. van Leer and K. Powell, $750,000 from W. M. Keck Foundation to establish the Laboratory of Computational Fluid Dynamics in 1994 and recently he was part of a team selected by NASA Goddard for a $1,500,000 contract to develop a computational model of solar wind. He was elected AIAA Fellow in 1996. A complete list of his publications and professional activities are included in the next article.

Prof. Phil Roe has influenced many people besides his students, his colleagues and his friends. He is a remarkable intellectual and a scholar of highest calibre and his pleasant personality and deep insight are simply outstanding. We wish Prof. Roe an active and productive career for many good years to come.

M. M. Hafez
J.-J. Chattot

Contributions of Philip Roe

Prof. Roe was born on May 4, 1938 in Derby, U.K. He received his B.A. in 1961 and Diploma in Aeronautics in 1962 from Cambridge University, Department of Engineering. He worked at the Royal Aerospace Establishment, Bedford, U.K. from 1962 to 1984. He joined the Cranfield Institute of Technology as a professor of aeronautics from 1984 to 1990 and he has been a professor in the Department of Aerospace Engineering, University of Michigan since 1990. In the following, his professional activities, lists of graduate students he supervised as well as his publications are included.

Professional Activities

- Organising Committee, International Conference on Computational Fluid Dynamics, Kyoto 2000 and Sydney 2002.
- Joint organiser, American Mathematical Society Symposium on Simulation of Transport in Transition regimes, May 2000.
- Visiting Research Fellow, University of Reading, 1998-1999
- Advisory Editor-Journal of Computational Physics,
- Editor-in-Chief-Journal of Computational Physics, 1992-1994.
- Consultant, ICASE, NASA Langley.
- Reviewer for numerous journals and funding agencies.
- Organiser, short course on Computational Fluid Dynamics, Cranfield, 1984-1989.
- External examiner for Ph.D. candidates in over twenty British, French and Swiss universities.
- Visiting Scientist, NASA Ames, 1989.
- Visiting Professor, University of Bari, 1988.
- Consultant, European Space Agency.

Honours and Awards

- NASA Group Achievement Award, 'for work.. which has formed the foundation of modern computational fluid dynamics', 1993.
- Departmental Research Award, Aerospace Engineering, University of Michigan, 1994
- Award of $750,000 from W.M. Keck Foundation to establish Laboratory in Computational Fluid Dynamics, 1994 (jointly with B. van Leer and K. G. Powell).
- Elected AIAA Fellow, 1996,
- 1981 paper 'Approximate Riemann solvers, parameter vectors and difference schemes' (cited over 500 times) selected for reprinting in 25th Anniversary issue of *Journal of Computational Physics.*
- Part of team selected by NASA Goddard for $1,500,000 contract to develop a computational model of solar wind.
- Honored by 60th Birthday Symposium "Innovative Numerical Methods for Partial Differential Equations", Arcachon, France, June, 1998.
- University of Michigan College of Engineering Research Excellence Award 2000-2001 (shared with John P. Boyd)

Current Research Interests

High-Resolution Methods Exploitation of developed computational methods (based on Riemann solvers, limiters, finite volumes) in new areas such as magnetohydrodynamics, rarefied flows, sound generation, elastodynamics, micromanufacturing.

High-Order Methods Development of techniques offering improved accuracy for long-range propagation of linear or low-amplitude waves. Candidate methods include Discontinuous Galerkin and Upwind Leapfrog methods. Also high-order (Hermite) cell-vertex schemes.

Multidimensional Algorithms Development of algorithms directly modelling genuinely multidimensional aspects of the governing equations, including the division into elliptic, parabolic and hyperbolic modes, and methods especially adapted to vortical flows.

Magnetohydrodynamics Algebraic structure of the MHD equations, nonlinearities, degeneracies and their computational implications.

Robust Algorithms Development of codes guaranteed never to violate physical criteria such as positivity of mass or energy. I am looking to merge ideas from Godonov-type schemes and Bolzmann-type schemes.

Adaptive Grid Generation Cell-vertex methods implemented on grids which minimise local truncation error. These methods may form a natural link with techniques of automatic design and shape optimisation.

