PUBLIC INVESTMENT, THE RATE OF RETURN, AND OPTIMAL FISCAL POLICY

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FOREWORD

Much of the work supported by RFF aims to provide a better factual and analytical basis for weighing the merits of alternative policies. These endeavors may take the form of the description of an emerging problem area, such as Herfindahl and Kneese's *Quality of the Environment;* the formulation of an analytical framework for a major problem of resource management, such as Krutilla and Eckstein's *Multiple Purpose River Development;* or the development of new theoretical structures required for the analysis of a general problem, such as that presented in Ayres and Kneese's article, "Production, Consumption, and Externalities."[1] From time to time, a specific need is uncovered for an extension of basic economic or social theory. It was a mutual awareness of the inadequacy of the existing theory of intertemporal criteria for resource allocation that first brought Professor Arrow and later Professor Kurz together with RFF.

The static theory of optimal resource allocation characterizes intertemporal value differences by one sole parameter: the discount rate. The selection of the optimum discount rate in a static setting itself poses many conceptual problems which have been discussed by leading economists, among them Baumol, Eckstein, Harberger, and Marglin. Professor Arrow made an earlier contribution in this field in "Discounting and Public Investment Criteria," published in *Water Research,*[2] which concerned the handling of risk by government.

[1] Orris C. Herfindahl and Allen V. Kneese, *Quality of the Environment: An Economic Approach to Some Problems in Using Land, Water, and Air* (Resources for the Future, 1965). John V. Krutilla and Otto Eckstein, *Multiple Purpose River Development: Studies in Applied Economic Analysis* (The Johns Hopkins Press for Resources for the Future, 1958). Robert U. Ayres and Allen V. Kneese, "Production, Consumption, and Externalities," *American Economic Review*, June 1969.

[2] Allen V. Kneese and Stephen C. Smith, eds., *Water Research* (The Johns Hopkins Press for Resources for the Future, 1966).

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Finally, we wish to thank the secretary of Serra House, Laura Staggers, for her dedicated and patient help during all phases of our work.

Some of the material in the following volume has appeared or will appear in somewhat different form in the following articles by one or both of the authors: "Criteria for Social Investment," *Water Resources Research,* Vol. 1, No. 1 (1965); "Discounting and Public Investment Criteria," in *Water Research,* ed. Allen V. Kneese and Stephen C. Smith (Baltimore: The Johns Hopkins Press, 1966); "Optimal Consumer Allocation over an Infinite Horizon," *Journal of Economic Theory,* Vol. 1, No. 1 (1969); "Optimal Public Investment Policy and Controllability with Fixed Private Savings Ratio," *Journal of Economic Theory,* Vol. 1, No. 2 (1969); and "The Social Discount Rate," paper presented at the North American Seminar on Cost-Benefit Analysis of Manpower Programs, held at the University of Wisconsin (May 1969).

THE FORMAT OF THE BOOK

The basic format has been selected to facilitate cross references.

Chapters are designated by roman numerals. Within each chapter we designate sections, propositions, theorems, and lemmas in numerical sequence. Thus,

"section II.3" means section 3 of chapter II;

"Proposition III.2" means Proposition 2 of chapter III.

Equations appear in parentheses in numerical order which begins again in each section. Thus,

"III.2.(5)" means equation (5) in section 2 of chapter III.

Equations appearing *within* a proposition use lower case letters. Thus,

"Proposition III.2.(b)" means equation (b), which appears in Proposition 2 of chapter III.

Assumptions are in numerical sequence within each section preceded by the letter A. Thus,

"Assumption IV.3.A.2" means Assumption 2, section 3, chapter IV.

If a reference in a chapter is made to any material in that chapter, the chapter designation will be dropped. Thus, in chapter VI we refer to section 2, not section VI.2.

When a reference in a section is made to an equation within the same chapter but in a different section, the chapter designation is omitted although the section designation is maintained. Thus, for example, in section 8 of a chapter we might refer to "equation 2.(17)" to mean equation (17) in section 2 of the same chapter. Similarly, within each section a reference to an equation will be designated by a number in parentheses. Thus, in section 3, for example, we might refer to equation (12), and not to equation 3.(12).

SUMMARY

In view of the lengthy and technical nature of the following discussion, it seems worthwhile to expose the leading points of view in a more informal manner. In a short compass, the recapitulation is necessarily very incomplete. This summary is intended to be a self-contained statement of the ground covered in the book and draws in some cases on the exact words of the subsequent text which, in turn, depends in no way on the summary. The reader may therefore feel a certain jarring discontinuity, for which we apologize, if he passes immediately from this summary to the beginning of chapter I.

References in parentheses are to chapters and sections of the text.

1. The Basic Issues

In the social evaluation of a competitive economy, it is customary and proper to start by using market prices as an estimate of social costs. Why, then, are many of us not content to use the market rate of return on capital as the measure of its opportunity cost? Several related difficulties have long been pointed out or can be abstracted from economic theory (I.1–2, V.1–2).

a) There is not one but a whole spectrum of market rates of return. Which is the appropriate rate to use? Indeed, the spectrum is even greater than is apparent, for the prevalence of credit rationing means that numerous shadow rates exist that are unobserved on the market.

b) Since there are very few futures or other forward markets, the welfare-theoretical argument for the allocative role of market prices, and in particular for the interest rate, is gravely undermined. Only if price anticipations were reasonably accurate would the classical case remain valid.

c) The future is risky, and existing risk-bearing markets are not, in principle, sufficiently complex and differentiated for optimal allocation. This raises the question of how uncertainty is to be introduced into the evaluation of future income streams from government investment, possibly by adjustments in the discount factor.

d) We take it that government investment is primarily investment in public or collective goods; i.e., in goods that, because of inappropriability or increasing returns, cannot suitably be left to the market. Hence, the costs of these goods and, by implication, the returns to public investment cannot be fully recaptured. The problem of financing those investments (through taxes or borrowing) then arises and its solution may in turn have repercussions on the appropriate rate of discount.

e) There is a widespread feeling that the government, or the general public for which it is a trustee, has a special responsibility for the future above and beyond that expressed by actors in the current market. In formal economic-theoretical terms, a full optimization over time would require obviously impossible trading between unborn and living generations. The government is then thought of as acting implicitly on behalf of the unrepresented.

