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Third International Conference, CPAIOR 2006
Cork, Ireland, May/June 2006
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Preface

This volume contains the proceedings of the Third International Conference on Integration of AI and OR Techniques in Constraint Programming for Combinatorial Optimization Problems (CPAIOR 2006). The conference was held in Cork, Ireland, from May 31 to June 2, 2006. Information about the conference can be found at `http://tidel.mie.utoronto.ca/cpaior/`. Previous meetings in this series include two international conferences held in Nice (2004) and Prague (2005) and five international workshops held in held in Ferrara (1999), Paderborn (2000), Ashford (2001), Le Croisic (2002), and Montreal (2003).

The goal of these meetings is to provide a forum for researchers to present approaches which highlight the integration of CP, AI, and OR techniques. An additional important goal is to allow researchers from diverse backgrounds to learn about techniques in other areas that are used for solving combinatorial optimization problems and therefore to encourage cross-fertilization. One measure of the success that has been enjoyed by these meetings is the number of publications outside this conference series (e.g., at the International Conference on the Principles and Practice of Constraint Programming) that directly explore integrated approaches to solving large and difficult combinatorial problems.

CPAIOR 2006 received 67 submissions. In order to streamline the reviewing process, a subcommittee of the Programme Committee, consisting of Michael Trick, Pascal Van Hentenryck, and the Programme Chairs, evaluated each submission to ensure relevance to the conference aims. The subcommittee unanimously judged that 41 of the submissions were sufficiently relevant to proceed to the full review stage. Each of these submissions received three reviews by members of the Programme Committee. The reviews were extensively discussed during an online Programme Committee meeting. As a result, the Programme Committee chose 20 (29.9%) to be included in the proceedings and presented at the conference.

The authors of the papers in this volume have been invited to submit extended versions to a special issue of the *Annals of Operations Research* entitled "Constraint Programming, Artificial Intelligence and Operations Research." All papers submitted will be subject to an additional rigorous review process and we expect the special issue to be published in early 2008.

In addition to the technical sessions, three invited talks were given by leading researchers. These diverse talks address the uses of optimization technology in visual art (Robert Bosch, Oberlin College, USA); the growing interest in the AI planning community in solving mixed discrete/continuous problems by exploiting existing CP and OR techniques (Maria Fox, University of Strathclyde, UK); and the issue of duality, a central issue in both traditional OR and CP solution approaches (John Hooker, Carnegie Mellon University, USA).

CPAIOR 2006 continued the tradition of holding a Master Class on a focused topic as part of the conference. This year's Master Class, organized by Ken Brown and Armagan Tarim, consisted of six tutorial sessions on the topic of "Modelling and Solving for Uncertainty and Change." The speakers at the Master Class were Nesim Erkip (Bilkent University, Turkey), Hélène Fargier (IRIT, Toulouse, France), Alexei Gaivoronski (Norwegian University of Science and Technology, Norway), Brahim Hnich (Izmir University of Economics, Turkey), Pascal Van Hentenryck (Brown University, USA), and Gérard Verfaillie (ONERA/CERT, France).

We would like to thank the Programme Committee for their careful work over the past few months in ensuring a high-quality programme for the conference. We would also like to thank everyone involved in the organization of the conference, notably Barry O'Sullivan, the Conference Chair; Ken Brown and Armagan Tarim, the Chairs of the Master Class; Ian Miguel, the Publicity Chair; Michela Milano, the Sponsorship Chair; Tom Carchrae, the Webmaster; and Eleanor O'Riordan for her administrative support. It would have been impossible to hold CPAIOR 2006 without their significant contributions of time and effort.

Finally, we would like to thank the institutions listed below who helped to sponsor the conference. Their generosity enabled the conference to attract invited speakers and instructors for the Master Class as well as to fund student participation. These funds, therefore, greatly contributed to the success of the conference.

