

DIALECTICAL THOUGHT IN NATURAL SCIENCE

**Speculations in Einstein's Theory of
Relativity and Non-Euclidean Geometry**

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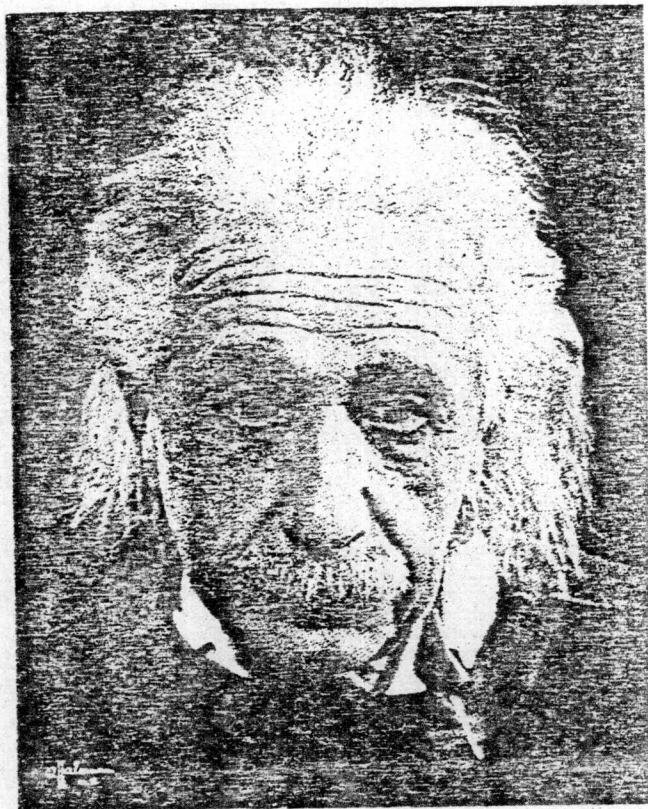
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To the memory of
my dear mother



A. Einstein



A. Einstein



N. I. Lobachevsky

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Chapter 1. The Special Theory of Relativity



N. I. Lobachevsky

PREFACE

Aside from being a vehicle for communicating my joy in the subject, this book is intended to supply a wide range of topics for course "dialectical thought in natural science". My object in writing the book is to explore the ways of improving mode of thinking. The book is rather more conceptually and mathematically than experimentally oriented, and only should be suitable from the upper undergraduate level onwards. But the book might well be used autodidactically by a somewhat more advanced reader. It assumes no prior knowledge of tensor analysis and Hyperbolic geometry. Much emphasis has been laid on developing the student's intuition for space-time geometry and four-tensor calculus. The book strikes a proper balance in the length, width, and depth of the presentation. The entire book can be covered in a one-year course, a three-hour week. For a two-quarter or one semester course, several plans are possible using this book. Interdependency between various chapters is kept at a minimal level.

I should perhaps say a word on the genesis of the book. More than thirty years ago, when I was an assistant in Nanjing university, Leopold Infeld, the world-famous relativist, arrived in China and gave a lecture on theory of relativity and Riemann geometry. Having had the privilege of being a listener, I was greatly excited by his dialectical thought. Later, I became a postgraduate student in the Academy of Sciences of China. Since then, I have read numerous relativity books and have made numerous reading notes. The present book arose primarily out of notes of lectures that I have been giving to the postgraduate students at the Southeast University; I believe, it should be of interest to anyone whose curiosity extends to the nature of the physical world and, in particular, to physicists, astronomer, mathematicians and philosophers of science.

The methodology of scientific researches is always the first-class important problem. Tracing the historical progress of natural science back to times long past, we found that a few tremendous break-throughs were all accompanied by violent controversies in philosophical area, and that such controversies continuously proved the importance and rightness of dialectical thought. Every progressing step in natural science, always negated idealism and metaphysics and contributed powerful proofs for materialism and dialectics.

