calculus and its applications

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preface

Students in the biological, social, and management sciences need something different from a condensed version of a traditional calculus course in which applications to physics and engineering are replaced by examples in biology and economics. In teaching a calculus course for such students at the University of Maryland during the past six years, we have found that the best text for these students would be one which:

—Contains as many relevant applications as possible;
—Is heavily stocked with exercises;
—Approaches calculus intuitively;
—Assumes a minimum of prerequisite knowledge in order to accommodate students of diverse backgrounds,
—But avoids spending one or more chapters on preliminaries before getting to the derivative;
—Is organized so that a student meets all the essential ideas of calculus in a first semester,
—And yet contains sufficient material for those students who require a one-year course.

With these characteristics in mind, we have written the present text.

The process of developing the material for this book has required several years. We met with faculty in various disciplines to find out which calculus topics are most appropriate for their students and to learn about interesting applications of these topics. We prepared many applications as supplementary problem sheets and distributed them to our classes, later modifying them based on student reactions. Most of the text has undergone this process of testing and revision. A prepublication edition which included all twelve chapters was class tested during the past year. A few minor revisions were made as a result of this experience. One of the authors, sixteen other instructors, and over fifteen hundred students were involved. We were pleased that the response was enthusiastic.

APPLICATIONS We have included as many applications of calculus as possible. The reader will find that many of the applications are integrated into the introduction of new material. This is because of our conviction that students appreciate mathematics best in concrete situations. To be included, an application had to pass two tests: Is it realistic? Can it be explained using a minimum of specialized terms to someone who has had no previous technical acquaintance with the subject? For the reader's convenience we have included an index of applications at the back of the book.

EXERCISES The nearly 2000 exercises comprise more than one-quarter of the text. They are divided into three categories.

1. The exercises at the ends of sections include both drill calculations and applied problems. They are usually arranged in the order in which the text proceeds, so that homework assignments may easily be made after only part of a section is discussed.
2. Supplementary exercises at the end of each chapter expand the other exercise sets and provide cumulative exercises that require skills from earlier chapters.
3. A Chapter Test reviews the important new concepts in the chapter. Also, just before each test, there is a Chapter Checklist that a student may use for review before taking the test.

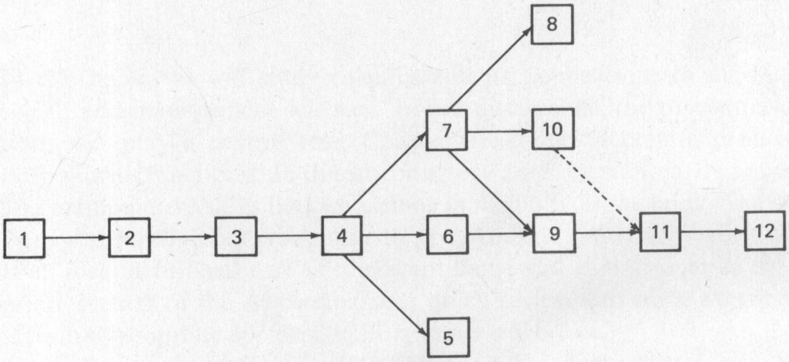
Answers to all odd-numbered exercises and complete solutions to chapter tests are given at the end of the text.

EXAMPLES AND ILLUSTRATIONS There are an unusually large number of examples in this text—more than an instructor can normally present. They are designed to be read by the students. Most algebraic manipulations are explicitly given, particularly in the first half of the text. We have tried to present only one new idea in each example. In order to better appeal to our students' geometric intuition, we have also included more drawings than are found in the average text.

PREREQUISITES The prerequisites have been deliberately kept to an absolute minimum in order to accommodate students' diverse backgrounds. We have summarized them in the Note to the Student. If any of the prerequisites are unfamiliar (or, more likely, forgotten), the student should turn to the Appendix, where a quick review may be found.

APPROACH Our approach to calculus is intuitive. Rather than introduce the traditional δ-ε approach, we have relied almost exclusively on graphical analysis. We have deliberately been informal with regard to "proofs." Our purpose here is to make formulas and theorems reasonable and clear, using arguments that can be made rigorous if desired. The basic guide in choosing explanations has been: Do they work for the students? We have used our students' reactions to refine our explanations over the years.

ORGANIZATION OF THE TEXT We have developed calculus in a spiral fashion, returning to most topics more than once. Some topics and many applications may be omitted without loss of continuity. Entire sections with this property are starred in the table of contents. As a result, an instructor of a one-quarter or one-semester course has considerable freedom in the selection of topics. It is even possible to emphasize applications to a single discipline (such as business) and thereby use the book for a specialized calculus course. There is a certain amount of flexibility in the order in which chapters can be covered, as indicated by the following table of chapter dependence.