Stiff Flow Problems Efficient computation of flows in which the timescale of reaction, relaxation, etc differs greatly from the residence time.

Two-Phase Flow Mathematical modelling of two-phase flows such as bubbly liquids, with special attention to possible ill-posedness and the implications for computation.

Radiation Transport Application of new advection schemes to radiative flows. News versions of, and alternatives to, discrete-ordinate methods.

Mathematical modelling of debris dispersal (with K. G. Powell) Probabilistic description of the dispersal of debris from airborne explosions, leading to partial differential equations for probability of encounter.

Graduate Student Supervision

Masters Projects with date of Completion

Smadar Karni Numerical solutions of the Euler equations in a non-conservative formulation. University of Tel-Aviv, 1985.

George Vrizalas Redesign of a leading-edge slat to avoid compressibility effects associated with high suction peaks. College of Aeronautics, Cranfield, 1986.

Hong-Chia Lin Comparison of two computational methods for the Euler equations. College of Aeronautics, Cranfield, 1987.

Nikola Gagovic Computation of flow fields with forward blowing. College of Aeronautics, Cranfield, 1987.

Robert Townshend Design of submarine control surfaces. (jointly supervised with A. Boyd) College of Aeronautics, Cranfield, 1988.

Mark Bannister Computing the effect of wingtip devices. College of Aeronautics, Cranfield, 1988.

Rolf Reinelt The accuracy of Euler codes for transonic flow. College of Aeronautics, Cranfield, 1988.

Detlef Schultz Experiments with far-field boundary conditions. College of Aeronautics, Cranfield, 1988.

Steven Rham Development of a edge-centered scheme for the Euler equations. College of Aeronautics, Cranfield, 1989.

Martin Clark A first-order 3D Euler code for hypersonic waverider design using an upwind space marching technique. College of Aeronautics, Cranfield, 1989.

Christophe Corre Experiments on cell-centre and cell-vertex schemes in the case of the Ringleb flow, University of Michigan, 1991.

Doctoral Theses with Dates of Completion and Current Employment

Doctoral Students Advised

Peter K. Sweby Flux-difference splitting methods for the Euler equations. (jointly supervised with M.J. Baines) University of Reading, 1982.
(Senior Lecturer, University of Reading)

Smadar Karni Far-field boundary conditions in aerodynamics. College of Aeronautics, Cranfield, 1989.
(Associate Professor, Mathematics, University of Michigan)

Hong-Chia Lin Topics in the computation of hypersonic viscous flow. College of Aeronautics, Cranfield, 1990.
(Lecturer, Nan-Rong Institute of Technology, Taiwan)

David W. Levy Use of a rotated Riemann solver for the two-dimensional Euler equations. (jointly supervised with K.G. Powell, B. van Leer) University of Michigan, 1990.
(Design Engineer, Cessna Aircraft)

Christopher L. Rumsey Development of a grid-independent Riemann solver. (jointly supervised with B. van Leer, K.G. Powell) University of Michigan, 1990
(Research Scientist, NASA Langley)

James J. Quirk Adaptive mesh refinement for steady and unsteady shock hydrodynamics. College of Aeronautics, Cranfield, 1991.
(Research Scientist, Los Alamos National Laboratory)

Karim Mazaheri Numerical wave propagation and steady-state solutions. University of Michigan, 1992.
(Lecturer, Sharif University, Teheran)

George T. Tomaich A genuinely multi-dimensional upwinding algorithm for the Navier-Stokes equations on unstructured grids using a compact, highly-parallelizable spatial discretization, University of

Michigan, 1995.
(Exa Corporation, Boston)

Jens-Dominik Müller On triangles and flow, (jointly with H. Deconinck, von Karman Institute, Brussels) University of Michigan, 1995.
(Research Fellow, University of Oxford)

Lisa-Marie Mesaros Multi-dimensional fluctuation-splitting schemes for the Euler equations, University of Michigan, 1995.
(Team Leader, FLUENT, Ann Arbor)

Shawn L. Brown Approximate Riemann solvers for moment models of dilute gases, University of Michigan, 1995.
(Lecturer, Wright State Unuversity)