Matter in the universe is in constant motion and develops in space and time. Since time immemorial, time and space, being the most fascinating subject, have occupied special position. Space-time is not only the objects for scientific research, but is itself philosophical category. From the copernican theory of earth motion to Newton's mechanics and the theory of relativity; from Euclidean space geometry to Non-Euclidean geometry represented by Lobachevsky; from simple classical models of universe to relativistic cosmology, the process of human knowledge and practice during the long years flashed with dialectical thought everywhere.

Both scientists and technicians need the guidance of dialectical thought. We hope that this teaching material can provide help and guidance for those advanced students who are interested in the subject.

I strongly feel that one of the essential aspects of teaching is to arouse the curiosity of the students in materialist dialectics. Probably, the most effective way of doing this is to discuss with them the salient features of the circumstances that led to the emergence of the epoch-making discoveries. In the case of a subject like relativity or Non-Euclidean geometry, such features appear all the more exciting; because, in the main, they have been carried out by men of the highest calibre (character) in physics; One is, therefore, tempted to stop time and again in order to see rather minutely as to how such and such developments actually came about.

As for the material contents of the book, there is hardly any pure philosophical subject in it. This book will not be concerned with such topics. Whenever necessary, the reader should consult a reference book of that kind.

Let us now give a brief description of some of the features of the four chapters in this book. Chapter 1 presents a detailed, self-contained discussion of the basic space-time concepts of special relativity, with emphasis on a through treatment of a variety of specific examples. A particularly important development was the discovery of the Lorentz transformation, a mathematical substitution that leaves Maxwell's equations invariant. It was the genius Einstein who told the physicists the precise meaning of that transformation.

Einstein's contribution was revolutionary because it was formally simple, and deep at the conceptual level. Aside from permitting observers in different state of motion to have different scales of distance and of time, Einstein demonstrated that the simultaneity of distant event would be observer or frame-dependent if one accepted the proposition that the speed of light can not be exceeded by any signaling device. He showed that once the notion of absolute time marks is dropped, two moving observers each can perceive the other's clock to be slow, and each can perceive the other's yardsticks to be contracted. The paradox was resolved by a profound modification of classical space and time concepts.

Poincaré and Lorentz were the most brilliant physicists at that time. Neither dared to take the decisive and revolutionary step to re-examine our concept of simultaneity, a concept perhaps inherited through a million years of evolution.

A very few years later Minkowski discovered the natural mathematical formulation of Einstein's new physics, the four-dimensional space time model, so the space-time of special relativity is sometimes referred to as Minkowsky space. It is a space in the mathematical sense of the word. It must not be supposed from this that space is really four-dimensional, or that time is really a form of space. The theory of relativity simply recognizes the fact that the properties of space and those of time are closely interwoven, and separate models of each can not be constructed.

Most students approaching the theory of relativity require an introduction to tensors, which provide the language of relativity, and these are dealt with in chapter 2.

Tensors are of great importance in connection with coordinate transformations. And since relativity is much concerned with coordinate transformations it is not surprising that tensors have been found to be the ideal mathematical tool for its study. Special relativity, in particular, is essentially concerned only with linear coordinate transformation, while general relativity retains no such restriction.

Chapter 3 treats Einstein's field equations and cosmology. The main purpose of this chapter is to establish the field equations of general relativity, which

couple the gravitational field with its source.

One of the most salient features of Einstein's equations is the extreme difficulty in solving them. Indeed, in the many years since their discovery, only a handful of exact solutions are known. The first exact solution was found by -- Karl Schwarzschild (German astronomer (1873-1916), and is known as schwarzschild solution, representing the gravitational field of spherically symmetric massive body. This solution forms the basis of our subsequent discussion, such as red shift, radar sounding and black holes. This chapter finishes with an introduction of Friedmann cosmological models.