INSTRUCTOR'S MANUAL An Instructor's Manual is available for use with this text. The manual contains suggestions for presenting each chapter, additional applications and illustrations, sample tests, and answers to the even-numbered exercises.

ACKNOWLEDGMENTS Many people have assisted us in writing this book. We would like to express our appreciation to the following reviewers for their critical evaluation of the manuscript at various stages: Russell Lee, Allan Hancock College; Donald Hight, Kansas State College of Pittsburg; Ronald Rose, American River College; W. R. Wilson, Central Piedmont Community College; Bruce Swenson, Foothill College; Samuel Jasper, Ohio University; Carl David Minda, University of Cincinnati; H. Keith Stumpff, Central Missouri State University; Claude Schochet, Wayne State University; and James E. Huneycutt, North Carolina State University. Especially, we wish to thank our students and colleagues at the University of Maryland. Their constant advice over the years has been most appreciated and has certainly been of great help in preparing this book. We should also like to thank David Harrington for his incisive critique, Holly Andrews and Pat Berg for their magnificent job of typing the manuscript, and the staff of Prentice-Hall, Inc., especially Edward Lugenbeel, Zita de Schauensee, and Dudley Kay, for their patient editorial assistance and advice.

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note to the student

In this book, we will study calculus and its applications to the biological, social, and management sciences. In our discussions, the properties of functions will play a central role. Chapter 1 assumes a certain proficiency in dealing with functions. In this introductory note, we summarize the concepts and manipulative skills that we assume in the body of the book. If everything discussed in this note is familiar to you, you are ready to begin Chapter 1. If an idea mentioned here is somewhat hazy, you should refer to the appropriate section of the Appendix for a quick review and some exercises. (The section appropriate for each skill is indicated below.)

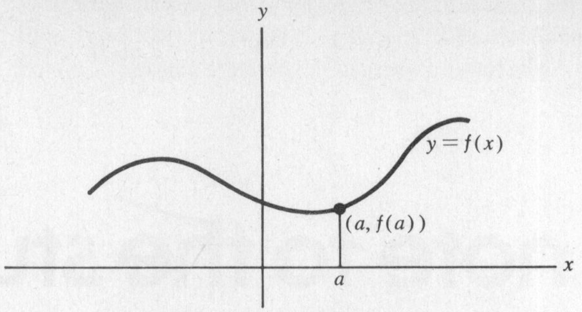
1. *The concept of a function* (Section A-I) A *function* f is a rule that associates to each value of a variable x a number $f(x)$, the *value of the function f at x*. As far as this book is concerned, a very simple concept of a function is sufficient. Namely, a function is a formula (such as $2x + 3$, $x^3 - 3x$, $1/x$), which can be evaluated at a given number x.

2. *Evaluating functions* (Section A-I) We assume the ability to evaluate a function at a given value of x. For example, if $f(x) = x^2$, then $f(0) = 0^2 = 0$ and $f(-3) = (-3)^2 = 9$. On occasion, it will be necessary to evaluate a function when x equals an algebraic expression. For example, if $f(x) = x^2$, then $f(a + 1) = (a + 1)^2 = a^2 + 2a + 1$ and $f(a + h) = (a + h)^2 = a^2 + 2ah + h^2$.

3. *Manipulating algebraic expressions* (Section A-I) We assume that you can perform algebraic calculations, such as adding, subtracting, multiplying, and dividing algebraic expressions.

4. *Graphing functions* (Section A-II) For our purposes, the most important feature about a function $f(x)$ is its *graph*, obtained by graphing, for each value a of x, the point in the x-y plane whose coordinates are $(a, f(a))$. Thus, for example, if $f(x) = x^2$, then $(0, 0)$, $(-3, 9)$, (a, a^2) (any a) are all

on the graph. The graph of any $f(x)$ is a curve (see the figure). For each number a on the x-axis, the height of the curve equals $f(a)$, the value of the



function at a. We assume that you can graph a function $f(x)$ by plotting points on the graph and drawing a smooth curve through the plotted points. Especially, we require the fact that the graph of a linear function (i.e., a function of the form $mx + b$) is a straight line.

5. *Solving equations* (Section A-III) Throughout the text, we need to solve equations such as $x^2 - 2x - 15 = 0$. The usual method will involve factoring. Occasionally, the quadratic formula will be needed.