The first three factors are undoubtedly very closely interrelated. Since risk bearing is complementary to investment, the paucity of risk-bearing markets inhibits the making of forward contracts. The multiplicity of interest rates and the wide extent of credit rationing are clearly also substitutes for nonexistent separate markets for insuring credit risks. There is, strictly speaking, no single risk-free rate of interest, so long as there is uncertainty about price movements and about the future of the rate of interest itself (variations in the rate of interest give rise to capital gains and losses). The latter source of uncertainty is absent from time deposits and very nearly so from short-term bills, but long-term rates are, unfortunately, most natural for discounting projects of long duration. Even if there were some interest rate that could be regarded as essentially risk free, it would not necessarily follow that it measured the time preference of individuals. Such an interest rate would clearly be inappropriate for an individual who neither held nor issued any of the riskless security. Further, if an individual planning over an extended period of time knows that in the future he cannot borrow at the riskless rate, his current consumption and portfolio decisions will be affected in complicated and not easily understood ways. There is a presumption that the riskless rate would understate the true marginal time preference.

There seem, then, to be three basic problems in identifying the social discount rate with market rates: (a) the divergence between private values and market behavior because of capital market imperfections; (b) the divergence between social values and private costs in the products of government investment activity; (c) the divergence between social and private values with regard to perspectives for the future. A fourth, more specific, problem has been mentioned prominently: the imperfections of the capital market that are a direct result of the corporate income tax.

2. Economic Policy in a Mixed Economy (V)

In a perfectly and ideally centralized economy, the divergences just listed would be irrelevant; the central planning board could produce an optimum investment policy and enforce it. In the course of computing this policy, it might be useful to determine some Lagrange multipliers that can be interpreted as discount rates.

In a world in which both a government and a private sector are involved in economic decision making, the problem is in principle more complicated. After all, the decisions on public investment should be made in light of those on consumption and private investment, and vice versa, both because all these activities compete for the same resources and because there are complementarity relations between public and private investment and between investment of either type today and consumption tomorrow. The government cannot directly control private investment or consumption, but it can influence them through its *instruments*, such as taxes and creation or retirement of debt. Hence, the government decision on public investment should be made jointly with a choice of instruments. Since a decision on the volume of public investment is implicitly a decision on its marginal productivity—i.e., on its rate of discount—this position is equivalent to the more usual formulation that the social rate of discount depends on the mode of financing.

The clearest way of posing the issues is to regard them as the dynamic analogue of Tinbergen's theory of economic policy. The government is assumed to have certain ends and to be endowed with a given set of instruments for their accomplishment. It is possible to calculate what may be termed the *publicly optimal policy*, the policy with regard to all variables (in this case, public investment, private investment, and consumption) that would be adopted by a perfectly altruistic government with unlimited powers. Then the first question that might be asked is whether or not the publicly optimal policy can be achieved by a suitable choice of values of the instruments by the government. Formally and more generally, we will say that a given allocation policy is *controllable* by a given set of instruments if there exist values of the instruments, varying over time in general, which cause the private and government sectors together to realize that policy (V.0, V.3).

The controllability of the publicly optimal policy will of course depend on the number and power of the instruments available. When the specified instruments are insufficient to control the publicly optimal policy, it is necessary to seek a "second-best" policy, the best that can be achieved with the given set of instruments (V.4).

3. Investment Policy as Optimization over Time (I.3)

Some problems encountered in understanding the question of the social rate of discount are common to all problems of investment policy. They inhere in the stock-flow relations between capital, which determines output, and investment, which is that part of output that goes to the increase of capital. Capital is a stock, measured in quantity units; investment, like consumption and output, is a flow, measured in quantity per unit time. If there were only one commodity and one kind of capital, we would have, in the absence of depreciation, the basic relation of capital accumulation,

$$\dot{K} = I, \tag{1}$$

where K is the stock of capital, I is the rate of investment, and the dot denotes the rate of change of K over time. If Y is output and C is consumption, we then supplement (1) with the identity,

$$Y = C + I, \tag{2}$$

and with some production relation linking Y to K. In a perfectly centralized economy, the instruments of the government at any moment of time are C and I, to be chosen subject to (2) to maximize some appropriately chosen utility functional which evaluates the entire consumption path.

This simple formulation has significant implications for analysis. Many previous analyses have simply evaluated the direct return from government investment. More recent writers have recognized that the return to government investment (the so-called throw-off) is available for future consumption, private investment, or government investment. However, even these writers have assumed an undue rigidity in future allocations. In particular, it is sometimes assumed that the proportion of the throw-off devoted to further government investment in the future will be fixed. But this reflects an inconsistency of viewpoint. The whole purpose of investment policy is to determine optimal decisions at a given moment; but then it should also be assumed that future investment is optimal.

A consistent simultaneous optimization of present and future investment decisions requires the use of mathematical techniques which, in their modern form, are known as "dynamic programming" and "optimal control theory" (II).

