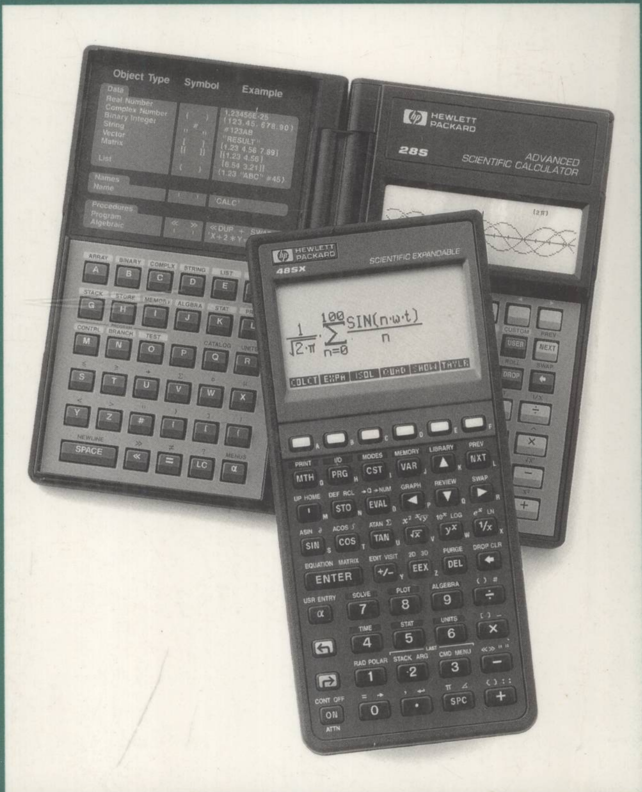
DISCOVERING CALCULUS

WITH THE

HP-28 AND THE HP-48



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DISCOVERING CALCULUS WITH THE HP-28 AND THE HP-48

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To the women in my life:
my daughter, Katherine,
my wife, Pam, and
my mother, Anne

R.T.S.

To my parents, Paul and Mosse;
for my children, Kelly and Greg;
with my wife, Jan

R.B.M.

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Preface

Calculus and Calculators

Over the last ten years, microcomputer technology has revolutionized business and industry. Similarly, the wide accessibility of computing power is changing the way in which mathematics is learned and applied. We would estimate that the HP-28S could perform 95 percent of the calculations on a typical calculus exam. However, the point of learning the calculus is not simply to generate correct answers to problems. Indeed, the purpose is to learn how to think mathematically. In this book, we show how the graphical, symbolic and numerical capabilities of the HP-28 and HP-48 families of calculators can help students think about and understand mathematics in new ways.

As we complete this book, the HP-28S is barely three years old. Within the last year, it has been joined by its cousin, the HP-48SX, and most recently by the HP-48S. These graphing calculators are incredibly powerful, each with a tremendous amount of built-in memory and a sophisticated programming language. Indeed, with the ability to perform symbolic and graphical as well as numerical manipulations, these machines might be more correctly referred to as hand-held computers. Used properly, the HP-28S/48SX/48S can be a valuable tool in exploring the concepts of calculus.

What you find in this book is the result of our experiences in the classroom as well as in computing in general. Our discussion does not focus on the HP-28S/48SX/48S. Rather, we concentrate our efforts on the main concepts of the calculus, using the calculator as a tool. We have therefore emphasized only those features of the HP-28S/48SX/48S that aid the study of calculus. There are many other features of these machines which we give only passing mention to or ignore altogether. For those interested in a more thorough discussion of the calculators' features, we suggest the excellent book HP-28 Insights by William Wickes (see the bibliography starting on page 267).

Using This Book

As its size alone should indicate, this book is certainly not a complete calculus text. The reader should have a standard calculus text as well, for although much of our discussion is self-contained, we make frequent reference to material found in such a text. Our choice of topics reflects those most often found in the first two semesters of calculus which we feel most benefit from the introduction of the HP-28S/48SX/48S. We also provide more realistic problems as well as the means of solving them.

You can use this book in a variety of ways. You will find, as our own students have, that you can sit down with this book and your calculator and learn calculus with the fresh approach of a new technology. Work through the book to gain that new perspective, use our exercises to supplement those of your standard calculus text, or – and this is our preferred choice – do both. No experience with graphing calculators is needed to use this text. We start from scratch in Chapter 1. You need only have the desire to learn some new mathematics using a slightly different approach.

