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SESSION 1

Multisensor Fusion, Tracking, and Resource Management

An IMM architecture for track fusion

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ABSTRACT

A numeric solution for the fusion of multiple tracks produced from an arbitrary number of asynchronous measurements has been recently developed. This track fusion solution is a weighted sum of the values associated with the previous fused estimate and the multiple individual estimates. This Optimal Asynchronous Track Fusion Algorithm (OATFA) has properties that are identical to the Kalman filter. However, the deficiencies of the Kalman filter when tracking maneuvering targets are also exhibited by the OATFA but can be overcome with the use of the Interacting Multiple Model (IMM) algorithm. Consequently, it should be possible to replace the Kalman filter commonly employed in a conventional IMM algorithm with the OATFA to form the IMM-OATFA. The IMM-OATFA will be developed and simulation results will be used to compare its performance with a conventional IMM tracker.

Keywords: Track fusion, IMM algorithm, Kalman filter, Asynchronous, Optimal, Feedback

1. INTRODUCTION

An analytic solution for the fusion of track estimates produced from two asynchronous measurements has been recently developed [1-3]. The fusion process can occur at any time in the interval between the arrival of the final (*i.e.*, second) measurement of a fusion interval and the next measurement (*i.e.*, the first measurement of the next fusion interval). The track fusion solution was stipulated to be a weighted sum of the values associated with the previous fused estimate and the two individual estimates. The matrix weights are the unknowns for which a solution was formulated. Even though this technique was a breakthrough, it is restricted to the fusing of only two estimates.

A numeric solution to this problem with an arbitrary number of asynchronous measurements is provided by the Optimal Asynchronous Track Fusion Algorithm (OATFA) with feedback [4]. Two properties of the OATFA are identical to those of the Kalman filter. First, the OATFA solution is optimal when the dynamics of the target match those employed in the fusion model. Second, the OATFA uses the previous fused state estimate as feedback when computing a new state estimate. The OATFA can address several critical issues [4]. Unlike the Kalman filter, a state estimate need not be computed as soon as a measurement arrives. The measurement can be processed at any time before the end of the fusion interval. The data can be buffered and processed at a convenient time. Thus, the OATFA can greatly reduce the adverse effects of latent data and lessen peak processing requirements. The deficiencies of the Kalman filter when tracking maneuvering targets are also exhibited by the OATFA but can be overcome with the use of a multiple model tracking approach such as the Interacting Multiple Model (IMM) algorithm [5,6].

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The IMM algorithm uses multiple models that interact through state mixing to track a target through an arbitrary maneuver. The state estimates are mixed according to their model probabilities and model switching probabilities. The output estimate is a probabilistic sum of the individual filter estimates and represents the relative performance of each model. The IMM algorithm provides a flexible method for tracking maneuvering targets and is considered a nearly-consistent estimator since its output error covariance reflects the actual tracking performance. The IMM provides more accurate state estimates when compared to traditional methods and the error covariance can be reliably employed for adjusting the tracking parameters, making system decisions, and performing resource management [5-8]. Consequently, it should be possible to replace the Kalman filter commonly employed in a conventional IMM tracker with the OATFA to form the IMM-OATFA. The purpose of this paper is to present and discuss the feasibility of the IMM-OATFA. Simulation results will be employed to compare the performance of the IMM-OATFA and a conventional IMM tracker.

The paper is organized as follows. Section 2 presents background material and Section 3 outlines the IMM algorithm. Sections 4 and 5 describe the OATFA and IMM-OATFA respectively while Section 6 presents the simulation results. Concluding remarks are provided in Section 7.

2. BACKGROUND

The conventional discrete-time model for target tracking is typically a linear (or linearized) stochastic system given by

$$X_k = \phi_{k-1}^k X_{k-1} + W_{k-1}^k \quad \text{and} \quad Z_k = H_k X_k + V_k \tag{2.1}$$

where W_{k-1}^k is a process noise vector, V_k is a measurement error vector, X_k is a state vector, Z_k is a measurement vector, and

$$E\left[V_{k_i} V_{k_j}^T\right] = \delta_{ij} R_{k_i} \tag{2.2}$$

$$E\left[W_{k_i-1}^{k_i} \left(W_{k_j-1}^{k_j}\right)^T\right] = \delta_{ij} Q_{k_i-1}^{k_i} \tag{2.3}$$

$$W_{k-1}^k = \int_{t_{k-1}}^{t_k} \phi_\tau^{t_k} G \overline{W}(\tau) d\tau \tag{2.4}$$

$$Q_{k-1}^k = \int_{t_{k-1}}^{t_k} \phi_\tau^{t_k} G q(\tau) \left(\phi_\tau^{t_k} G\right)^T d\tau \tag{2.5}$$

with δ_{ij} being the Kronecker delta function and $E[\cdot]$ denoting the expectation value. The Kalman filter algorithm is commonly used to estimate the state and error covariance of the system from the measurements. The equations for the Kalman filter are outlined as follows.