March 2006

J. Christopher Beck
Barbara M. Smith
Programme Chairs
CPAIOR 2006

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Invited Talks

Technical Papers

Opt Art

Robert Bosch

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Abstract. Optimization deals with finding the best way to complete a task—creating a schedule for a tournament, matching professors with courses, constructing an itinerary for a traveling salesman. It has been applied successfully to such a great number of diverse disciplines that one could argue that it can be put to good use in *every* imaginable field. In this talk, we will showcase its amazing utility by describing some applications in the area of art: portraits constructed out of complete sets of dominoes (via integer programming) mosaics comprised of abstract geometric tiles (via integer programming and various heuristics), and continuous line drawings (via the "solution" of large-scale instances of the traveling salesman problem).

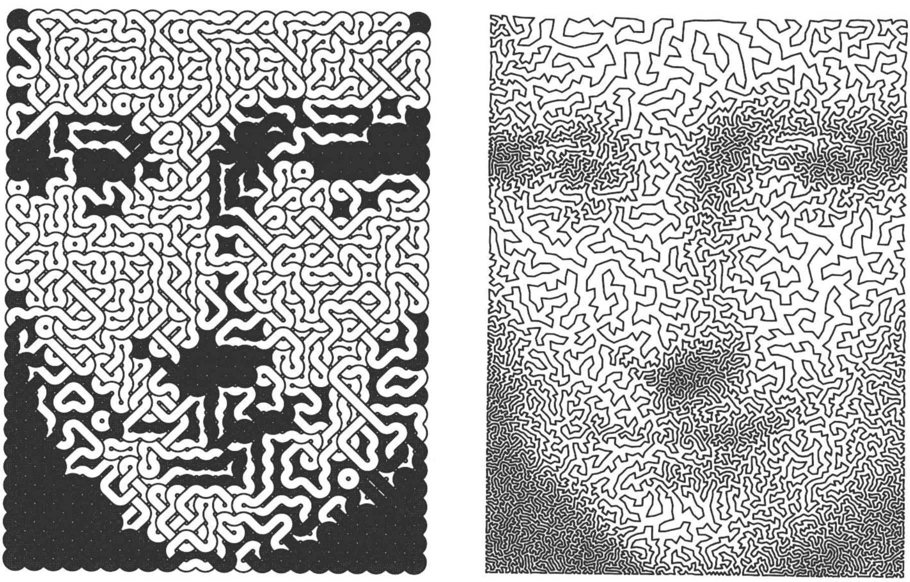

Fig. 1. Examples of Opt Art

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Planning for Mixed Discrete Continuous Domains

Maria Fox

Department of Computer Science and Information Systems
University of Strathclyde, U.K.

Abstract. Mixed discrete-continuous systems are hybrid systems that exhibit both discrete changes of state, describable in terms of their logical and metric properties, and continuous numeric change describable in terms of differential equations. Continuous change occurs within a state as a consequence of one or more continuous processes being active in that state, whilst discrete change results in state transitions. Such hybrid systems are well-understood in the formal verification and real-time control communities.

Many real planning problems involve interaction with continuously changing values that directly affect both the validity and efficiency of plans. The problem of planning with continuous effects is harder than planning under the assumption of discrete change. The planner must be capable of reasoning about the evolution of continuous processes and their interactions with discrete state changes. For this reason, the standard approach to handling complex continuous effects in planning is to abstract them out of the domain model by lifting the representation to a level where all change can be seen as discrete.

In this talk we discuss progress we have made towards planning in mixed discrete-continuous domains. We begin by arguing that there are problems of critical interest to potential users of planning technology that cannot be adequately modelled under the assumption of discreteness. We then discuss an approach to planning in these domains that relies on the integration of a discrete planner with a continuous non-linear constraint solver. We present some results taken from a range of planning domains featuring continuous change. We discuss the future of this branch of planning and relate our work to the AI and OR literature.

J.C. Beck and B.M. Smith (Eds.): CPAIOR 2006, LNCS 3990, p. 2, 2006.