We strongly recommend that you work carefully through Chapter 1 before going on to the later chapters. The material on graphing found there is prerequisite for almost everything that follows. This is particularly true if you are using the HP-28S, whose graphics are more difficult to deal with than the HP-48SX.

The HP-28S and the HP-48SX are in some ways very similar machines and we most often discuss their use making no distinction between the two. As far as the programming goes, the two machines are almost identical. In those few instances where there are differences, we have tried to explain them carefully and when there is any chance for confusion, we have discussed the two machines separately. The most noticeable difference is in the handling of graphics and so, in Chapter 1, we have provided easy-to-use programs which allow the HP-28S to mimic some of the graphics functions of the HP-48SX. We make no distinction at all between the HP-48SX and the HP-48S, since the two are identical, except for the expandability of the HP-48SX. For the purposes of this text, then, all references to the HP-48SX apply equally to the HP-48S.

We have tried to write programs that are as easy to follow as possible. By doing so, we sometimes sacrifice a certain level of computational efficiency or sophistication, but we hope that we have better enabled you to understand the mathematics. We have included many examples throughout the text, as well as a wide variety of exercises. Of particular note are the "Exploratory Exercises" found at the end of every section. These provide good experience in tackling some extended, open-ended problems. We have also provided numerous tips on using the HP-28S/48SX/48S more efficiently.

For your convenience, when we refer to a special key (or a "softkey" located in one of the many menus), we put that key in a box. For instance, [ENTER] indicates that you should press the ENTER key, rather than type the letters E-N-T-E-R. Likewise, [CRDIR] indicates that you should press the CRDIR softkey located in the Memory menu. In order to help you find the commands in the numerous menus, we have included a listing inside the front cover of the book (for the HP-28S) and inside the back cover (for the HP-48SX/48S).

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Contents

CHAPTER 1

Overview of the HP-28 and HP-48

1.1 Introduction

Calculus is a very broad and tremendously deep study with many and varied applications. As the name implies, it is filled with calculation. You will compute velocities, areas, volumes, etc. The skills that you learn in calculus are basic tools for studying (and yes, practicing) engineering, physics, chemistry, economics and many other diverse fields. But, calculus is more than just necessary background work for the sciences. It is a fascinating subject in its own right, where geometry and algebra come together into a powerful problem solver. The mathematical theory developed here will allow you to make connections between seemingly unrelated real world problems, ultimately leading to a deeper understanding of the world in which we live.

These are some pretty strong statements that we've made. Indeed, what we have described above is the ideal. Unfortunately, the ideal and the reality are not always quite the same. The often messy details of algebra and computation can sometimes obscure the calculus that's behind them. We cannot, nor would we want to, get rid of all the algebra and computation involved in studying calculus. Indeed, these things are necessary. However, we also don't want you to get so lost in the details that you miss the larger picture.

By giving you easy access to fast calculations and graphics, your HP-28S/48SX can help you to discover important relationships. Because of its speed and easy interface, you can ask those "What if I did this..." questions, without a large investment of time. In short, you can experiment. You can focus on the calculus questions, and leave many of the details to your HP-28S/48SX assistant.

As is the case throughout the book, most of what follows is applicable to users of both the HP-28S and the newer HP-48SX graphing supercalculators. Except for the handling of the graphics and the layout of keys and location of programs in subdirectories, there are few differences in the functionality of the two machines that affect their use here. In the instances where there are differences, we first give the HP-28S keystrokes and displays and make comments regarding the modifications needed for the HP-48SX. We also end the section with a set of notes for HP-48SX users.

ARITHMETIC ON THE HP-28S/48SX

The first features that you are likely to notice on the HP-28S/48SX are the Reverse Polish Notation (RPN) and the stack. When you first turned your machine on, you probably tried to add or multiply two numbers. If you tried to enter

$$3+5$$

the calculator beeped at you before you could even type the 5. Most calculators use algebraic notation, where expressions are entered in the same way as they are written (as above). This is not so for RPN machines such as the HP-28S/48SX. For instance, to compute 3+5, you would enter

3 [ENTER] 5 + OR 3 [SPACE] 5 +

To compute $3 - 5/21$, press

3 [ENTER] 5 [ENTER] 21 / −

At first, if you are used to algebraic calculators, this seems rather awkward, but with practice, RPN will become quite natural. In fact, you'll find in the examples below that for longer calculations, RPN has significant advantages over algebraic notation.

