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Analysis and Design of Hybrid Systems 2006

A Proceedings Volume from the 2nd IFAC Conference,
Alghero, Italy,
7-9 June 2006

Edited by
CHRISTOS CASSANDRAS
ALESSANDRO GIUA
CARLA SEATZU
JANAN ZAYTOON

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ANALYSIS AND DESIGN OF HYBRID SYSTEMS 2006

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Published for the
International Federation of Automatic Control
By
ELSEVIER LTD

Elsevier
The Boulevard, Langford Lane, Kidlington, Oxford OX5 1GB, UK
30 Corporate Drive, Suite 400, Burlington, MA 01803, USA

First edition 2006

British Library Cataloguing in Publication Data
A catalogue record for this book is available from the British Library

Library of Congress Cataloging-in-Publication Data
A catalog record for this book is available from the Library of Congress

ISBN–13: 978-0-08-044613-4
ISBN–10: 0-08-044613-2

Printed and bound in Great Britain
06 07 08 09 10   10 9 8 7 6 5 4 3 2 1

ADHS'06

2nd IFAC Conference on Analysis and Design of Hybrid Systems (Alghero, Italy) - June 7-9, 2006

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Preface

This volume contains the proceedings of ADHS'06: the 2nd IFAC Conference on Analysis and Design of Hybrid Systems, organized in Alghero (Italy) on June 7-9, 2006.

ADHS is a series of triennial meetings that aims to bring together researchers and practitioners with background in control and computer science in order to provide a survey of the advances in the field of hybrid systems and of their ability to take up the challenge of analysis, design and verification of efficient and reliable control systems. ADHS'06 is the second Conference of this series after ADHS'03 in Saint Malo. The ADHS series follows the successful conference series on the Automation De Processus Mixed / Automation of Mixed Processes (ADPM'92 in Paris, ADPM'94 in Brussels, ADPM'98 in Reims, ADPM'2000 in Dortmund). The continuity between the ADHS conferences is guaranteed by the IFAC Technical Committee on Discrete Event and Hybrid Systems (TC 1.3) that is the main technical sponsor of the series.

We express our gratitude to the authors of the papers submitted to ADHS'06. Out of 96 submissions, 65 papers were selected after a careful reviewing process during which each paper received, on the average, three reviews. The high quality of the program is due to the authors of the submitted papers, to the members of the International Program Committee and to the referees who provided accurate and extensive reports.

Among the 65 papers accepted and published in this volume, 52 are coming from Europe, 7.5 from US/Canada, 2.5 from Latin America, 2 from Japan, 0.5 from Egypt, 0.5 from Australia.

Three distinguished speakers have been invited to give a one hour plenary talk during the conference. They are: Vadim I. Utkin, whose talk is titled *"Chattering Problem for Sliding Mode Control";* Alberto Sangiovanni-Vincentelli, whose talk is titled *"System Theory Approaches for Embedded Systems";* Claude Iung, whose talk is titled *"Optimal Control in Hybrid Systems"*. The invited speakers have contributed to the proceedings with an extended abstract of their talk.

Some of the papers presented at the conference have been submitted as part of invited sessions. All of them went through the normal review procedure. We are grateful to the organizers of these sessions that have timely put into focus interesting new research topics. They are: E. De Santis organizer of a session on *"Structural Analysis and Approximation of Hybrid Systems"*; S. Engell and O. Stursberg organizers of a session on *"Controller Design Based on Hybrid Models of Industrial Plants"*; A. Bemporad and F. Lamnabhi-Lagarrigue organizers of a session on *"Applications of Hybrid Control"*.

A special thank goes to C.G. Cassandras and P. Mosterman, organizers of an invited session on *"Hybrid Simulation Tools: Principles, Challenges and Applications"* that was also complemented by an interactive tool presentation. This session was composed by five presentations whose speakers have also contributed to the proceedings with an extended abstract of their talk.

We are grateful to the members of the Organizing Committee who took care of all important logistic and administrative details. In particular, we thank Maria Paola Cabasino for her help in preparing these proceedings. This conference benefits of support from the *Università di Cagliari*, from the *Comune di Alghero* and from *Akhela s.r.l.*

The Editors
C.G. Cassandras, A. Giua, C. Seatzu, J. Zaytoon

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CHATTERING PROBLEM IN SLIDING MODE CONTROL SYSTEMS

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ABSTRACT

In practical applications of sliding mode control, engineers may experience undesirable phenomenon of oscillations having finite frequency and amplitude, which is known as 'chattering'. At the first stage of sliding mode control theory development the chattering was the main obstacle for its implementation. Chattering is a harmful phenomenon because it leads to low control accuracy, high wear of moving mechanical parts, and high heat losses in power circuits. There are two reasons which can lead to chattering.

