THEORETICAL NAVAL ARCHITECTURE

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PREFACE

THE present work is an expansion of the Author's "Text-Book of Theoretical Naval Architecture" originally published in 1899. The 1922 edition has been revised with the assistance of Professor Pengelly of the Royal Naval College, Greenwich The book is intended to provide students and draughtsmen engaged in Shipbuilders' and Naval Architects' drawing offices with a text-book which should explain the calculations which have continually to be carried out. It is intended also that the work should form a text-book for the various examinations which are held in this subject. The subject of Naval Architecture is continually growing, and it is impossible to deal satisfactorily with it in any one book, but the object of the Author has been to show how all the ordinary ship calculations can be intelligently carried out and to give the student a groundwork of knowledge on which further progress can be based.

A special feature of the book is the large number of examples given in the text and at the ends of the various chapters. By means of these examples, the student is enabled to test his grasp of the principles and processes given in the text. It has been found that this feature is much appreciated, especially by students who have to work without the aid of a teacher.

A bibliography of works on Naval Architecture and cognate subjects is given at the end with short notices as to the scope and value of the various works.

The Author ventures to hope that the work may be found useful to that large class which desires to obtain a working knowledge of the first principles of Theoretical Naval Architecture.

E. L. ATTWOOD.

REMARKS ON EDUCATION IN NAVAL ARCHITECTURE

For the bulk of those who study the subject of Naval Architecture, the only instruction possible is obtained in evening classes, and this must be supplemented by private study. The institutions in which systematic instruction in day courses is given are few in number, viz. (1) Armstrong College, University of Durham, Newcastle-on-Tyne; (2) University of Glasgow; (3) University of Liverpool; (4) Royal Naval College, Greenwich; and students who can obtain the advantage of this training are comparatively few in number. An account of the course at Glasgow is to be found in a paper before the I.N.A. in 1889 by the late Prof. Jenkins, and at the Royal Naval College, in a paper before the I.N.A. in 1905 by the writer; see also a paper by Professor Welch on the scientific education of naval architects before N.E. Coast Institution, 1909. There are scholarships to be obtained for such higher education, particulars of which can be had by application to the Glasgow, Liverpool, and Newcastle Colleges, to the Secretary of Admiralty, Whitehall, S.W., and to the Secretary of the Institution of Naval Architects, Adam St., Strand. In these courses it is recognized that the study of other subjects must proceed concurrently with that of Naval Architecture.

The Naval Architect has to be responsible for the ship as a complete design, and in this capacity should have some familiarity with all that pertains to a ship. Thus he should know something of Marine Engineering (especially of propellers); of Electricity and Magnetism; of armour, guns and gun-mountings in warships; of masts, rig, etc., in sailing vessels; of the work of the stevedore in cargo vessels; of questions relating to the docking and undocking of ships; of appliances for loading and unloading of ships; of the regulations of the Registration Societies and the Board of Trade regarding structure, freeboard, and tonnage; of appliances for navigating, as well as having a thorough knowledge of the practical work of the shipyard. In the early stages of a design, the naval architect frequently has to proceed independently in trying alternatives for the desired result, and it is not until the design is somewhat matured that he can call in the assistance of specialists in other departments. The naval architect should, therefore, have an interest in everything connected with the type of ship he has to deal with, and he will continually be collecting data which may be of use to him in his subsequent work.

For the average student of Naval Architecture, in addition to the work he does and observes in the shipyard, mould loft, and drawing office, it is necessary to attend evening classes in Naval Architecture and other subjects. The apprentice should systematically map out his time for this purpose. In the first place, a good grounding should be obtained in mechanical drawing and in elementary mathematics. Both of these subjects are now taught by admirable methods. The drawing classes are usually primarily intended for Engineering students, but this is no drawback, as it will familiarize the student with drawings of engineering details which he will find of considerable service to him in his subsequent work. Some institutions very wisely do not allow students to take up the study of any special subject, as Naval Architecture, until they have proved themselves proficient in elementary drawing and mathematics. The time thus spent is a most profitable investment.