Next, try computing $4(2 + 3/\sqrt{11})$. Note that the square root symbol on the HP-28S is printed in red above the $-$ key, so that you need to press the red (shift) key followed by the $-$ key to get $\sqrt{x}$.

There are a number of ways to compute $4(2+3/\sqrt{11})$, depending on how much pencil-and-paper work you do first and how you choose to use the stack. One sequence is

3 [ENTER] 11 [ENTER] [$\sqrt{x}$] / 2 [ENTER] + 4 [ENTER] *

The result is 11.6181361349.

NOTE: Throughout the text we will denote multiplication by * and division by /. You should note that although the keys are marked by "×" and "÷," respectively, the operations are displayed as "*" and "/" on the screen. Further, we will denote exponentiation by ∧ (e.g., x^3 will correspond to "X ∧ 3"). This usage is consistent with the displays of both the HP-28S and the HP-48SX and the labeling of the HP-28S key. The corresponding HP-48SX key is labeled y^x.

You can think of the calculator's stack as a (almost endless) scratchpad on which values are stored. Each line of the stack is numbered, with the bottom line labeled line 1. Although you can see only 3 or 4 lines of the stack at any one time, there may be many, many numbers stored on lines which are not presently visible. For the moment, think of the stack as a tremendously long list of numbers. We will discuss the uses and manipulation of numbers on the stack shortly.

Try computing $6\sqrt{5/3} + 7\sqrt[3]{5/8}$. To do this on an algebraic calculator, you would need to do several side calculations, recording the results of each by storing them in a memory or by writing them down for use in completing the calculation. On the HP-28S/48SX, this can be done with great ease, seamlessly. For instance, we can break the calculation up into three parts:

(1) 5 [ENTER] 3 / [$\sqrt{x}$] 6 *

(2) 5 [ENTER] 8 / 1 [ENTER] 3 / ∧ 7 *

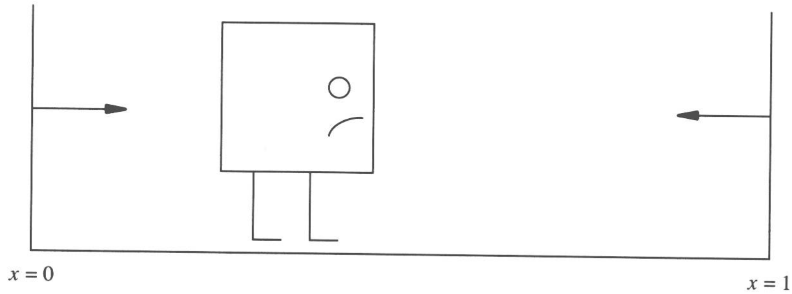
(3) Press + to add the results of the calculations in (1) and (2)

The result of this calculation is 13.7308825058.

The following examples will introduce you to some of the most commonly used features of the HP-28S/48SX, while exploring some interesting situations.

Example 1. The Stack

Consider the case of the trapped video game robot. A robot starts at one end of a corridor ($x = 0$) and moves toward the other end ($x = 1$). At the same time, the walls at the ends of the corridor begin closing in, moving at half the robot's speed.



The robot runs into a wall at $x = 2/3$ (why?) then immediately turns and goes the other way. Since the walls are now separated by a distance of 1/3, the next collision occurs after the robot has gone (2/3)(1/3) back to the left. The second collision, then, is at $x = 2/3 - 2/9$. The walls are now separated by a distance of 1/9. (Why?) The third collision will be at $x = 2/3 - 2/9 + 2/27$, and so on. Where exactly will the robot be trapped?

We have analyzed the problem enough to see the pattern of where the collisions occur. Let's use the HP-28S/48SX to crunch some numbers. First, compute 2/3 (press 2 [ENTER] 3 /). Then compute 2/9 (press 2 [ENTER] 9 /) and subtract the two values (press −). Then add 2/27 (press 2 [ENTER] 27 / +), and continue in this way to generate the table below.