- The chattering can be caused by fast dynamics which were neglected in the ideal model. These 'unmodeled' dynamics with small time constants are usually disregarded in models of servomechanisms, sensors and data processors.
- The second reason of chattering is utilization of digital controllers with finite sampling rate, which causes so called 'discretization chatter'. Theoretically the ideal sliding mode implies infinite switching frequency. Since the control is constant within a sampling interval, switching frequency can not exceed that of sampling, which lead to chattering as well.

Mechanism of chattering generating is demonstrated for control of inverted an pendulum with unmodeled actuator dynamics.

The efficient recipe for chattering suppression is use of asymptotic observers. The main idea of using an asymptotic observer to prevent chattering is to generate ideal sliding mode in the auxiliary loop including the observer. In the observer loop, sliding mode is generated in the control software; therefore, any unmodeled dynamics which cause chattering can be excluded.

As follows from analysis, based on the describing function method, the amplitude of chattering is proportional to magnitude of discontinuous control. Therefore the methods of chattering suppression can be developed such that the magnitude is decreased properly holding the establishment of sliding mode. First option is to decrease the magnitude along with the system states. The second one implies that the magnitude is the function of an equivalent control derived by a low-pass filter u_{eq}. The method can be applied for the plants subjected to unknown disturbances.

"Discretization chattering" in discrete-time systems is caused by discontinuities in control. Increasing a sampling frequency to decrease the chattering amplitude seems unjustified, since using a computer is adequate to control system dynamics if a sampling frequency corresponds to average, slow system motion rather than to a high frequency component.

First the definition of sliding mode is introduced embracing both discrete- and continuous-time systems. Then free of chattering design methodology is proposed for discrete-time systems. The fundamental difference is that the control should be a continuous function of the state.

Experimental results for sliding mode control of inductions motors with observers are discussed. *Copyright © 2006 IFAC.*

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CHALLENGES AND OPPORTUNITIES FOR SYSTEM THEORY IN EMBEDDED CONTROLLER DESIGN

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Abstract: Embedded controllers are essential in today electronic systems to assure that the behaviour of complex systems as cars, airplanes, trains, building security management systems, is compliant to strict safety constraints. I will review the evolution of embedded systems and the challenges that must be faced in their design. I will also present methodologies aimed at simplifying and speeding the design process. The role of hybrid systems in the development of embedded controllers will be outlined. Future applications such as wireless sensor networks in an industrial plant will also be presented. *Copyright © 2006 IFAC.*

Keywords: Embedded Systems, Systems Design, Systems Methodology, Control Applications, Distributed Control.

EXTENDED ABSTRACT

The ability of integrating an exponentially increasing number of transistors within a chip, the ever-expanding use of electronic embedded systems to control increasingly many aspects of the "real world", and the trend to interconnect more and more such systems (often from different manufacturers) into a global network, are creating a nightmarish scenario for embedded system designers. Complexity and scope are exploding into the three inter-related but independently growing directions mentioned above, while teams are even shrinking in size to further reduce costs. In this scenario the three challenges that are taking center stage are:

- *Heterogeneity and Complexity of the Hardware Platform.* The trends mentioned above result in exponential complexity growth of the features that can be implemented in hardware. The integration capabilities make it possible to build real complex system on a chip including analog and RF components. The decision of what to place on a chip is no longer dictated by the amount of circuitry that can be placed on the chip but by reliability, yield and ultimately cost (it is well known that analog and RF components force to use more conservative manufacturing lines with more processing steps than pure digital ICs). Even if manufacturing concerns suggest to implement hardware in separate chips, the resulting package may still be very small given the advances in packaging technology yielding the concept of System-in-Package (SiP). Pure digital chips are also featuring an increasing number of components. Design time, cost and manufacturing unpredictability for deep submicron technology make the use of custom hardware implementations appealing only for products that are addressing a very large market and for experienced and financially rich companies. Even for these companies, the present design methodologies are not yielding the necessary productivity forcing them to increase beyond reason the size of design and verification teams. These IC companies (for example Intel, AMD and TI) are looking increasingly to system design methods to allow them to assemble large chips out of pre-designed components and to reduce validation costs. In this context, the adoption of design models above RTL and of communication mechanism among components with guaranteed properties and standard interfaces is only a matter of time.