The Board of Education formerly held examinations in two stages, a "lower" and a "higher," see Appendix C. Arrangements are now made for the award of "ordinary" and "higher" National Certificates in Naval Architecture by the Institution of Naval Architects and the Worshipful Company of Shipwrights in conjunction with the Board of Education. Conditions governing the award of these certificates are contained in a pamphlet published by the Board of Education; paragraphs 2 and 3 of the conditions are as follows:—

2. Part-time courses for purposes of a certificate under the scheme are of the following types:—

A. *Courses* adapted to the needs of students who have had full-time continuous education up to the age of 15 or 16 years or who have passed through a two years' preliminary part-time course. Such courses should normally extend over three years. Certificates relating to these courses will be termed "Ordinary Certificates."

B. *Courses* adapted to the needs of more advanced students. Certificates relating to such courses will be termed "Higher Certificates."

3. No scheme will be approved for the award of certificates unless the part-time grouped course under the scheme is carried on for at least 150 hours in each year. If the instruction is given exclusively in Evening Classes, the course should, as a rule, be carried on for three evenings a week during the school session. Course A for the Ordinary Certificate should as a rule extend over at least three years; and Course B for the Higher Certificate, over an additional two years.

Technical Schools and Colleges may submit for approval proposed time-tables of studies and syllabuses to a National Certificates Committee, consisting of representatives of the Institution of Naval Architects, the Worshipful Company of Shipwrights and the Board of Education. The syllabus of former examinations in Naval Architecture held by the Board of Education has been retained in Appendix C, and specimen examination questions retained in Appendix D, as being typical of those approved under the new scheme.

We will suppose that a student starts definitely with the lowest class in Naval Architecture. With this subject he should also take up Elementary Applied Mechanics, and, if possible, some Mathematics. The next year may be devoted to Naval Architecture, with a course in more advanced Applied Mechanics, and a course in Magnetism and Electricity or Chemistry would form a welcome relief. The next year may be devoted to further study in Mathematics, Theoretical and Applied Mechanics, Electricity and Magnetism. The next year may be devoted to another class in Naval Architecture, with more advanced Mathematics, including the Differential and Integral Calculus. This latter branch of mathematics is essential in order to make any progress in the higher branches of any engineering subject. If the student is fortunate enough to live in a large shipbuilding district, he will be able to attend lectures preparing him for the National Certificates in Naval Architecture. A "higher" certificate is worth having, and in preparing for the examination, the student must to a large extent read on his own account, and he will be well advised to devote considerable attention to the subject. Much will depend on the particular arrangements of teaching adopted in a district as to how the work can be best spread over a series of years.

In making the above remarks, the writer wishes to emphasize the fact that a student cannot be said to learn Naval Architecture by merely attending Naval Architecture classes. Teachers in this subject have not the time to teach Geometry, Applied Mechanics, or Mathematics, and unless these subjects are familiar to the student, his education will be of a very superficial nature. Teachers of the subject are always ready to advise students as to the course of study likely to be most beneficial in any given case.

Students are strongly advised to make themselves familiar with the use of the "slide rule," which enables ship calculations to be rapidly performed.

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THEORETICAL NAVAL ARCHITECTURE

CHAPTER I.

AREAS, VOLUMES, WEIGHTS, DISPLACEMENT, ETC.

Areas of Plane Figures.

A Rectangle.—This is a four-sided figure having its opposite sides parallel to one another and all its angles right angles. Such a figure is shown in Fig. 1. Its area is the product of the length and the breadth, or AB × BC. Thus a rectangular plate 6 feet long and 3 feet broad will contain—

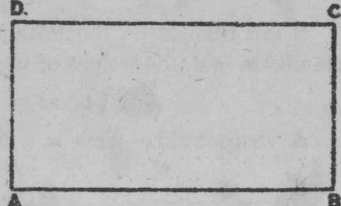

FIG. 1.