Creigh McNeil Efficient upwind algorithms for solution of the Euler and Navier-Stokes equations, (jointly with N. Qin) Cranfield University, 1995.
(Researcher, Centre for Turbulence Research, Stanford University)

Robert B. Lowrie Compact higher-order numerical methods for hyperbolic conservation laws. (jointly with B. van Leer), University of Michigan, 1996 (Research Scientist, Los Alamos)

Mohit Arora Explicit Characteristic-based high-resolution algorithms for hyperbolic conservation laws with stiff source terms, University of Michigan, 1996 (Morgan Stanley Bankers, Houston)

Rho-Shin Myong Theoretical and computational investigations of nonlinear waves in magnethydrodynamics. University of Michigan, 1996
(Assistant Professor, Gyeongsang National University, Korea)

Brian T. Nguyen Three-level time-reversible schemes for acoustic and electromagnetic waves. University of Michigan, 1996
(Research Scientist, Lawrence Livermore National Laboratory)

Jeffrey P. Thomas Investigation of upwind leapfrog schemes for acoustics and aeroacoustics, University of Michigan, 1996.

(Assistant Professor, Duke University)

Cheolwan Kim High-order upwind leapfrog schemes for advection,acoustics and aeroacoustics, University of Michigan, 1997.
(General Motors Research Laboratory, Detroit)

Timur Linde A three-dimensional adaptive multifluid model of the heliosphere, (jointly supervised with T. I. Gombosi, awarded University of Michigan distinguished dissertation prize) University of Michigan, 1998
(Research Fellow, University of Chicago)

Dawn D. Kinsey Toward the Direct Design of Waveriders, University of Michigan, 1998.
(Team Leader, MathSoft, Seattle)

Jeffrey A. F. Hittinger Foundations for the extension of the Godunov method to hyperbolic systems with stiff relaxation, (jointly supervised with A. Messiter) University of Michigan, 2000
(Postdoctoral Fellow, Lawrence Livermore National Laboratory)

Current Supervision of Research Students

Suichi Nakazawa Dissipation-free algorithm for elastic wave propagation. (jointly supervised with P. D. Washabaugh)

Hiroaki Nishikawa Simultaneous flow solver and mesh optimisation.

Mani Rad Genuinely multidimensional flow solver

Edward Wierzbicki Partial differential equations modelling the probability of debris encounters. (jointly with K. G. Powell)

List of Publications

1 Books

P.L.Roe(ed), *Numerical Methods in Aeronautical Fluid Dynamics*, Academic Press, 1982.

2 Review Articles

P.L.Roe, Characteristic-based schemes for the Euler equations, in **Annual Review of Fluid Mechanics, 1986**, eds M.van Dyke, J.V.Wehausen, J.L.Lumley, Annual Reviews,Inc., 1986.

P.L.Roe, A survey of upwind differencing techniques, 11th International Conference on Numerical Methods in Fluid Dynamics, Williamsburg, 1989, in **Lecture Notes in Physics, vol 323**, eds D.L.Dwoyer, M.Y.Hussaini, R.G.Voigt, Springer, 1989.

P.L.Roe, Modern numerical methods applicable to stellar pulsation, NATO Advanced Study Institute, Les Arcs, France, 1989, in **The Numerical Modelling of Nonlinear Stellar Pulsations-Problems and Prospects**, ed J.R.Buchler, Kluwer, 1990.

P.L.Roe, Beyond the Riemann problem, in *Algorithmic Trends in Computational Fluid Dynamics*, eds M.Y.Hussaini, A. Kumar, M.D. Salas, Springer 1993.

K.G.Powell, **P.L.Roe**, J.J.Quirk, Adaptive-Mesh Algorithms for Computational Fluid Dynamics, in *Algorithmic Trends in Computational Fluid Dynamics*, eds M.Y.Hussaini, A. Kumar, M.D. Salas, Springer 1993.

P.L.Roe, A brief introduction to high resolution schemes, Technical introduction to *Upwind and High-resolution Schemes* eds M.Y.Hussaini, B. van Leer, J van Rosendale, Springer, 1997.