- *Embedded Software Complexity.* Given the cost and risks associated to developing hardware solutions, an increasing number of companies is selecting hardware platforms that can be customized by reconfiguration and/or by software programmability. In particular, software is taking the lion's share of the implementation budgets and cost. In cell phones, more than 1 Million lines of code is standard today, while in automobiles the estimated number of lines by 2010 is 100 Millions. The number of lines of source code of embedded software required for defense avionics systems is also growing exponentially. However, as this happens, the complexity explosion of the software component causes serious concerns for the final quality of the products and the productivity of the engineering forces. In transportation, the productivity of embedded software writers using the traditional methods of software development ranges in the few tens of lines per day. The reasons for such a low productivity are in the time needed for verification of the system and long redesign cycles that come from the need of developing full system prototypes for the lack of appropriate virtual engineering methods and tools for embedded software. Embedded software is substantially different from traditional software for commercial and corporate applications: by virtue of being embedded in a surrounding system, the software must be able to continuously react to stimuli in the desired way, i.e., within bounds on timing, power consumed and cost. Verifying the correctness of the system requires that the model of the software be transformed to include information that involve physical quantities to retain only what is relevant to the task at hand. In traditional software systems, the abstraction process leaves out all the physical aspects of the systems as only the functional aspects of the code matter.

- *Integration Complexity.* A standard technique to deal with complexity is decomposing "top-down" the system into subsystems. This approach, which has been customarily adopted by the semiconductor industry for years, has limitation as a designer or a group of designers has to fully comprehend the entire system and to partition appropriately its various parts, a difficult task given the enormous complexity of today's systems. Hence, the future is one of developing systems by *composing* pieces that all or in part have already been pre-designed or designed independently by other design groups or even companies. This has been done routinely in vertical design chains for example in the transportation vertical, albeit in a heuristic and ad hoc way. The resulting lack of an overall understanding of the interplay of the sub-systems and of the difficulties encountered in integrating very complex parts causes system integration to become a nightmare in the system industry. For example, Jurgen Hubbert, then in charge of the Mercedes-Benz passenger car division, publicly stated in 2003: "*The industry is fighting to solve problems that are coming from electronics and companies that introduce new technologies face additional risks. We have experienced blackouts on our cockpit management and navigation command system and there have been problems with telephone connections and seat heating.*"

I believe that in today's environment this state is the rule for the leading system OEMs let them operate in the transportation domain, in multimedia systems, in communication, rather than the exception. The source of these problems is clearly the increased complexity but also the difficulty of the OEMs in managing the integration and maintenance process with subsystems that come from different suppliers who use different design methods, different software architecture, different hardware platforms, different (and often proprietary) Real-Time Operating Systems. Therefore, there is a need for standards in the software and hardware domains that will allow plug-and-play of subsystems and their implementation while the competitive advantage of an OEM will increasingly reside on novel and compelling functionalities.

I will present a methodology to cope with some of these problems and that can use hybrid system modeling. I will review how this methodology can be applied to the design of embedded controllers for the automotive industry. Finally I will present the application of the methodology and of hybrid systems to the design of wireless sensor networks in an industrial environment.

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OPTIMAL CONTROL IN HYBRID SYSTEMS

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Abstract: Necessary conditions for optimality have been established for hybrid systems. Unfortunately, these conditions lead to multi-point boundary value problems and they do not prevent from the combinatorial explosion when no constraints are given on the transitions. Different approaches have been proposed to approximate the solution, such as relaxed dynamic programming, non linear programming and sensitivity functions. *Copyright* © 2006 IFAC

Keywords: Hybrid systems, optimal control, sensitivity functions

EXTENDED ABSTRACT

In hybrid systems context, the necessary conditions for optimal control are now well known. These conditions mix discrete and continuous classical necessary conditions on the optimal control. The discrete dynamic involves dynamic programming methods whereas between the a priori unknown discrete values of time, optimization of the continuous dynamic is performed using the maximum principle (MP) or Hamilton Jacobi Bellmann equations(HJB). At the switching instants, a set of boundary tranversality necessary conditions ensure a global optimization of the hybrid system.