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the derivative

1

Often it is possible to give a succinct and revealing description of a situation by drawing a graph. For example, Fig. 1-1 describes the amount of money

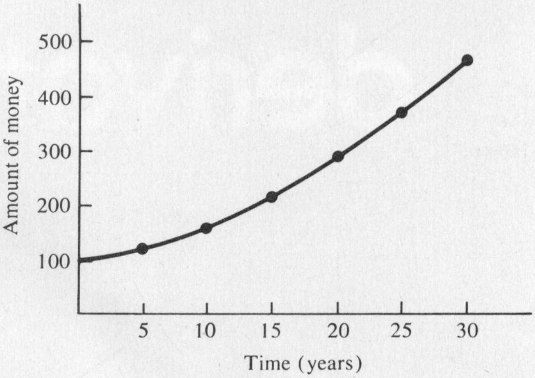

Figure 1-1

in a bank account drawing 5% interest, compounded daily. The graph shows that as time passes, the amount of money in the account grows. In Fig. 1-2 we have drawn a graph that depicts the weekly sales of a breakfast cereal at

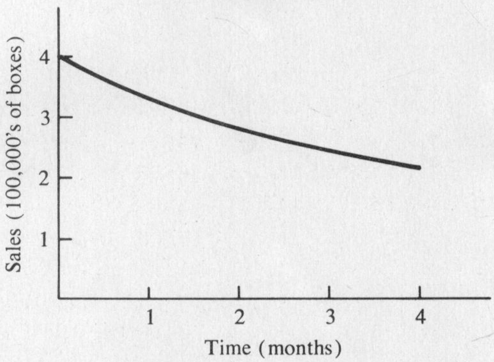

Figure 1-2

various times after advertising has ceased. The graph shows that the longer the time since the last advertisement, the fewer the sales. Figure 1-3 shows the size of a bacteria culture at various times. The culture grows larger as time passes. But there is a maximum size that the culture cannot exceed. This maximum size reflects the restrictions imposed by food, space, and

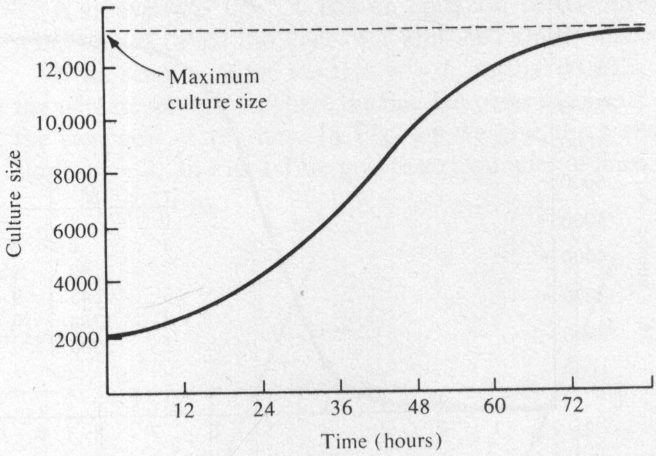

Figure 1-3

similar factors. The graph in Fig 1-4 describes the decay of the radioactive isotope iodine 131. As time passes, less and less of the original radioactive iodine remains.

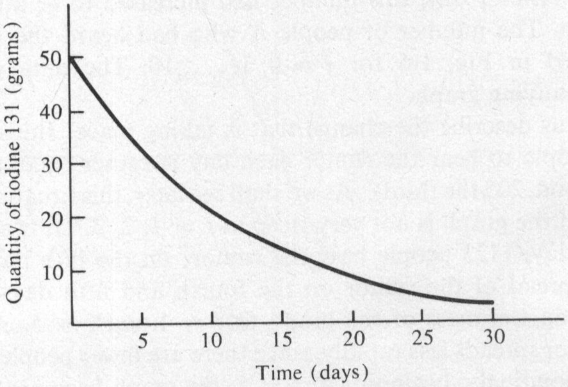

Figure 1-4

Each of the graphs in Figs. 1-1 to 1-4 describes a change that is taking place. The amount of money in the bank is changing, as are the sales of cereal, the size of the bacteria culture, and the amount of the iodine. Calculus provides mathematical tools to study each of these changes in a quantitative way. In this chapter we introduce the notion of a *derivative* to measure the steepness of a graph at a given point. This notion will provide us with a measure of how fast a graph is changing at each point on it. In order to explain what is meant by "how fast the graph is changing," consider a concrete example.

In Fig. 1-5 we have drawn a graph that describes the spread of a rumor*

* The graph used here is derived from a mathematical model used by sociologists. See Section 5-3 and J. Coleman, *An Introduction to Mathematical Sociology*, Collier-Macmillan Ltd., London, 1964, p. 43.