$$6 \times 3 = 18 \text{ square feet}$$

and if of such a thickness as to weigh $12\frac{1}{2}$ lbs. per square foot, will weigh—

$$18 \times 12\frac{1}{2} = 225 \text{ lbs.}$$

A Square.—This is a particular case of the above, the length being equal to the breadth. Thus a square hatch of $3\frac{1}{2}$ feet side will have an area of—

$$3\frac{1}{2} \times 3\frac{1}{2} = \frac{7}{2} \times \frac{7}{2} = \frac{49}{4}$$
$$= 12\frac{1}{4} \text{ square feet}$$

A Triangle.—This is a figure contained by three straight lines, as ABC in Fig. 2. From the vertex C drop a perpendicular on to the base AB (or AB produced, if necessary). Then the area is given by half the product of the base into the height, or—

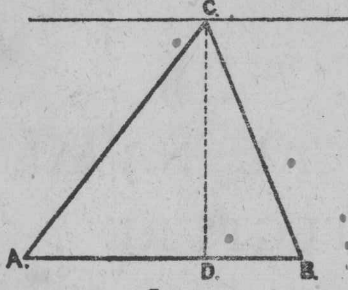

FIG. 2.

$$\frac{1}{2}(AB \times CD)$$

If we draw through the apex C a line parallel to the base AB, any triangle having its apex on this line, and having AB for its base, will be equal in area to the triangle ABC. If more convenient, we can consider either A or B as the apex, and BC or AC accordingly as the base.

Thus a triangle of base $5\frac{1}{2}$ feet and perpendicular drawn from the apex $2\frac{1}{4}$ feet, will have for its area—

$$\frac{1}{2} \times 5\frac{1}{2} \times 2\frac{1}{4} = \frac{1}{2} \times \frac{11}{2} \times \frac{9}{4} = \frac{99}{16}$$
$$= 6\frac{3}{16} \text{ square feet}$$

If this triangle be the form of a plate weighing 20 lbs. to the square foot, the weight of the plate will be—

$$\frac{99}{16} \times 20 = 123\frac{3}{4} \text{ lbs.}$$

A Trapezoid.—This is a figure formed of four straight lines, of which two only are parallel. Fig. 3 gives such a figure, ABCD.

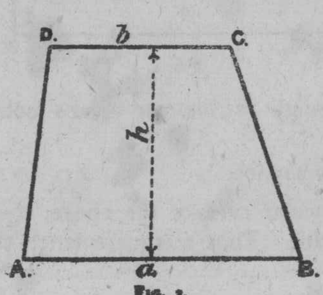

FIG. 3.

If the lengths of the parallel sides AB and CD are a and b respectively, and h is the perpendicular distance between them, the area of the trapezoid is given by—

$$\frac{1}{2}(a + b) \times h$$

or one-half the sum of the parallel sides multiplied by the perpendicular distance between them.

Example.—An armour plate is of the form of a trapezoid with parallel sides 8′ 3″ and 8′ 9″ long, and their distance apart 12 feet. Find its weight if 6 inches thick, the material of the armour plate weighing 490 lbs. per cubic foot.

First we must find the area, which is given by—

$$\left(\frac{8'\,3'' + 8'\,9''}{2}\right) \times 12 \text{ square feet} = \frac{17}{2} \times 12$$
$$= 102 \text{ square feet}$$

The plate being 6 inches thick = $\frac{1}{2}$ foot, the cubical contents of the plate will be—

$$102 \times \frac{1}{2} = 51 \text{ cubic feet}$$

The weight will therefore be—

$$51 \times 490 \text{ lbs.} = \frac{51 \times 490}{2240}$$
$$= 11{\cdot}15 \text{ tons}$$

A Trapezium is a quadrilateral or four-sided figure of which no two sides are parallel.