4. The Production and Valuation Assumptions

We assume now that there are two types of capital—private and government—and that output, Y, at any moment is a function of the quantities of these two types of capital, K_p and K_g, respectively, and of the labor force, L. We assume further that the labor force is growing at a constant rate, π. First, suppose that constant returns to all variables prevail and that technological progress is labor-augmenting at a constant rate, τ.

$$Y = F(K_p, K_g, e^{\tau t}L), \tag{1}$$

where t is time. The function F is concave and homogeneous of degree 1. If we define the natural rate of growth, $\gamma = \pi + \tau$, and let

$$y = e^{-\gamma t}Y, \qquad k_p = e^{-\gamma t}K_p, \qquad k_g = e^{-\gamma t}K_g, \tag{2}$$

then we can write

$$y = f(k_p, k_g), \tag{3}$$

where $f(k_p, k_g)$ is strictly concave (I.5, III.1, IV.1–2).

What may not be so well known is that certain cases of increasing returns can be written the same way. Basically, it is assumed that government capital is labor-augmenting, a case which certainly covers manpower programs very well. Assume that government capital and labor cooperate, with possible increasing returns, to produce an intermediate good, which might be termed "trained labor," which, in turn, cooperates with private capital under constant returns to produce output. The production function for trained labor is in particular taken to be of the form, $H(K_g, L^\delta)$, where H is concave and homogeneous of degree 1. The case $\delta > 1$ displays increasing returns.

We may generalize this formulation by assuming in addition labor-augmenting technological progress at a rate τ'. The production assumptions now take the form

$$Y = G[K_p, H(K_g, L_E^\delta)] \tag{4}$$

for some $\delta > 0$, where G and H are both homogeneous of degree 1 and concave, and $L_E = Le^{\tau' t}$. (This form is the most general which admits the possibility of balanced growth.) If we now define $\gamma = \delta(\pi + \tau')$, $\tau = \gamma - \pi$, and use the definitions (2), we can again arrive at (3), together with the relation, $\gamma = \pi + \tau$. (I.5, III.5, IV.8).

The criterion function that evaluates alternative policies is here taken to be

$$\int_0^\infty e^{-\rho t}P(t)U[\tilde{c}(t)]dt, \tag{5}$$

where ρ is a discount factor for utilities (not necessarily for commodities), P is population, $\bar{c}$ is per capita consumption, and U is the current flow of utility (or *felicity*). A detailed defense of this criterion function will not be attempted here but we hope that it will be accepted as plausible and will add a few remarks. (a) Expression (5) does ignore distributional considerations, from which we are abstracting. (b) The additivity over time and the stationarity of the discounting process seems eminently reasonable in the analysis of long-run consequences. (c) The infinite horizon is an idealization of the fundamental point that the consequences of investment are very long-lived; any short horizon requires some method of evaluating end-of-period capital stocks, and the only proper evaluation is their value in use in the subsequent future. (d) Utility attaches to per capita consumption, since that is what the representative individual receives, but if different generations have different numbers of individuals, society should maximize the sum of discounted total utilities for all generations; otherwise, the more numerous generations are discriminated against. (e) Some government capital contributes directly to consumer satisfaction, and it would be more general to let U depend on $\bar{k}_g$, per capita government capital, as well as $\bar{c}$. The more general assumption is made in the following text; only the simpler assumption is made in this summary (I.4).

Assume in addition that $U(\bar{c})$ is homogeneous of degree $1 - \sigma (\sigma > 0)$; the family of such functions is broad and flexible. If we define

$$c = P(t)\bar{c}e^{-\gamma t} = P(0)\bar{c}e^{-\tau t} \tag{6}$$

(i.e., total consumption adjusted for the natural rate of growth), then some elementary manipulation shows that

$$e^{-\rho t}P(t)U(\bar{c}) = [P(0)]^{\sigma}e^{-\gamma t}U(c)\,,$$

where

$$\omega = \rho + \sigma\tau\,, \qquad \lambda = \omega - \gamma. \tag{7}$$

The constant, $[P(0)]^{\sigma}$, can be ignored in maximization problems, so that the criterion function can be written

$$\int_0^{\infty} e^{-\lambda t}U[c(t)]dt. \tag{8}$$

We assume $\lambda > 0$.

By definition, we have

$$Y = C + I_p + I_g\,, \qquad \dot{K}_p = I_p\,, \qquad \dot{K}_g = I_g\,,$$

where I_p and I_g are the rates of investment in private and government capital, respectively, and the dot over a symbol denotes its derivative with respect to time. If we now introduce the growth-normalized investment rates,

$$i_p = e^{-\gamma t}I_p\,, \qquad i_g = e^{-\gamma t}I_g\,, \tag{9}$$

then from (2) and (6) we can say,

$$y = c + i_p + i_g\,, \qquad \dot{k}_p = i_p - \gamma k_p\,, \qquad \dot{k}_g = i_g - \gamma k_g. \tag{10}$$

It is probably easiest, after these reductions, to assume that we are dealing with a stationary population and technology, with future utilities discounted at λ and both kinds of capital depreciating at a rate γ. From the preceding discussion and, in particular, formulas (8), (2), and (10), such a static model is completely isomorphic to the dynamic one we are primarily concerned with (III.1–2, IV.1–2).

5. The Publicly Optimal Policy (IV)

It is useful as a starting point to consider the intertemporal allocation policies that would be followed by a fully centralized economy which can choose consumption and both kinds of investment. We start from some initially given quantities of the two kinds of capital. The problem then is to choose the instruments—consumption, c, and the two kinds of investment, i_p and i_g—as functions of time so as to maximize the criterion function, (8) or (5). Notice that at any moment of time the future from then on has the same structure; it is therefore clear that the choice of instruments is a function only of k_p and k_g.

An optimal policy will, under the hypotheses made, tend to a stationary equilibrium, a pair of values for k_p and k_g (the growth-adjusted stocks of the two kinds of capital); call them k_p^{∞} and k_g^{∞}, respectively. Under the hypotheses made, if the initial values of k_p and k_g are k_p^{∞} and k_g^{∞}, respectively, the optimal policy calls for keeping these constant; then the growth-adjusted values of consumption and the two kinds of investment, c, i_p and i_g, respectively, are also constant. However, the following discussion is not necessarily confined to balanced growth paths; the initial conditions may be arbitrary.