Key Sequence	Result
2 [ENTER] 3 /	.666666666667
2 [ENTER] 9 / −	.444444444445
2 [ENTER] 27 / +	.518518518519
2 [ENTER] 81 / −	.493827160494
2 [ENTER] 243 / +	.502057613169
2 [ENTER] 729 / −	.499314128944

It looks like we're *homing in* on .5. We'll examine this idea of "homing in" (called a *limit*) very carefully in Chapter 2. We'll look further into limits involving sums in Chapter 6. For now, let's take .5 as an educated guess and rethink the problem somewhat. Is .5 a reasonable solution? The answer is "yes." Since the walls are traveling at the same speed, they will meet in the middle at $x = .5$, with the robot trapped between them. ■

You should notice several things about the preceding problem before going on to Example 2. We worked this problem in three stages: basic analysis, calculations to estimate the solution and an evaluation of the estimate. You should follow this process whenever possible. A numerical answer is of little value without understanding its meaning.

Example 2. The Solve Menu

In Example 1, we were able to guess what turned out to be the precise solution by computing several collision points and recognizing the pattern. We will often search for an answer by repeatedly computing values of a function. For instance, we know from algebra that for

$$f(x) = \frac{1}{x^2 + 120x - 22}$$

$f(x)$ gets steadily smaller as x gets larger (for $x > 1$). Let's find a "smallness threshold," e.g., find the smallest positive integer n such that $f(n) < .001$. We'll start by computing $f(2)$ and then we'll see how the HP-28S/48SX can make our task easier. The keystroke sequence

$$2 \ \boxed{x^2} \ 120 \ \boxed{\text{ENTER}} \ 2 * + 22 - \boxed{1/x}$$

gives us $f(2) = .0045045045$, which is not small enough.

It looks like it will take a lot of typing to generate all of the function values we need. Fortunately, the HP-28S/48SX has some features to minimize this work. We can put the expression for $f(x)$ onto the stack by enclosing the expression in single quote marks. The ' tells the HP-28S/48SX to delay execution of the commands. Type

' 1 / (X ∧ 2 + 120 * X − 22) ' [ENTER]

Note that the parentheses are *not* optional here. (Why not?)

Next, activate the Solve menu by pressing SOLV (or SOLVE on the HP-48SX). The top row of keys are now "soft keys" whose function is described by the labels appearing on the screen directly above them. The soft key labeled STEQ (for "store equation") will take the expression on line 1 of the stack and store it for later use in a variable named EQ. The soft key labeled RCEQ (for "recall equation" on the HP-28S; use ↪ STEQ on the HP-48SX) will recall the expression stored in EQ and return it to line 1 of the stack. Since we presently want to store our function, we press STEQ . Notice that the expression has now been removed from the stack.

Now, activate the Solver menu (press the SOLVR soft key). To evaluate $f(2)$, first set $x = 2$ by pressing 2 followed by the soft key X . (Note that pressing the usual X key will not have the same effect.) The top of the screen should show X: 2 Now press the soft key EXPR= . The value $f(2) = .0045045045$ should be returned to the stack. Try computing $f(3)$: press 3 X EXPR= and we get $f(3) = .00288184$. Continue by computing $f(4)$, $f(5)$, and so on, until you have $f(n) < .001$. You should get $f(8) = .00099800 < .001$. The entire sequence of calculations follows.

Key Sequence	Result (to 8 places)
2 X EXPR=	.00450450
3 X EXPR=	.00288184
4 X EXPR=	.00210970
5 X EXPR=	.00165837
6 X EXPR=	.00136240
7 X EXPR=	.00115340
8 X EXPR=	.00099800

■

We should mention at this point how to correct an expression that has been mistyped. Rather than retype the entire expression, you should use the EDIT command, built into the HP-28S/48SX. Suppose that, instead of 'X∧2+120*X−22', you had accidently typed

' X ∧ 2 + 122 * X − 22'

If this expression is on line 1 of the stack, you can edit it by pressing EDIT (located above the ENTER key on the HP-28S and above the +/− key on the HP-48SX). The four arrow keys (in the top row of keys on the HP-28S) will now move a blinking cursor through the expression. If you are using an HP-28S, pressing any key replaces the current character with the one pressed. (This is called *replace mode.*) On The HP-48SX, you must first press the soft key [INS] to enter replace mode. Try this now by moving the cursor over to the second 2 in 122 and pressing 0. Press [ENTER] and the original expression appears corrected on line 1 of the stack.