Such a figure is ABCD (Fig. 4). Its area may be found by drawing a diagonal BD and adding together the areas of the triangles ABD, BDC. These both have the same base, BD. Therefore from A and C drop perpendiculars AE and CF on to BD. Then the area of the trapezium is given by—

$$\tfrac{1}{2}(AE + CF) \times BD$$

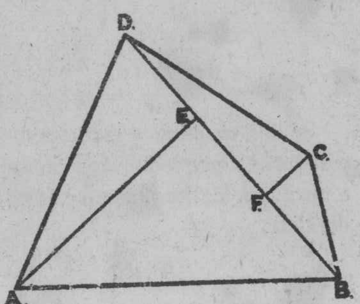

Fig. 4.

Example.—Draw a trapezium on scale $\frac{1}{8}$ inch = 1 foot, where four sides taken in order are 6, 5, 6, and 10 feet respectively, and the diagonal from the starting-point 10 feet. Find its area in square feet.

Ans. 40 sq. feet.

A Circle.—This is a figure all points of whose boundary are equally distant from a fixed point within it called the *centre*. The boundary is called its *circumference*, and any line from the centre to the circumference is called a *radius*. Any line passing through the centre and with its ends on the circumference is called a *diameter*.

The ratio between the circumference of a circle and its diameter is called π,[1] and $\pi = 3\cdot1416$, or nearly $\frac{22}{7}$

Thus the length of a thin wire forming the circumference of a circle of diameter 5 feet is given by—

$$\pi \times 5 = 5 \times 3\cdot1416 \text{ feet}$$
$$= 15\cdot7080 \text{ feet}$$

or using $\pi = \frac{22}{7}$, the circumference $= 5 \times \frac{22}{7}$

$$= \frac{110}{7} = 15\tfrac{5}{7} \text{ feet}$$

The circumference of a mast 2′ 6″ in diameter is given by—

$$2\tfrac{1}{2} \times \pi \text{ feet} = \frac{5}{2} \times \frac{22}{7}$$
$$= \frac{55}{7} = 7\tfrac{6}{7} \text{ feet}$$

The *area of a circle* of diameter d is given by—

$$\frac{\pi \times d^2}{4} \qquad (d^2 = d \times d)$$

Thus a solid pillar 4 inches in diameter has a sectional area of—

$$\frac{\pi \times 4^2}{4} = \frac{22}{7} \times 4$$
$$= 12\tfrac{4}{7} \text{ square inches}$$

A hollow pillar 5 inches external diameter and $\frac{1}{4}$ inch thick will have a sectional area obtained by subtracting the area of a circle $4\frac{1}{2}$ inches diameter from the area of a circle 5 inches diameter

$$= \left(\frac{\pi(5)^2}{4}\right) - \left(\frac{\pi(4\frac{1}{2})^2}{4}\right)$$
$$= 3\cdot73 \text{ square inches}$$

The same result may be obtained by taking a mean diameter of the ring, finding its circumference, and multiplying by the breadth of the ring.

Mean diameter $= 4\frac{3}{4}$ inches

Circumference $= \frac{19}{4} \times \frac{22}{7}$ inches

Area $= (\frac{19}{4} \times \frac{22}{7}) \times \frac{1}{4}$ square inches

$= 3\cdot73$ square inches as before

[1] This is the Greek letter *pi*, and is always used to denote 3·1416, or $\frac{22}{7}$ nearly; that is, the ratio borne by the circumference of a circle to its diameter

Trapezoidal Rule.[1]—We have already seen (p. 2) that the area of a trapezoid, as ABCD, Fig. 5, is given by $\frac{1}{2}(AD + BC)AB$, or calling AD, BC, and AB y_1, y_2, and h respectively the area is given by—

$$\tfrac{1}{2}(y_1 + y_2)h$$

If, now, we have two trapezoids joined together, as in

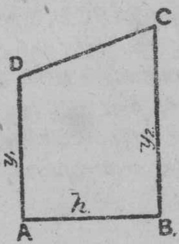

FIG. 5.

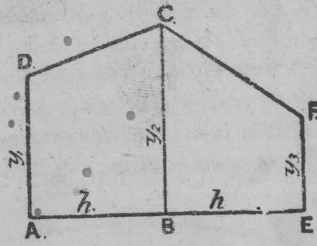

FIG. 6.