P.L.Roe, Est-ce-qu'il-y-a une fonction de flux ideale pour les lois de

conservation hyperboliques? CANUM 98.

P.L.Roe, Shock Capturing (90 page chapter) in *Handbook of Shock-waves*, G. Ben-Dor *et.al, eds,* Academic 2000.

3 Refereed Articles in Journals

J.G.Jones, K.C.Moore, J.Pike, **P.L.Roe**, A method for designing lifting configurations for high supersonic speeds, using axisymmetric flow fields. *Ingenieur Archiv*, **37** no.1, 1968.

P.L.Roe, Approximate Riemann solvers, parameter vectors, and difference schemes, *J. Comput. Phys*, **43**, no.2, 1981.

J.Pike, **P.L.Roe**, Accelerated convergence of Jameson's finite volume Euler scheme using van der Houven integrators, *Computers and Fluids*, **13**, 1985.

P.L.Roe, Discrete models for the numerical analysis of time-dependent multi-dimensional gas dynamics, *J. Comput. Phys*, **63** no.2, 1986.

P.L.Roe, Remote boundary conditions for unsteady multidimensional aerodynamic calculations, *Computers and Fluids*, **17**, 1989.

B.Einfeldt, C.D.Munz, **P.L. Roe**, B. Sjögreen, On Godonov-type methods near low densities, *J. Comput. Phys.*, **92** no.2 1991.

P.L.Roe, Discontinuous solutions to hyperbolic problems under operator splitting, *Numerical Methods for Partial Differential Equations*, **7** pp 277-297, 1991.

P.L.Roe, Sonic flux formulae, *SIAM J. Sci. Stat. Comput.*, **13**, no. 2, 1992.

P.L.Roe, D.Sidilkover, Optimum positive linear schemes for advection in two and three dimensions, *SIAM J.Num.Anal*, **29** No 6, 1992.

B.van Leer, W-T Lee, **P.L.Roe**, K.G. Powell, C-H. Tai, Design of Optimally-Smoothing Multi-Stage Schemes for the Euler Equations,

Comm Appl. Num. Math, 8, p 761, 1992.

P.L.Roe, M. Arora, Characteristic-based schemes for dispersive waves I. The method of characteristics for smooth solutions, *Num Meth for PDEs*, **9**, p 459, 1993

H.Deconinck, **P.L.Roe**, R.Struijs, A multidimensional generalisation of Roe's flux difference splitter for the Euler equations, *Computers and Fluids*, **22**, p215, 1993.

C.L. Rumsey, B. van Leer, **P.L.Roe**, A multidimensional flux function with applications to the Euler and Navier-Stokes equations, *J.Comput.Phys*, **105**, p 306, 1993.

J.F. Clarke, S. Karni, J.J. Quirk, **P.L.Roe**, L.G. Simmonds, E.F. Toro, Numerical computation of two-dimensional unsteady detonation waves in high-energy solids, *J.Comput.Phys*, **106**, p 215, 1993.

R.B.Lowrie, **P.L.Roe**, On the numerical solution of conservation laws by minimizing residuals, *J.Comput.Phys*, **113**, p 304, 1994.

P.L.Roe, Reduction of certain wave operators to locally one-dimensional form, *Applied Math Letters*, **8**, 3, 1995.

J-C Carette, H. Deconinck, **P.L.Roe**, Multidimensional Upwinding- Its Relation to Finite Elements, *Int. J. Num. Meth. in Fluids*, **20**, 8/9, p 935, 1995.

P.L.Roe, D.S. Balsara, Notes on the eigensystem of magnetohydrodynamics, *SIAM J. App. Math.*, **56**, p 57, 1996.

M. Arora, **P.L.Roe**, A well-behaved limiter for high-resolution calculations of unsteady flow. *J.Comput.Phys*, **128**, p 1, 1997.

M. Arora, **P.L.Roe**, On post-shock oscillations due to shock-capturing schemes in unsteady flow. *J.Comput.Phys*, **130**, p 25, 1997.

K. Mazaheri, **P.L.Roe**, Numerical Wave Propagation and Steady-State Solutions- Soft Wall and Outer Boundary Conditions, *AIAA*

Jnl. **35**, no 8, p 965, 1997.