Duality in Optimization and Constraint Satisfaction

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Abstract. We show that various duals that occur in optimization and constraint satisfaction can be classified as inference duals, relaxation duals, or both. We discuss linear programming, surrogate, Lagrangean, superadditive, and constraint duals, as well as duals defined by resolution and filtering algorithms. Inference duals give rise to nogood-based search methods and sensitivity analysis, while relaxation duals provide bounds. This analysis shows that duals may be more closely related than they appear, as are surrogate and Lagrangean duals. It also reveals common structure between solution methods, such as Benders decomposition and Davis-Putnam-Loveland methods with clause learning. It provides a framework for devising new duals and solution methods, such as generalizations of mini-bucket elimination.

1 Two Kinds of Duals

Duality is perennial theme in optimization and constraint satisfaction. Well-known optimization duals include the linear programming (LP), Lagrangean, surrogate, and superadditive duals. The constraint satisfaction literature discusses constraint duals as well as search methods that are closely related to duality.

These many duals can be viewed as falling into two classes: inference duals and relaxation duals [12]. The two classes represent quite different concepts of duality. This is perhaps not obvious at first because the traditional optimization duals just mentioned can be interpreted as both inference and relaxation duals.

Classifying duals as inference or relaxation duals reveals relationships that might not otherwise be noticed. For instance, the surrogate and Lagrangean duals do not seem closely related, but by viewing them as inference duals rather than relaxation duals, one sees that they are identical except for a slight alteration in the type of inference on which they are based.

A general analysis of duality can also unify some existing solution methods and suggest new ones. Inference duals underlie a number of nogood-based search methods and techniques for sensitivity analysis. For instance, Benders decomposition and Davis-Putnam-Loveland methods with clause learning, which appear unrelated, are nogood-based search methods that result from two particular inference duals. Since any inference method defines an inference dual, one can

J.C. Beck and B.M. Smith (Eds.): CPAIOR 2006, LNCS 3990, pp. 3–15, 2006.

in principle devise a great variety inference duals and investigate the nogood-based search methods that result. For example, filtering algorithms can be seen as inference methods that define duals and give rise to new search methods, such as decomposition methods for planning and scheduling.

Relaxation duals underlie a variety of solution methods that are based on bounding the objective function. A relaxation dual solves a class of problem relaxations that are parameterized by "dual variables," in order to obtain a tight bound on the objective function value. The LP, surrogate, Lagrangean, and superadditive duals familiar to the optimization literature are relaxation duals. A constraint dual is not precisely a relaxation dual but immediately gives rise to one that generalizes mini-bucket elimination methods.

Inference and relaxation duals are precise expressions of two general problem-solving strategies. Problems are often solved by a combination of search and inference; that is, by searching over values of variables, which can yield a certificate of feasibility for the original ("primal") problem, and by simultaneously drawing inferences from constraints, which can yield a certificate of optimality by solving the dual problem. A problem belongs to NP when the primal solution has polynomial size and to co-NP when the dual solution has polynomial size.

Problems can also be solved by a combination of search and relaxation; that is, by enumerating relaxations and solving each. The relaxation dual is one way of doing this, since it searches over values of dual variables and solves the relaxation corresponding to each value.

2 Inference Duality

An *optimization problem* can be written

$$\min_{x\in D}\{f(x)\mid \mathcal{C}\} \tag{1}$$

where $f(x)$ is a real-valued function, $\mathcal{C}$ is a constraint set containing variables $x=(x_1,\ldots,x_n)$, and D is the *domain* of x. A solution $\bar{x}\in D$ is *feasible* when it satisfies $\mathcal{C}$ and is *optimal* when $f(\bar{x})\le f(x)$ for all feasible x. If there is no feasible solution, the optimal value of (1) is ∞. If there is no lower bound on $f(x)$ for feasible x, the problem is *unbounded* and the optimal value is $-\infty$.[1] A value $\bar{z}$ is a feasible value of (1) if $f(x)=\bar{z}$ for some feasible x, or if $\bar{z}=\infty$, or if $\bar{z}=-\infty$ and the problem is unbounded.