If the two kinds of capital are freely transferable between the two sectors, then a publicly optimal policy would regard the two kinds of capital as one. Specifically, let $k = k_p + k_g$; then at any given moment of time, the total k should be reallocated between the two sectors so as to maximize output. (Under the assumption made, capital can be reallocated in the future as desired; hence, maximization of current output clearly dominates any other policy.) Maximization of output requires that

$$f_g = f_p; \tag{11}$$

i.e., the familiar doctrine that the rate of return on government capital should equal that on private capital.

However, it is important to note that even when this position holds, it is not a complete description of policy; by itself it does not in any way determine the volume of investment to be undertaken. We need a complete description of the optimal policy.

In this case, the process is fairly simple. Since we are choosing k_p, k_g, to maximize output for given k, the output is now determined, from (3), by

$$y = g(k) = \max_{k_p + k_g = k} f(k_p, k_g). \tag{12}$$

From (10), we note,

$$\dot{k} = i - \gamma k, \qquad y = c + i, \tag{13}$$

where $i = i_p + i_g$ is total growth-adjusted investment. We choose instruments, c, i, as functions of time, or, better, of k, so as to maximize the criterion function (5) subject to the constraints (12) and (13) for a specified initial value of $k = k_0$.

Associated with any proposed consumption stream, there is an implicit rate of interest, as will be shown in the next paragraph; then an optimal accumulation policy is one for which the marginal product of capital [the same for both kinds according to (11)] equals the consumption rate of interest (III.1–2, IV.1–2).

For a given consumption stream, the consumption discount factor for time t is simply the marginal rate of substitution between future and present consumption; in view of (5) or (8) it is the ratio of their marginal felicities, modified by the discount on future felicities.

$$\text{Consumption discount factor} = (\text{felicity discount factor}) \times \left(\frac{\text{marginal felicity of future consumption}}{\text{marginal felicity of present consumption}}\right).$$

Since the (proportional) rate of change of a product is the sum of the rates of change of the factors and an interest rate is the negative of the rate of change of a discount factor,

$$\text{consumption rate of interest} = (\text{felicity rate of interest}) - (\text{rate of change of marginal felicity of consumption}).$$

Let us apply this statement to (8),

rate of change of marginal felicity of (growth-adjusted) consumption

$$= \frac{1}{U'(c)} \frac{dU'(c)}{dt} = \frac{U''(c)}{U'(c)} \dot{c} = \frac{U''(c)c}{U'(c)} \frac{\dot{c}}{c} = -\sigma \frac{\dot{c}}{c}.$$

Thus, the rate of interest appropriate to growth-adjusted consumption

is $\lambda + \sigma(\dot{c}/c)$. Since growth-adjusted consumption increases at a rate that is lower than that of consumption itself by γ,

$$\text{consumption rate of interest} = r_c = \omega + \sigma(\dot{c}/c), \tag{14}$$

from (7).

Then by the usual arguments, optimality demands that

$$f_p = f_g = g' = r_c; \tag{15}$$

i.e., the rate of interest used in evaluating either kind of investment should be that implicit in the individuals' evaluations of their changing consumption stream.

A very important and not always understood implication of these elementary remarks is that the rate of investment (in this case, either type of investment) is not determined merely by the rate of interest. What (15) tells us is that the stock of capital is at any moment related to the rate of interest. Changes in the growth-adjusted capital stock (i.e., investment other than that needed to maintain the normal growth of the capital stock) require changes in the rate of interest.

Indeed, in (15), the line of causation in the short run goes from the existing capital stock to the rate of interest rather than vice versa. The latter in turn determines not consumption (and therefore investment) but the rate of change of consumption. From (14) and (15),

$$\dot{c}/c = [g'(k) - \omega]/\sigma. \tag{16}$$

This equation, together with

$$\dot{k} = g(k) - c - \gamma k, \tag{17}$$

deducible from (13) and (12), constitute a pair of differential equations governing the evolution over time of the capital stock and the consumption level. The initial stock of capital is given, but that of consumption is not, since it is an instrument.

In fact, the initial value of consumption has to be determined by the condition that the two time paths must converge to their stationary values. These in turn are found by setting $\dot{c}$ and $\dot{k}$ both equal to 0 in (16) and (17).

$$f_p^\infty = f_g^\infty = g'(k^\infty) = \omega, \tag{18}$$

$$c^\infty = y^\infty - \gamma k^\infty. \tag{19}$$

That is, in the long run the marginal productivities of the two kinds of capital have to equal the subjective time preference parameter; investment is that needed to increase the stocks of capital at the natural rate of growth; and consumption is whatever is left out of output.

From (16) it follows that consumption is increasing so long as the rate of interest is above long-run subjective time preference; i.e., if the initial stock of capital is low, then consumption is low and gradually increases to its steady-state value as capital increases to its steady-state value. The relation between investment and present and future rates of return is complex; it can be said that, to a first approximation, growth-adjusted investment is proportional to the discrepancy between the current rate of return and the steady-state time preference, ω.

To repeat, the optimal level of investment, apart from normal growth, is not determined by the rate of interest but primarily by its future changes. An interest rate determination for optimal public investment policy makes sense only when joined to an appropriate level of investment activity.

6. Controllability with Fixed Savings Ratio in the Private Sector (VI)

We turn now to the controllability of the publicly optimal policy and to the possibility of being forced to seek second-best policies if the publicly optimal policy is not controllable in the sense of section 2. The possibility of controllability depends upon the workings of the private markets and upon the range of instruments open to the government.