Fig. 6, having BE = AB, the area of the added part will be given by—

$$\tfrac{1}{2}(y_3 + y_2)h$$

The area of the whole figure is given by—

$$\tfrac{1}{2}(y_1 + y_2)h + \tfrac{1}{2}(y_2 + y_3)h = \tfrac{1}{2}h(y_1 + 2y_2 + y_3)$$

If we took a third trapezoid and joined on in a similar manner, the area of the whole figure would be given by

$$\tfrac{1}{2}h(y_1 + 2y_2 + 2y_3 + y_4) = h\left(\frac{y_1 + y_4}{2} + y_2 + y_3\right)$$

***Trapezoidal rule for finding the area of a curvilinear figure, as ABCD*, Fig. 7.**

Divide the base AB into a convenient number of equal parts, as AE, EG, etc., each of length equal to h, say. Set up perpendiculars to the base, as EF, GH, etc. If we join DF, FH, etc., by straight lines, shown dotted, the area required will very nearly equal the sum of the areas of the trapezoids ADFE, EFHG, etc. Or using the lengths y_1, y_2, etc., as indicated in the figure—

$$\text{Area} = h\left(\frac{y_1 + y_7}{2} + y_2 + y_3 + y_4 + y_5 + y_6\right)$$

[1] The Trapezoidal rule is largely used in France and in the United States for ship calculations.

In the case of the area shown in Fig. 7, the area will be somewhat greater than that given by this rule. If the curve however, bent towards the base line, the actual area would be somewhat less than that given by this rule. In any case, the closer the perpendiculars are taken together the less will be the error involved by using this rule. Putting this rule into words, we have—

To find the area of a curvilinear figure, as ABCD, Fig. 7, by means of the trapezoidal rule, divide the base into any convenient number of equal parts, and erect perpendiculars to the base meeting the curve; then to the half-sum of the first and last of these add the sum of all the intermediate ones; the result multiplied by the common distance apart will give the area required.

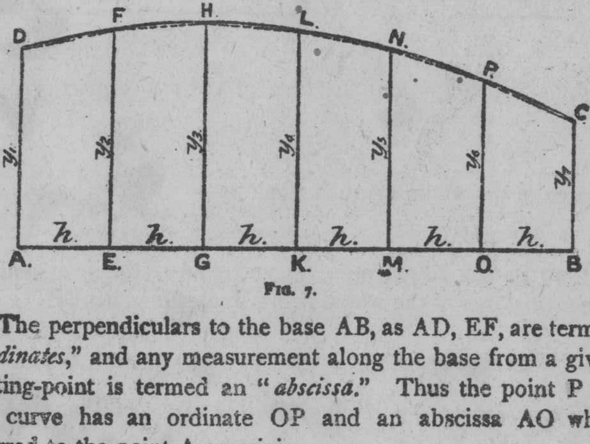

Fig. 7.

The perpendiculars to the base AB, as AD, EF, are termed "*ordinates*," and any measurement along the base from a given starting-point is termed an "*abscissa*." Thus the point P on the curve has an ordinate OP and an abscissa AO when referred to the point A as origin.

Simpson's First Rule.[1]—This rule assumes that the curved line DC, forming one boundary of the curvilinear area ABCD, Fig. 8, is a portion of a curve known as *a parabola of the second order.*[2] In practice it is found that the results given by its application to ordinary curves are very accurate, and it is

[1] It is usual to call these rules Simpson's rules, but the first rule was given before Simpson's time by James Stirling, in his "Methodus Differentialis," published in 1730.

[2] A "parabola of the second order" is one whose equation referred to co-ordinate axes is of the form $y = a_0 + a_1x + a_2x^2$, where a_0, a_1, a_2 are constants.

this rule that is most extensively used in this country in finding the areas of curvilinear figures required in ship calculations.