Chapter 4 describes the discovery of Non-Euclidean geometry. The mystery of why Euclid's parallel postulate could not be proved remained unsolved for over two thousand years, until the discovery of Non-Euclidean geometry and its Euclidean models revealed the impossibility of any such proof. This discovery shattered the traditional conception of geometry as the true description of physical space. Albert Einstein stated that without this new conception of geometry, he would not have been able to develop the theory of relativity. Indeed, the overthrow of Euclidean geometry is the most important event in the history of science for the epistemologist.

Among the Non-Euclidean geometries Hyperbolic geometry, discovered by Russian Lobachevsky (1793-1856), is the most remarkable one. Lobachevsky studied mathematics at the university of Kasan, he took his degree in 1813 and remained in the university, first as assistant, and then as professor. In the latter position he lectured upon mathematics in all its branches and also upon physics and astronomy.

As early as 1815 Lobachevsky was working at parallels, between 1823 and 1825, he had turned his attention to a geometry independent of Euclid's hypothesis where two parallels to a given line can be drawn through a point and where the sum of the angle of a triangle is less than two right angles.

** For a full account of the development of spacetime conception, the reader is referred to General relativity, an Einstein centenary survey (1979) by Hawking, S.W. et al. Here We give an extract.

Now let us take up the interesting subject on the development of space-time conception.** The structure of space and time lies at the very foundation of both physical science and our perceptual experience of the world. They are concepts so fundamental that in every day life we do not question their properties. Yet modern science has discovered situations in which space and time can change their character so drastically that remarkable and unexpected phenomena occur. Many of these situations owe their appearance to recent development in astronomy. The possibility of the existence of black holes or a big bang origin of the universe have stimulated detailed investigations into the behavior of space, time and matter, when gravity becomes overwhelmingly strong. The results indicate that space-time itself may collapse out of existence under some circumstances.

The Newtonian space and time are absolute space and absolute time. This concept remained unchallenged until the development of electromagnetic theory in the nineteenth century, principally by Faraday and Maxwell. Maxwell showed that electromagnetic waves ought to propagate at a speed of 186000 miles per second, the velocity of light. But what did this mean? What was the velocity relative to? The concept of a luminiferous aether was introduced as a medium in which electromagnetic waves propagate. This was, in fact, a return to absolute space. However, a famous series of experiments by Michelson and Morley failed to detect any existence of aether. In 1905, Einstein proposed the special theory of relativity as an answer to the paradox. In this theory neither space nor time was absolute but they were just regarded as coordinates or labels on a four-dimensional space time continuum or manifold. The structure of space time was represented by the Minkowski metric, which determined the proper distance or proper time interval between two events.

In special relativity the space time metric is a fixed quantity. In 1915, Einstein arrived at the general theory of relativity. In this theory the metric of space time was no longer flat as in special relativity, but was curved or distorted by the matter and energy contained in space time. Four years later in 1919, an eclipse expedition confirmed that, as predicted by Einstein, light passing near the sun is deflected through a small angle by the gravitational field. The world woke up to the fact that it lived in a curved universe.

In the year 1929 the American astronomer Edwin Hubble

(1889-1953) announced some results concerning measurements made on the light coming from distant galaxies. An examination of the frequency spectrum of this far-off starlight revealed that the spectral lines were systematically displaced toward the colour red (low frequency end of the optical spectrum). Accordingly, physicists started to study cosmic dynamics by constructing relativistic mathematical models of universe.

Einstein's model differed from earlier static models, based on Newton's theory of gravitation, in one novel and fascinating way. The Einstein model universe is finite, but still the same everywhere, that is, it is a universe limited in size but without an edge! Such a monstrosity is clearly impossible with Newton's model of space and time. However, the curved space of general theory of relativity does admit this possibility. An example of such was based on an analogy with the two-dimensional surface of the surface sphere. The spherical surface is finite, but has no edge or boundary anywhere—a finite, unbounded space.