Suppose, instead, that the coefficient of X should be 1200 instead of 120. Press [EDIT] and move the cursor to the spot where you want to insert the extra 0. On the HP-28S, first press the [INS] key (in the top row of keys; this puts the editor in insert mode) and then press 0 to insert the extra 0. Again, pressing [ENTER] returns the edited expression to line 1 of the stack.

Finally, suppose that you had wanted x^3 instead of x^2. Move the cursor over to the 2 and press the [DEL] key to delete the 2 and then replace it with a 3.

NOTE: On the HP-48SX, the editor is initially automatically in *insert mode* and you must press the [INS] soft key to switch back and forth between insert and replace mode.

As you edit more and more complicated expressions, you will find the need to switch back and forth between insert and replace mode while in the process of editing a single expression. This will become routine in a short time.

Example 3. The Stack Menu

At this point, if you've been following along with the calculations, your stack should contain a number of entries, not all of which are visible. Before clearing the stack, let's experiment with some of the commands which allow you to manipulate items on the stack. Activate the Stack menu by pressing [STACK] on the HP-28S (it is a red label located above the G; on the HP-48SX, press [PRG] and then [STK]). Press [DUP] [located on the second page of the HP-48SX Stack menu (press [NEXT] to get the next page)]: the .001 on line 1 is duplicated. Now, press [DROP] (or the [←] key on the HP-48SX; in both cases this is not a soft key): the entry on line 1 of the stack is removed.

In the Stack menu, [DUP2] and [DROP2] work like [DUP] and [DROP] , but operate on 2 lines of the stack at the same time. Try these now. If you haven't already done so, go to the second page of the Stack menu by pressing [NEXT] . Now, enter 3 [DUPN] Although you may not be able to see it yet, something did happen. Press [VIEW↑] and the screen displays lines 2, 3, and 4. Press [VIEW↑] twice more and note that the entries on lines 4-6 are duplicates of those on lines 1-3. (On the HP-48SX, the display will show the first 4 lines of the stack and you use the up/down arrow keys to move around the stack.) That is, 3 [DUPN] copied the first 3 stack lines onto the next 3 lines.

Enter 3 [ROLLD] . The value on line 1 is rolled to line 3, with the values in lines 2 and 3 rolled down to lines 1 and 2, respectively. Try 3 [ROLL] . The value on line 3 is rolled to line 1, with the values on lines 1 and 2 rolled up. Finally, press [SWAP] The values on lines 1 and 2 are interchanged. We encourage you to discover the functions of the other stack commands on your own. When you are ready to move on, press [CLEAR] (or [CLR] on the HP-48SX) to clear the entire stack.

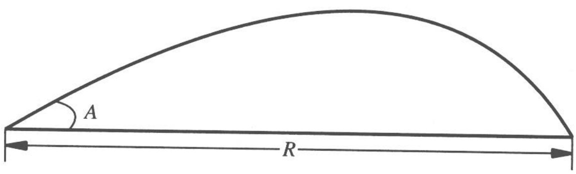
We summarize below the most frequently used stack commands. (The entries in parentheses indicate the corresponding HP-48SX commands.)

Command	Result
STACK (STK)	Activate the stack menu
DUP	Copy line 1
n DUPN	Copy first n lines
DROP	Delete line 1
n DROPN	Delete first n lines
VIEW↑ (△)	Move viewing window up
VIEW↓ (▽)	Move viewing window down
n ROLL	Move line n to line 1
n ROLLD	Move line 1 to line n
SWAP	Swap lines 1 and 2

■

In Example 2, we used the Solver to simplify repeated function evaluation. We now illustrate the use of the Solver for a more complicated example.

Example 4. Parameters and the Solver Menu



Suppose that a ball is thrown from ground level with initial speed S ft/sec and initial angle A above the horizontal. If air effects (such as lift and drag) are ignored,