Let ABCD, Fig. 8, be a figure bounded on one side by the curved line DC, which, as stated above, is assumed to be a parabola of the second order. AB is the base, and AD and BC are end ordinates perpendicular to the base.

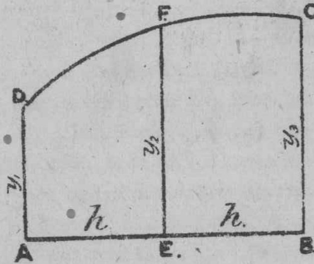

FIG. 8.

Bisect AB in E, and draw EF perpendicular to AB, meeting the curve in F. Then the area is given by—

$$\frac{1}{3}AE(AD + 4EF + BC)$$

or using y_1, y_2, y_3 to represent the ordinates, h the common interval between them—

$$\text{Area} = \frac{h}{3}(y_1 + 4y_2 + y_3)$$

Now, a long curvilinear area[1] may be divided up into a number of portions similar to the above, to each of which the above rule will apply. Thus the area of the portion GHNM of the area Fig. 7 will be given by—

$$\frac{h}{3}(y_3 + 4y_4 + y_5)$$

and the portion MNCB will have an area given by—

$$\frac{h}{3}(y_5 + 4y_6 + y_7)$$

Therefore the total area will be, supposing all the ordinates are a common distance h apart—

$$\frac{h}{3}(y_1 + 4y_2 + 2y_3 + 4y_4 + 2y_5 + 4y_6 + y_7)$$

Ordinates, as GH, MN, which divide the figure into the elementary areas are termed "*dividing ordinates.*"

Ordinates between these, as EF, KL, OP, are termed "*intermediate ordinates.*"

[1] The curvature is supposed continuous. If the curvature changes abruptly at any point, this point must be at a dividing ordinate.

Notice that the area must have an *even* number of *intervals*, or, what is the same thing, an *odd* number of *ordinates*, for Simpson's first rule to be applicable.

Therefore, putting Simpson's first rule into words, we have—

Divide the base into a convenient even number of equal parts, and erect ordinates meeting the curve. Then to the sum of the end ordinates add four times the even ordinates and twice the odd ordinates. The sum thus obtained, multiplied by one-third the common distance apart of the ordinates, will give the area.

Approximate Proof of Simpson's First Rule.—The truth of Simpson's first rule may be understood by the following approximate proof:[1]—

Let DFC, Fig. 9, be a curved line on the base AB, and with end ordinates AD, BC perpendicular to AB. Divide AB equally in E, and draw the ordinate EF perpendicular to AB. Then with the ordinary notation—

$$\text{Area} = \frac{h}{3}(y_1 + 4y_2 + y_3)$$

by Simpson's first rule. Now divide AB into three equal parts by the points G and H. Draw perpendiculars GJ and HK to the base AB. At F draw a tangent to the curve, meeting GJ and HK in J and K. Join DJ and KC. Now, it is evident that the area we want is very nearly equal to the area ADJKCB. This will be found by adding together the areas of the trapezoids ADJG, GJKH, HKCB.

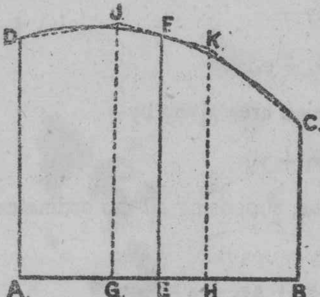

Fig. 9.

$$\text{Area of ADJG} = \tfrac{1}{2}(\text{AD} + \text{GJ})\text{AG}$$
$$\text{,, GJKH} = \tfrac{1}{2}(\text{GJ} + \text{HK})\text{GH}$$
$$\text{,, HKCB} = \tfrac{1}{2}(\text{HK} + \text{BC})\text{HB}$$

[1] Another proof will be found on p. 77. The mathematical proof will be found in Appendix A.