The idea of a closed finite and unbounded universe was certainly a strange idea of Einstein's. People often have a little difficulty in envisaging such an entity. A frequent question being what is "outside" a finite universe. This question is as meaningless to three-dimensional humans as the question of what is 'outside' the surface of a sphere—would be to a flat creature permanently restricted to live in the spherical surface. There can be no outside to the Einstein universe, because if there were an outside and inside, there would have to be a boundary between them. No such boundary exist in this mode. All points are equivalent to all others. None are near the 'center' or 'edge', there is no center or edge.

The first person to use the general theory of relativity to construct a range of mathematical models of an expanding universe was the Russian meteorologist Alexander Friedmann (1888-1925), who published his work unobtrusively in 1922. These models remain the basic theoretical framework for the discussion of nearly all modern cosmology. The vital feature of Friedmann's models is their assumption of spatial uniformity. Friedmann assumed an exact uniform distribution of matter, and then solved Einstein's equations of general relativity for such a matter distribution to find out how the geometry of space would change with time. Because of the uni-

formity, the only geometrical change which can occur is an over all change of scale, i.e. an expansion or contraction which is the same everywhere.

The idea of an infinite distribution of galaxies expanding is some times difficult for people to grasp. If the galaxies already fill all of space, what is there left for them to expand into? The reader should remember that the galaxies are not expanding through a fixed space, but are trapped in an expanding space. It is simply that the scale of all distance is everywhere increasing.

It should also be clear, from this analogy that the expansion of the universe is one of space itself, and must not be pictured as the migration of the galaxies out a pre-existing void.

Today, we have all heard of the space time geometry in Einstein's theory of relativity. In fact, the geometry of the space time continuum is so closely related to the Non-Euclidean geometries that some knowledge of it is an essential prerequisite for a proper understanding of relativistic cosmology. We must highly appraise Lobachevsky's work. It was Lobachevsky who openly challenged the Kantian doctrine of space as a subjective intuition. According to Immanuel Kant, the supreme European philosopher in the late eighteenth century and much of the nineteenth century, Euclidean space is inherent in the structure of our mind.

Today, according to Einstein, space and time are inseparable and the geometry of space-time is affected by matter, so that lightrays are indeed curved by the gravitational attraction of masses. Space is no longer conceived of as an empty Newtonian Box whose contours are unaffected by the rock put into it. The problem is much more complicated than Euclid or Lobachevsky ever imagined—neither of their geometries is adequate for our present conception of space. This does not diminish the historical importance of Non-Euclidean geometry. Einstein said: To this interpretation of geometry I attach great importance, for should I not have been acquainted with it, I never would have been able to develop the theory of relativity.

The general theory of relativity on which all studies of black holes and cosmological models depend, is only a theory. Its predictions have been reasonably

well checked only in gravitational fields in the solar system. Inside a black hole, gravity is billions of times stronger than this. No one knows how far the theory can be extended with any confidence, or which of its features might remain, - if a better theory were known. The theory of relativity is very beautiful and accepted by most physicists as the best description of gravity available. But all theories have their limits. The general theory of relativity actually predicts its own breakdown. It contains the essence of its own limitations. This is manifested in the occurrence of the so-called singularities. These regions are boundaries of space-time and the theory of relativity can not apply there. It follows that some new theory, a new model, is necessary. We may conclude that all of physics is not yet discovered. What this new theory will be like, can only be guessed. It may not even employ the old concepts of space and time at all. We must not forget that the criterion of practice can never, in the nature of things, either confirm or refute any human idea completely. The limits of approximation of our knowledge to objective absolute truth are historically conditional although we are approaching nearer to it.

My object in writing the book is to explore the way of improving mode of thinking. Of course, the extent to which I have succeeded in doing justice to this project can be gauged only by the reader. I, on my part, will be ever ready to receive, with gratitude, any suggestions that are intended to improve the warp and woof of this book.

Finally, I must thank all the people who have helped to make this publication possible: Associate Professor Li Zhensheng, the coauthor and my brother, for the contribution of his effective cooperation through all stages of this project; I also owe a considerable debt to my students; I earnestly appreciate the unfailing cooperation of my publishers, and the untiring efforts of the printing staff.

Nanjing, PRC, September 1988

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