Time Update:

$$X_{k|k-1} = \phi_{k-1}^k X_{k-1|k-1} \qquad P_{k|k-1} = \phi_{k-1}^k P_{k-1|k-1} \left(\phi_{k-1}^k\right)^T + Q_{k-1}^k \tag{2.6}$$

Measurement Update:

$$X_{k|k} = X_{k|k-1} + K_k \widetilde{Z}_k \qquad P_{k|k} = [I - K_k H_k] P_{k|k-1} \tag{2.7}$$

with

$$\widetilde{Z}_k = Z_k - H_k X_{k|k-1} = Z_k - Z_{k|k-1}, \qquad K_k = P_{k|k-1} H_k^T S_k^{-1}, \qquad S_k = H_k P_{k|k-1} H_k^T + R_k \tag{2.8}$$

where $X_{i|j}$ denotes the state estimate for time i given measurements through time j, and $P_{i|j}$ denotes the corresponding error covariance. An extended Kalman filter is employed in this paper since target measurements of range, bearing, and/or elevation are a nonlinear function of the state. For the extended Kalman filter, the measurement update is modified to reflect the nonlinear relation between the state and measurement according to

$$\widetilde{Z}_k = Z_k - h_k(X_{k|k-1}) \tag{2.9}$$

where $h_k(X_{k|k-1})$ is the expected measurement. The $H_k(X_{k|k-1})$ is computed as the gradient of h_k with respect to X_k. The extended Kalman filter readily accommodates track updates with measurements from a multitude of dissimilar sensors [5,6,8].

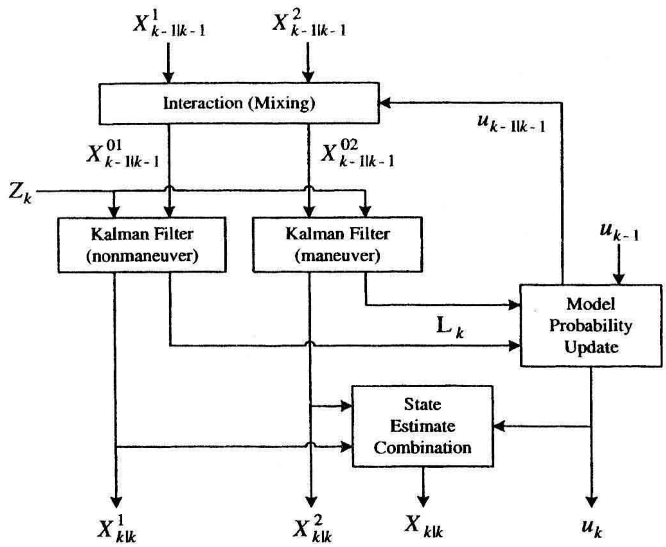

Figure 3.1 IMM Algorithm for Two Models

3. IMM ALGORITHM

The IMM algorithm consists of a filter for each model, a model probability evaluator, an estimate mixer at the input of the filters, and an estimate combiner at the output of the filters. A flow diagram of an IMM algorithm with two models is given in Fig. 3.1, where $X_{k|k}$ is the output state estimate based on both models, $X^j_{k|k}$ is the state estimate for time k based on model j, L_k is the vector of model likelihoods, and u_k is the vector of model probabilities. The mixer uses the model probabilities and the model switching probabilities to compute a mixed estimate for each filter, $X^{0j}_{k-1|k-1}$. Each filter uses a mixed estimate and a measurement, Z_k, to compute a new estimate and a likelihood for the model within the filter. The likelihoods, prior model probabilities, and the model switching probabilities are then used to compute new model probabilities. The output state estimate is then computed with the new state estimates and their model probabilities. The IMM algorithm for tracking with N models is outlined in the following 5 steps. A derivation and detailed explanation of the IMM algorithm are given in [9].