R.S. Myong, **P.L. Roe**, Shock waves and rarefaction waves in magnetohydrodynamics, I. The model system, *J. Plasma Physics*, **58-3**, pp 485-519, 1997.

R.S. Myong, **P.L. Roe**, Shock waves and rarefaction waves in magnetohydrodynamics, II. The MHD system, *J. Plasma Physics*, **58-3**, pp 521-552, 1997.

T. J. Linde, T. I. Gombosi, **P.L.Roe**, K. G. Powell, D. L. DeZeeuw, Heliosphere in the magnetized local interstellar medium: results of a three-dimensional MHD simulation, *J. Geophys. Res.* **103, A2**, pp 1889-1904, 1998.

P.L.Roe, Linear bicharacteristic schemes without dissipation, *SIAM J. Sci.Comp.* **19**,5, p 1405, 1998

T. Linde, **P.L. Roe**, On a mistaken notion of "proper upwinding", *J.Comput.Phys*, **142**, pp 611-614, 1998.

R.S. Myong, **P.L. Roe**, Godunov-type schemes for magnetohydrodynamics, I. a model system, *J.Comput.Phys*, **147**, pp. 545-567 1998.

K. Mazaheri, **P.L.Roe**, Numerical Wave Propagation and Steady-State Solutions- Artificial Bulk Viscosity, *AIAA Jnl.*, submitted.

K. G. Powell,*P. L. Roe*, T. J. Linde, T. I. Gombosi, D. L. De Zeeuw , Solution-Adaptive Upwind Scheme for Ideal Magnetohydrodynamics, *J.Comput.Phys*, **154**, pp. 284-309, 1999.

M.Hubbard, **P.L. Roe**, An algorithm for high-resolution advection on unstructured grids *Int. J. Num. Meth, in Fluids*, **33** p 711-736, 2000.

K. W. Morton, **P.L. Roe**, Vorticity-preserving Lax-Wendroff schemes for the system wave equation, *SIAM J. Scientific Computing*, to appear

G. Toth, **P.L. Roe**, Divergence- and curl-preserving prolongation and

restriction operators, *J.Comput.Phys*, submitted.

4 Invited Conference Papers

P.L.Roe, Numerical modelling of shockwaves and other discontinuities, Institute of Mathematics and its Applications conference, Reading, U.K. March,1981, in **Numerical Methods in Aeronautical Fluid Dynamics**, ed. P.L.Roe, Academic Press, 1982.

P.L.Roe, Fluctuations and signals - a framework for numerical evolution problems, Institute of Mathematics and its Applications conference, Reading, U.K., March 1982, in **Numerical Methods for Fluid Dynamics**, eds. K.W.Morton, M.J.Baines, Academic Press, 1983.

P.L.Roe, Upwind schemes using various formulations of the Euler equations, INRIA Workshop, Rocquencourt, 1983, in **Numerical Methods for the Euler Equations of Fluid Dynamics**, eds F.Angrand, A.Dervieux, J.A.Desideri, R.A.Glowinski, SIAM, 1985.

P.L.Roe, Some contributions to the modelling of discontinuous flows, Am. Math.Soc Symposium, San Diego, 1983, in **Large-scale Computations in Fluid Mechanics**, eds B.E.Engquist, S.Osher, R.C.J. Somerville, Lectures in Applied Mathematics, vol 22, Am.Math.Soc.,1985.

P.L.Roe, A basis for upwind differencing of the two-dimensional unsteady Euler equations, Institute of Mathematics and its Applications conference, Oxford, 1986, in **Numerical Methods for Fluid Dynamics II**, eds K.W.Morton, M.J.Baines, Oxford University Press, 1986.

P.L.Roe, Finite-volume methods for the compressible Navier-Stokes equations, Montreal,1987, in **Numerical Methods for Laminar and Turbulent Flow**, eds C.Taylor, W.G.Habashi, M.M.Hafez, Pineridge Press, 1987.

P.L.Roe, Mathematical problems associated with computing flow of real gases, GAMNI/SMAI/IMA conference on Computational Aeronautical Dynamics, Antibes, 1989, Academic Press 1993.