We noted briefly in section 1 good reasons for believing that private savings behavior is not a decisive indicator of individual time preference. There is little evidence that savings are in fact responsive to rates of return, though it would be premature to say that the contrary is proved. For our purposes, the simple assumption is made that private savings are a fixed fraction, s, of disposable income.

$$s_p = sy_d, \tag{20}$$

where s_p and y_d are the growth-adjusted values of private savings and disposable income, respectively.

We consider several alternative hypotheses concerning the range of instruments available to the government. The first case most nearly reflects actual practice; financing of government investment is accomplished through the income tax alone. Borrowing, at least in peacetime, is relatively small compared with the total budget and is primarily motivated by considerations of employment, rather than allocation, policy. In this case, it is simplest to assume the absence of debt. Let x be the rate of income tax. Then,

$$y_d = (1 - x)y, \tag{21}$$

$$c = (1 - s)y_d. \tag{22}$$

Private capital formation equals private saving; adjustment for growth yields

$$k_p = sy_d - \gamma k_p, \tag{23}$$

while government capital formation equals taxes,

$$k_g = xy - \gamma k_g. \tag{24}$$

The aim of an optimal policy is to maximize the criterion function, (8), subject to the above constraints. First, we observe that the publicly optimal policy is not controllable except by chance. From (23) and (22),

$$(k_p + \gamma k_p)/c = s/(1 - s).$$

In the limit, then,

$$\gamma k_p^\infty / c^\infty = s/(1 - s).$$

If the publicly optimal policy could be achieved, the left-hand side would have a certain value which would not depend on s; hence, equality could hold only by accident.

Since the publicly optimal policy is not in general controllable, the optimal policy sought for is a "second-best" policy. We proceed heuristically as follows. For any given initial values of k_p and k_g, there is an optimal policy which yields a value of total discounted utility, (8); call this value $W(k_p, k_g)$. Then the shadow price in utility terms of k_p is

$$p_p = \partial W / \partial k_p$$

and similarly, for government capital,

$$p_g = \partial W / \partial k_g.$$

Any point of time could be regarded as the initial point; so two functions of time—$p_p(t)$ and $p_g(t)$, termed auxiliary variables in control theory—are associated with the optimal policy. Hence, an addition to either kind of capital (growth-adjusted) can be valued at these rates. At the same time, a consumption, c, yields a current utility return, $U(c)$, so that total national income in utility terms can be written

$$H = U(c) + p_p k_p + p_g k_g,$$

or, from (23) and (24),

$$H = U(c) + p_p(sy_d - \gamma k_p) + p_g(xy - \gamma k_g). \tag{25}$$

Since it is always desirable to increase H, the sole instrument, x, is to be chosen to maximize H. (Recall that c and y_d both depend on x.) Then setting $\partial H / \partial x = 0$ yields

$$(1 - s)U'(c) + sp_p = p_g, \tag{26}$$

which can be thought of as an equation to determine x in terms of k_p, k_g, p_p, and p_g. Note that the burden of an increment of government capital, financed by taxation, falls on consumption and private saving in the proportions $1 - s$ and s.

The evolution of the auxiliary variables over time remains to be determined. The equations used are the analogues in utilities and auxiliary variables to the usual equilibrium condition for the holding of an asset: the marginal productivity plus capital gains must equal the rate of interest times the price. Since we are dealing with utilities here, the "rate of interest" is simply λ. The "marginal productivity" is the contribution to H. Thus the equation for private capital is

$$(\partial H/\partial k_p) + \dot{p}_p = \lambda p_p$$

and a corresponding equation holds for government capital. From (25), we find, after some simplification,

$$\dot{p}_p = \omega p_p - p_g f_p, \tag{27}$$

$$\dot{p}_g = \omega p_g - p_g f_g. \tag{28}$$

These equations, together with (23), (24), and (26), form a complete dynamic system. The solution is then defined if the initial conditions are specified. The initial values of $k_p(0)$ and $k_g(0)$ are given historically; however, those of the auxiliary variables have to be so chosen that the solution converges to a stationary value.

It is not easy to give a simple interpretation of an interdependent system like this. For analysis we clearly need not only the rates of return, f_p and f_g, but also the auxiliary variables and their rates of change. These solutions are computable, however.

Some insight can be found by looking at the stationary values, which, as before, we denote by superscript ∞. The stationarity of the capital stocks, whose motion is defined by (23) and (24), implies,

$$sy_d^\infty = \gamma k_p^\infty,$$
$$x^\infty y^\infty = \gamma k_g^\infty.$$

Multiply the second equation by s, and add to the first, while recalling the definition of y_d.

$$sf(k_p^\infty, k_g^\infty) = \gamma(k_p^\infty + sk_g^\infty), \tag{29}$$

a form of the Harrod-Domar relation. The stationarity of the auxiliary variables, whose motion is defined by (27) and (28), implies

$$f_p(k_p^\infty, k_g^\infty) = (p_p^\infty/p_g^\infty)\omega, \tag{30}$$

$$f_g(k_p^\infty, k_g^\infty) = \omega. \tag{31}$$

Equations (29) and (31) involve only k_p^∞ and k_g^∞, which are therefore determined. Then the asymptotic ratio of the auxiliary variables is, from (30) and (31), in the same ratio as the marginal productivities of the two kinds of capital.

Equation (31) is a little surprising; the long-run rate of return on government capital is the social rate of time preference, even though the return on private capital may be quite different. This holds because the benefits from a government investment project increase national income and therefore are partly saved. Hence, indirectly, the returns from government investment include some benefit from private investment projects. It turns out that in balanced growth this benefit exactly offsets the loss of private investment due to the initial act of government investment.