A *constraint satisfaction problem* can be viewed the problem of determining whether the optimal value of

$$\min_{x\in D}\{0\mid \mathcal{C}\}$$

is 0 or ∞.

[1] We exclude problems that have no optimal value, such as $\min_{x\in\Re}\{x\mid x>0\}$.

2.1 The Inference Dual

The *inference dual* of (1) is the problem of finding the greatest lower bound on $f(x)$ that can be inferred from $\mathcal{C}$ within a given proof system. The inference dual can be written

$$\max_{P \in \mathcal{P}} \left\{ v \;\middle|\; \mathcal{C} \overset{P}{\vdash} (f(x) \geq v) \right\} \tag{2}$$

where $\mathcal{C} \overset{P}{\vdash} (f(x) \geq v)$ indicates that proof P deduces $f(x) \geq v$ from $\mathcal{C}$. The domain of variable P is a family $\mathcal{P}$ of proofs. A pair $(\bar{v}, \bar{P})$ is a feasible solution of (2) if $P \in \mathcal{P}$ and $\mathcal{C} \overset{\bar{P}}{\vdash} (f(x) \geq \bar{v})$, and it is optimal if $\bar{v} \geq v$ for all feasible (v, P). If $f(x) \geq v$ cannot be derived from $\mathcal{C}$ for any finite v, the problem is infeasible and has optimal value $-\infty$. If for any v there is a feasible (v, P), the problem is unbounded and has optimal value ∞. A value $\bar{v}$ is a feasible value of (2) if $(\bar{v}, P)$ is feasible for some $P \in \mathcal{P}$, or if $\bar{v} = -\infty$, or if $\bar{v} = \infty$ and (2) is unbounded.

The original problem (1) is often called the *primal* problem. Any feasible value of the dual problem is clearly a lower bound on any feasible value of the primal problem, a property known as *weak duality.* The difference between the optimal value of the primal and the optimal value of the dual is the *duality gap.*

The constraint set $\mathcal{C}$ *implies* $f(x) \geq v$ when $f(x) \geq v$ for all $x \in D$ satisfying $\mathcal{C}$. The proof family $\mathcal{P}$ is *complete* if for any v such that $\mathcal{C}$ implies $f(x) \geq v$, there is a proof $P \in \mathcal{P}$ that deduces $f(x) \geq v$ from $\mathcal{C}$. If $\mathcal{P}$ is complete, then there is no duality gap. This property is known as *strong duality.*

Solution of the inference dual for a complete proof family $\mathcal{P}$ solves the optimization problem (1), in the sense that a solution $(\bar{v}, \bar{P})$ of the dual provides a proof that $\bar{v}$ is the optimal value of (1). If $\bar{P}$ always has polynomial size, then the dual belongs to NP and the primal problem belongs to co-NP. Solution of the inference dual for an incomplete proof family may not solve (1) but may be useful nonetheless, for instance by providing nogoods and sensitivity analysis.

2.2 Nogood-Based Search

Nogoods are often used to exclude portions of the search space that have already been explicitly or implicitly examined. The inference dual can provide a basis for a nogood-based search.

Suppose we are solving problem (1) by searching over values of x in some manner. The search might proceed by splitting domains, fixing variables, or by adding constraints of some other sort. Let $\mathcal{B}$ be the set of constraints that have been added so far in the search. The constraint set has thus been enlarged to $\mathcal{C} \cup \mathcal{B}$. The inference dual of this restricted problem is

$$\max_{P \in \mathcal{P}} \left\{ v \;\middle|\; \mathcal{C} \cup \mathcal{B} \overset{P}{\vdash} (f(x) \geq v) \right\} \tag{3}$$

If $(\bar{v}, \bar{P})$ solves the dual, we identify a subset $\mathcal{N}$ of constraints that include all the constraints actually used as premises in the proof $\bar{P}$. That is, $\bar{P}$ remains a