Now, AG = GH = HB = $\frac{1}{3}$AB = $\frac{2}{3}$AE, therefore the total area is—

$$\frac{1}{2}\left(\frac{2AE}{3}\right)(AD + 2GJ + 2HK + BC)$$

Now, AE = h, and GJ + HK = 2EF (this may be seen at once by measuring with a strip of paper), therefore the total area is—

$$\frac{h}{3}(AD + 4EF + BC) = \frac{h}{3}(y_1 + 4y_2 + y_3)$$

which is the same as that given by Simpson's first rule.

Application of Simpson's First Rule.—*Example.*—A curvilinear area has ordinates at a common distance apart of 2 feet, the lengths being 1·45, 2·65, 4·35, 6·45, 8·50, 10·40, and 11·85 feet respectively. Find the area of the figure in square feet.

In finding the area of such a curvilinear figure by means of Simpson's first rule, the work is arranged as follows :—

Number of ordinate.	Length of ordinate.	Simpson's multipliers.[1]	Functions of ordinates.
1	1·45	1	1·45
2	2·65	4	10·60
3	4·35	2	8·70
4	6·45	4	25·80
5	8·50	2	17·00
6	10·40	4	41·60
7	11·85	1	11·85

117·00 sum of functions

Common interval = 2 feet
$\frac{1}{3}$ common interval = $\frac{2}{3}$ feet
area = 117 × $\frac{2}{3}$ = 78 square feet

The length of this curvilinear figure is 12 feet, and it has been divided into an *even* number of intervals, viz. 6, 2 feet apart, giving an *odd* number of ordinates, viz. 7. We are consequently able to apply Simpson's first rule to finding its area. Four columns are used. In the first column are placed the numbers of the ordinates, starting from one end of the figure. In the second column are placed, in the proper order, the lengths of the ordinates corresponding to the numbers in the first column. These lengths are expressed in feet and

[1] Sometimes the multipliers used are half these, viz. $\frac{1}{2}$, 2, 1, 2, 1, 2, $\frac{1}{2}$, and the result at the end is multiplied by two-thirds the common interval.

decimals of a foot, and are best measured off with a decimal scale. If a scale showing feet and inches is used, then the inches should be converted into decimals of a foot; thus, 6′ 9″ = 6·75′, and 6′ $3\frac{1}{2}$″ = 6·3′. In the next column are placed Simpson's multipliers in their proper order and opposite their corresponding ordinates. The order may be remembered by combining together the multipliers for the elementary area first considered—

$$\begin{array}{ccccccc} 1 & 4 & 1 & & & & \\ & & 1 & 4 & 1 & & \\ & & & & 1 & 4 & 1 \\ \hline 1 & 4 & 2 & 4 & 2 & 4 & 1 \end{array}$$

or 1 4 2 4 2 4 1

The last column contains the product of the length of the ordinate and its multiplier given in the third column. These are termed the "*functions of ordinates.*" The sum of the figures in the last column is termed the "*sum of functions of ordinates.*" This has to be multiplied by one-third the common interval, or in this case $\frac{2}{3}$. The area then is given by—

$$117 \times \frac{2}{3} = 78 \text{ square feet}$$

Simpson's Second Rule.—This rule assumes that the curved line DC, forming one boundary of the curvilinear area

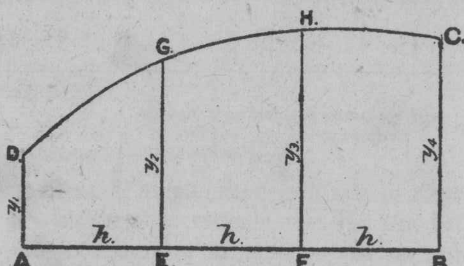

Fig. 10.

ABCD, Fig. 10, is a portion of a curve known as "*a parabola of the third order.*"[1]

Let ABCD, Fig. 10, be a figure bounded on one side by the curved line DC, which, as stated above, is assumed to be

[1] A "parabola of the third order" is one whose equation referred to co-ordinate axes is of the form $y = a_0 + a_1x + a_2x^2 + a_3x^3$, where a_0, a_1, a_2, a_3 are constants.