The uncontrollability of the publicly optimal policy and the need to resort to a second-best policy in the above discussion arose from the restriction to a single financing instrument, the income tax. To illustrate the possibility of controllability with more instruments, suppose that the tax rates on consumption can differ from those on savings, but that a balanced budget is still required (no borrowing) (VI.7). We shall understand the hypothesis of a fixed savings ratio to mean that post-tax savings and post-tax consumption are fixed fractions of total personal income. Thus, expenditures on savings are sY, but the government takes a fraction x_s of this so that private savings are reduced to $s(1 - x_s)Y$; similarly, consumption is $(1 - s)(1 - x_c)Y$. Government tax collections, and therefore government investment, are given by

$$[sx_s + (1 - s)x_c]Y.$$

Then,

$$\dot{k}_p = s(1 - x_s)y - \gamma k_p, \tag{32}$$

$$\dot{k}_g = [sx_s + (1 - s)x_c]y - \gamma k_g, \tag{33}$$

$$c = (1 - s)(1 - x_c)y. \tag{34}$$

Suppose the government wishes to control the publicly optimal or indeed any other feasible policy. Since

$$c + i_p + i_g = y,$$

where

$$\dot{k}_p = i_p - \gamma k_p, \qquad \dot{k}_g = i_g - \gamma k_g,$$

it is clear that if the government can choose its instruments—the two tax rates—to satisfy two of the three equations (32)–(34), the third is automatically satisfied. Given any policy determining consumption and the two kinds of capital as functions of time, it is then only necessary to solve (34) for x_c, the consumption tax, and x_s, the tax on savings. The consumption tax thus insures that the correct amount of aggregate investment is forthcoming, while the tax on savings allocates it between the two forms of capital formation.

Controllability is also achieved if the government can both borrow and impose either an income tax or a consumption tax. It may be worth noting that merely counting instruments is not a sufficient condition for controllability; borrowing plus a tax on savings will not achieve controllability (VI.8).

Thus, the rule that the rate of discount for government investment should equal that in the private sector requires not only that the appropriate level of investment be forthcoming, as argued in section 5, but also, in a mixed economy, that the financing be accomplished by the unique appropriate mixture of taxes (or of borrowing and taxes).

7. Controllability with Perfectly Rational Consumers (VII, VIII)

The fixed savings ratio assumption is perhaps extreme in emphasizing the shortsightedness of the consumer. The opposite extreme hypothesis is that the consumer chooses his consumption pattern over time to maximize the sum of discounted utilities in full knowledge of all future interest rates and wage levels. We can then ask under what conditions the publicly optimal policy is controllable.

Although more general cases can be analyzed, we confine ourselves here to the case in which there is no divergence between public and private values. It is assumed, that is, that the consumer is seeking to maximize the same criterion function, (5) or (8), as the government.

We will assume here that the government borrows (or lends) and can impose one kind of tax. We also assume that bonds are a perfect substitute for private capital from the viewpoint of the consumer, so that the rate of interest on government bonds equals the marginal productivity of private capital. (Government bonds are assumed to be short-term bonds, so that capital gains may be ignored.)

We only summarize the results here. The fact that there may be an initial debt is important, because the interest on it must be financed in any case, in addition to subsequent government investment. Actually, if the initial debt has an appropriate initial value, then choosing government investment according to the publicly optimal policy and financing it out of borrowing alone is optimal (VIII.2). An income tax in the ordinary sense cannot be used; it destroys optimality through the well-known double taxation of savings. But if the initial debt differs from the critical initial value just described, then an initial capital levy (which has no incentive effects) can be used to change the debt to the appropriate level, after which financing is again done only by borrowing. The initial capital levy might be thought of as a limiting form of the income tax (VIII.3).

The most interesting case is the one in which the sole tax is the consumption tax (VIII.4). It turns out that the publicly optimal policy can be carried out with a consumption tax whose rate is constant over time; the remainder of the government investment is financed by borrowing. Of course, this remainder might be negative, in which case the government retires debt or even lends to the private sector. Incidentally, the constant rate of the consumption tax has a simple interpretation. If *private wealth* is defined as the sum of government debt, private capital, and future wages discounted to the present according to the wages and interest rates implicit in the publicly optimal policy (wages equal to marginal product of labor), the consumption tax is then the ratio of private wealth to the total of future consumption discounted to the present. The consumption tax rate will thus depend, among other things, on the initial level of debt.

8. Risk and the Rate of Return

The analysis so far has been based on the implicit assumption that the returns to government investment are riskless. If they are not, then it is contended here that as a general rule the uncertain benefits and costs should be evaluated at their expected value and discounted at the rate of return appropriate to riskless investments. This statement is not applicable to all circumstances but is a proposition which follows from certain hypotheses that are approximately fulfilled, with some exceptions.

First, suppose that: (a) an optimal allocation of risk bearing occurs before the government investment in question, and (b) the random returns on the government investment are statistically independent of those in the economy before the investment takes place. Then it can be shown that the proposed new investment should be introduced if and only if its expected net return is positive (I.2).

The assumption of statistical independence appears not unreasonable as a first approximation. However, that of the optimality of risk bearing in the private sector is more dubious. But much the same result still holds if the assumption is dropped, for the allocation of risks among all taxpayers implies not only that the risk to any one is negligible but more strongly that the total of all risk premiums tends to zero as the number of individuals grows indefinitely large. If the government adopts an expected value criterion, while private industry does not, then a government investment may indeed displace a private investment of higher expected value; however, this is correct in the context, because the government is supplying a valuable complementary activity of risk bearing which is not being supplied by the private sector.

We do note, however, the possibility that the benefits accruing to an individual as a result of government investment may increase his uncertainty. An example of this is the introduction of irrigation with an uncertain water supply into a predictably dry climate. Then, indeed, the benefits should be discounted at a rate higher than the riskless rate; but the preceding analysis shows that it would be still better for the government to offer insurance against failure of the irrigated water and then evaluate the total package of water and insurance at the riskless rate.