These theoretical conditions were applied to minimum time problem and to linear quadratic optimization.

But it is practically very hard to perform such an optimization. The major raison is that discrete dynamic requires evaluating the optimal cost along all branches of the tree of all possible discrete trajectories.

Dynamic programming is then used, but the duration between two switchings and the continuous optimization procedure make the task really hard. This makes the complexity increasing and only problems with a poor coupling between continuous and discrete parts can be reasonably solved.

Nowadays, it seems obvious that only approximated solutions can be found. Various schemes have been imagined.

Recent works have proposed to solve optimal switching problems by using a fixed switching schedule. By switched systems we mean a class of hybrid dynamical systems consisting of a family of continuous (or discrete) time subsystems and a rule (to be determined) that governs the switching between them.

The optimization consists then in determining the optimal switching instants and the optimal continuous control assuming the number of switchings and the order of active subsystems already given.Then a nonlinear search method is used to determine the optimal solution.after the calculus of the derivatives of the value function with respect to the switching instants.

Relaxed Dynamic programming : a relaxed procedure based on upper and lower bounds of the optimal cost was recently introduced. It proved to give good results for piece-wise affine systems and to obtain a suboptimal state feedback solution in the case of a quadratic criteria

Algorithms based on the maximum principle for both multiple controlled and autonomous switchings with fixed schedule have been proposed. The algorithms use the transversality conditions at switching instants. Then, the authors develop a combinational search in order to determine the optimal switching schedule

Interesting results on state or output feedback have been given with the regions of the state space where an optimal mode switch should occur.

In complement of all the methods resulting from the resolution of the necessary conditions of optimality, we propose to use a multiple-phase multiple-shooting formulation which enables the use of standard constraint nonlinear programming methods. This formulation is applied to hybrid systems with autonomous and controlled switchings and seems to be of interest in practice due to the simplicity of implementation.

Sensitivity analysis is the key point of all the methods based on non linear programming. It can also be used to determine limit cycles and the optimal strategy to reach them.

All these items are discussed in the plenary session.

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CONVERGENT DESIGN OF SWITCHED LINEAR SYSTEMS

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Abstract: This paper deals with the design of switching rules for switched linear systems with inputs, in such a way that the resulting closed-loop system is exponentially convergent. Two types of switching rules are addressed, that is state-based and observer-based rules. The developed theory is illustrated by two examples. *Copyright © 2006 IFAC*

Keywords: Convergent systems, switched systems, continuous time systems, exponential stability, performance evaluation, observer.

1. INTRODUCTION

A switched linear system is a hybrid/nonlinear system which consists of several linear subsystems and a switching rule that decides which of the subsystems is active at each moment in time. These systems have been a subject of growing interest in the last decades, see e.g. (Liberzon and Morse, 1999; DeCarlo *et al.*, 2000) and references therein. Because of the combination of multiple linear systems/controllers, a well-tuned switched linear system can achieve better performance then a single linear system, or can achieve certain control goals that cannot be realized by linear systems (Morse, 1996; Narendra and Balakrishnan, 1997; Feuer *et al.*, 1997).

Besides these extended possibilities that switched linear systems have with respect to linear systems, the design of such a switched system also brings along difficulties. For example, if all the linear subsystems of a switched system are stable, this does not automatically guarantee the stability of that switched system. A good example of this apparent contradiction is given in (Branicky, 1998). Another property that a linear time invariant (LTI) system with asymptotically stable homogeneous part has, but is not natural for a nonlinear/hybrid system, is that any solution of an LTI system with a bounded input converges to a unique solution that depends only on the input. Nonlinear/hybrid system that *do* possess this property are referred to as convergent. Solutions of convergent system "forget" their initial conditions and after some transient depend only on the system input, which can be a command or reference signal.

Convergency of nonlinear/hybrid systems is an interesting property, since it results in a limit solution that is independent of the initial conditions of the system. This is useful in for example synchronization problems (Pogromsky *et al.*, 2002). Another possible area of interest is the performance analysis of nonlinear systems. For general nonlinear systems simulation-based analysis is quite impossible, since all possible initial conditions need to be evaluated in order to obtain a reliable analysis. For a convergent system, however, this problem does not exist, since all initial conditions lead to the same limit solution. Therefore simulation can be used to analyse and optimize performance of convergent systems. This