Note that no contention has been made that the government uses a riskless rate because it can spread its risks over a great many projects; Hirshleifer (1966, pp. 270–75) is certainly right in asserting that each project should be evaluated separately if the appropriate discount factors are used.

9. Some Loose Ends

Some aspects have been considered in the literature that have not been taken into account here. In effect, we have been assuming that, apart from divergence between private and social time preferences, the problem of determining the social discount rate arises from a particular set of imperfections in the market structure, the inappropriability of the products of government investment, the existence of an initial debt which needs financing, and, in section 6, the imperfection of private capital markets as reflected in the fixed savings ratio hypothesis. Implicitly, it has been assumed that the market structure is otherwise perfect. If, however, there are imperfections elsewhere in the market structure—such as monopolistic price distortions, excise taxes, or the corporate income tax that falls on the fruits of some but not all private investment—then the analysis becomes far more complicated. Any suggested policy must be evaluated in terms of all sorts of cross-effects.

Another neglected aspect is the relation between government bond rates and return on private capital. In the preceding discussion and indeed in virtually all analysis, the two are assumed equal, whenever government borrowing is considered as a possible means of financing. In fact, this is very far from the case. This raises the possibility that if public investment is financed by bonds, the government is in effect producing a joint product; the investment creates the opportunity to produce bonds, which are preferred to private capital. This suggests the appropriateness of discounting by the government bond rate rather than by the average observed rate of return.

PUBLIC INVESTMENT, THE RATE OF RETURN, AND OPTIMAL FISCAL POLICY

It has become increasingly clear with the passage of time that the criteria used in the allocation of resources must be derived from dynamic rather than static notions of efficiency. It has also become clear that these criteria have not been adequately related to the macroeconomic problems of stability and growth.

Professors Arrow and Kurz in this book have produced a path-breaking formulation of the problems of public expenditure in the context of modern economic growth theory. Their work ties the determination of the allocation of resources between public and private sectors to the goal of getting the economic system onto an efficient growth path. While the models utilized are highly abstract, there are provocative implications for the determination of the appropriate discount rate and, indeed, the entire process of economic planning.

The authors first present a clear but rigorous analysis of optimal growth, distinguishing public and private capital as separate decision variables. This section of the book (chapters I–IV) constitutes an exceptionally fine exposition of modern growth theory.

Following this exposition, the authors develop an entirely new body of theory, a theory of "controllability." This seminal work systematically explores the question of whether or not the instruments of general fiscal control which the government has available (taxation, expenditure, and debt management) are sufficient to bring about an optimal intertemporal allocation of resources in the presence of the constraints imposed by private sector behavior. Private decisions may not lead to the optimum implied by the government's social welfare function even when there is no conflict between public and private objectives. The injection of the question of "controllability" into the fields of public finance and growth theory constitutes a major theoretical advance.

The findings of this study promise to influence the theoretical developments in the public expenditure field for a long time to come.

JOSEPH L. FISHER, *President,*
Resources for the Future, Inc.

CONTENTS

I

BASIC CONCEPTS FOR THE THEORY OF PUBLIC INVESTMENT

0. Introduction

One of the major economic phenomena of the twentieth century has been the emergence of the public sector as the most powerful economic agent in all modern societies. The economic strength of this sector varies from complete control, as is the case in Communist societies, to subtle and indirect power, as is the case in the more decentralized economies of the West.

The economic-theoretical analysis that has dealt with the functioning of the government sector has mostly been focused on two major aspects: on one hand, fiscal and monetary policies directed toward stability and growth and, on the other, the theory of public finance dealing with the welfare implications of alternative ways of financing the public sector. A limited amount of theoretical work has been done in the area of the behavior of the public sector as an investing agent. It may be contended that this behavior ought not to differ from the behavior of any private firm; we shall argue that this is not the case and that specialized criteria are needed to guide the investment decisions of the public sector.

Faced with a lack of conceptual development, the various agencies of the government have developed rules of thumb for their decision-making procedures. Under pressure from Congress and the public, the administration has sought to unify these criteria in order to have objective methods of evaluating alternative public projects. These efforts have resulted in what has come to be known as cost-benefit analysis. This analysis aims at measuring the cost of and evaluating the benefits from public projects so that they can be ranked according to their "desirability." The application of cost-benefit analysis is usually difficult since it involves quantities that have no market price. Thus, only further theoretical developments in these areas will clarify the issues and enable us to establish more reliable estimates of social returns from investments.

Most discussion of the nature of public goods has focused attention on the static allocation problem. The only serious work on criteria for public investments has been done in connection with the development of water resources (see Eckstein 1958; Hirshleifer, De Haven, and Milliman 1960). We shall not review this work but rather reconsider the problems afresh from a more fundamental point of view.

In the following sections of this chapter we shall consider several basic conceptual and technical matters that will serve to guide our investigation into the determination of the volume of public investments, with particular reference to its rate of return. In sections 1 and 2, the special nature of social investments is stressed, particularly those aspects relevant to the choice of the discount rate. In section 3, the general optimization principles of investment policy are discussed briefly, and some indication is given of how our approach differs from that of those who have already studied the field. In section 4, the criteria by which the optimality of alternative public investment programs is to be judged are discussed. Finally, in section 5, some of the basic ideas of the so-called neoclassical growth theory are summarized.

1. The Special Nature of Social Investments

Appropriability

Most investments yield their benefits in the form of identifiable goods that can be marketed or withheld. These benefits are in a very natural way *appropriable* in the sense that the organization producing them can without difficulty charge individual consumers for them, so that those who want and need the product can buy, and others can refrain. The production of food and clothing provides, perhaps, the purest example of appropriable benefits. The future benefits from such an investment can be fairly measured by the output evaluated at the price at which it can all be sold, less, of course, all current production costs (wages and materials).

But a wide and important class of investments yields benefits which, in their very act of production, inure to a wide class of individuals. These people cannot be excluded from the benefits and, hence, a price cannot be charged that will effectively discriminate between those who want the service and those who do not. Water purification provides a simple example: if it is decided to install equipment that will improve the purity of the water, all users will receive the benefits over the lifetime of the equipment whether or not they would be willing to bear the cost in a free choice. (This choice is not only a matter of individual taste for pure water; some of the uses of household water, such as gardening, have much lower purity requirements than others, so that some individual consumers may, in fact, derive very little additional benefit.) The price system is not operative, for it would require that each consumer be given the freedom to buy water at both the older and newer levels of purity or, at the very least, be given his option between the two, with price differences reflecting cost differences. Water purification is really of the same order as the general run of collective services provided by the government. In this context it is differentiated from the rest only in that there is an investment component; i.e., the benefits and costs do not accrue at the same point of time.

The example of water purification indicates the common nature of all public investments: they generate a great many benefits which inure to individual consumers and firms in such a way that the normal market mechanism would not price these benefits correctly, and this leads to the incorrect allocation of resources to their production. This does not mean that one cannot design methods for pricing these benefits. In the example of water purification, it is possible to construct two water systems and let each consumer have two water outlets; in this way he can be offered each kind of water at a different price. However, in fact, there is generally only one water system, not two; thus, neither a competitive mechanism nor public decisions could lead to this double price structure for water.

There are other instances in which pricing of benefits would be technically feasible, but for other reasons it is not regarded as performing an appropriate social function. Elementary, secondary, and, to a considerable extent, higher education have begun to belong to this category. The public schools could charge pupils or their parents for the cost of education, but in the first place there may be a divergence of interest between the parents who are capable of paying and the children who are receiving the benefit. This is part of a wider class of cases in which the beneficiaries are incapable of appreciating the benefit, either because of natural limitations of understanding (as in children or mental patients) or because the benefits would not really be understood until they have been experienced. The second reason, in the case of education, is that the benefits of education accrue not merely to the students but to the society of which they are a part.

In general, the line between appropriability and inappropriability cannot be drawn very sharply. There are very few acts, even of private consumption, that do not have some direct effect on the welfare of others. It is a matter partly of empirical evaluation and partly of value judgment as to when the external effects of benefits are sufficiently widespread to set aside the principle that the individual is the best judge of his own welfare.

Another and very important reason, rooted in the facts of technology, exists for the treatment of wide-scale classes of benefits as inappropriable, even though it would be technically feasible to set prices; namely, in cases where there are increasing returns to the scale of operation. In that circumstance, a collective agreement to undertake a productive enterprise and to share the costs in some way may benefit everyone, yet any ordinary pricing system would fail. For example, competition among electricity systems would certainly not ensure an optimal allocation of resources but instead would probably reduce the supply of electricity to small proportions. It is, to be sure, often possible (for example, in irrigation) to determine the benefits through a pricing calculation, but the supply must nevertheless be arranged through a monopoly; because of the dangers of monopoly in certain circumstances, the investment must actually be provided socially.

For our present purposes it is not useful to examine in detail either the problem of measuring benefits or the use of pricing of government services to increase efficiency in their use. The main point to be made here is that the services derived from government investment may not be charged for at all, or, if they are, the rate need not correspond to their marginal usefulness to society.

Divergence between Social and Private Benefits

A classic in economic theory is the case in which there is a direct beneficiary from whom the product can be withheld, but his act of consumption, or the act of production in order to achieve this consumption, yields benefits to other parties against whom no exclusion is possible. The water purification example cited above is an extreme case. To supply pure water to even one individual, it is necessary to supply it to everyone. Milder interactions are very common. Thus, treatment of infectious diseases is beneficial to the patient but, in addition, the possibility that other individuals may be infected is also reduced. Under these conditions it would be necessary to take into account the fact that the aggregate benefits are greater than the part that can be allocated privately with ease. Hence, to be justified, it is not necessary for an investment in public health, including medical care of infectious diseases, to be fully repaid from fees charged to patients.

A special possibility of this divergence may arise from the relation between successive generations; this point is discussed briefly in the following section.

Production and Consumption Benefits

Although, in the last analysis, all benefits accrue to individuals whom we may think of as consumers, the relation may be direct or it may be an indirect result of the facilitation of the production of goods desired by consumers. Most social investment activities yield benefits of both types. A highway increases the convenience of private automobile travel, a direct benefit to consumers; it also decreases the cost of trucking operations, thus ultimately decreasing the cost or increasing the supply of consumers' goods.

Consumption benefits are those whose immediate beneficiaries are individuals in their capacities as consumers; production benefits are those whose immediate beneficiaries are economic units engaged in production for a market. Thus, government investment may affect both the output of goods and the satisfactions of individual consumers.

2. Decentralization and the Discount Factor Appropriate for Public Investments

In a totally centralized society, the problem of optimal public investments is rather simply defined: The central planning board sets up its objectives and then seeks those investment criteria that will maximize these objectives subject to the technological constraints and resource availability. This kind of procedure will be discussed in chapters III and IV.

In a decentralized society, where individuals and firms are free to maximize their own objectives subject to their own private constraints, the investment and financing behavior of the public sector becomes a more delicate matter. Under these conditions, what can be said about the discount factor appropriate to public investment decisions?

One approach to this question is to ask: Why not use the market rate of interest?

Social and Private Rates of Time Preference

If the private capital market were perfect, and if there were no divergences between social and private benefits (in the static and intertemporal senses) on it, the discount rate on public investments would be the same as that found on the private market. The argument for this proposition is the standard one in welfare economics and need not be repeated in detail here: If the public investment program were determined by evaluating benefits according to some rate different from that of the market, then there would always be a way to increase total output by some reallocation of investment resources as between the public and private sectors.