Step 1: Mixing of State Estimates

The filtering process starts with *a priori* state estimates $X^j_{k-1|k-1}$, state error covariances $P^j_{k-1|k-1}$, and the associated probabilities μ^j_{k-1} for each model. The mixed state estimate for model j at time k, M^j_k, is computed as

$$X^{0j}_{k-1|k-1} = \sum_{i=1}^{N} \mu^{i|j}_{k-1|k-1} X^i_{k-1|k-1} \tag{3.1}$$

where

$$\mu^{i|j}_{k-1|k-1} = \frac{1}{\bar{c}_j} p_{ij} \mu^i_{k-1}, \qquad \text{with} \qquad \bar{c}_j = \sum_{i=1}^{N} p_{ij} \mu^i_{k-1} \tag{3.2}$$

and p_{ij} is the probability of switching from model i to model j. The mixed error covariance for M^j_k is computed as

$$P^{0j}_{k-1|k-1} = \sum_{i=1}^{N} \mu^{i|j}_{k-1|k-1} \left[P^i_{k-1|k-1} + \left(X^i_{k-1|k-1} - X^{0j}_{k-1|k-1}\right)\left(X^i_{k-1|k-1} - X^{0j}_{k-1|k-1}\right)^T \right] \tag{3.3}$$

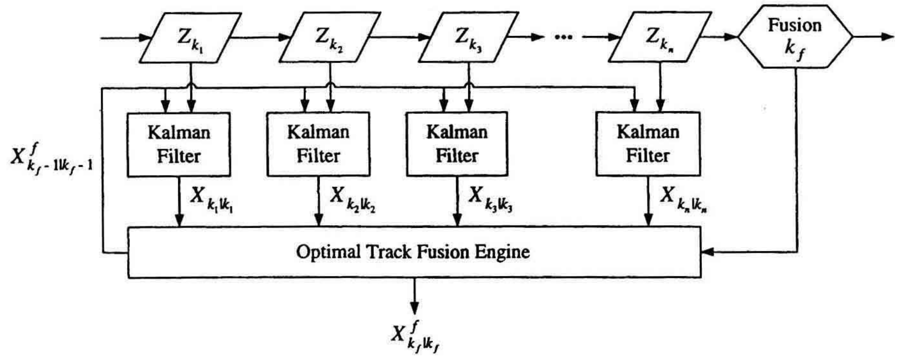

Figure 4.1 Optimal Asynchronous Track Fusion Algorithm

Step 2: Model-Conditioned Updates

The Kalman filtering equations provide the model-conditioned updates.

Step 3: Model Likelihood Computations

The likelihood of M_k^j, L_k^j, is given by

$$L_k^j = \frac{1}{\sqrt{|2\pi S_k^j|}} \exp\left[-0.5(\tilde{Z}_k^j)^T (S_k^j)^{-1} \tilde{Z}_k^j\right] \tag{3.4}$$

Step 4: Model Probabilities Update

Each model probability, μ_k^j, is updated as

$$\mu_k^j = \frac{1}{c} L_k^j \bar{c}_j, \qquad \text{where} \qquad c = \sum_{i=1}^{N} L_k^i \bar{c}_i \tag{3.5}$$

Step 5: Combination of State Estimates

The state estimate and error covariance for the IMM algorithm output, $X_{k|k}$ and $P_{k|k}$, are given by

$$X_{k|k} = \sum_{i=1}^{N} \mu_k^i X_{k|k}^i \qquad \text{and} \qquad P_{k|k} = \sum_{i=1}^{N} \mu_k^i \left[P_{k|k}^i + (X_{k|k}^i - X_{k|k})(X_{k|k}^i - X_{k|k})^T\right] \tag{3.6}$$

4. OPTIMAL ASYNCHRONOUS TRACK FUSION ALGORITHM

The OATFA is presented in this section. The OATFA employs a feedback architecture to optimally solve the asynchronous track fusion problem. A flowchart of the OATFA is presented in Fig. 4.1, where k_f is the time to perform the fusion operation, $X_{k_f|k_f}$ is the fused state estimate at k_f, Z_{k_i} is the sensor measurement at time k_i, and $X_{k_i|k_i}$ is the state estimate based on Z_{k_i} and the previously fused estimate $X_{k_f-1|k_f-1}$. The OATFA is outlined in the following 4 steps. A derivation and detailed explanation and implementation of the OATFA are given in [1,3,4].

4.1. Formulation of the Fused State Estimate

For the feedback process, the fused state estimate at time k computed from n state estimates in the time interval $k-1$ to k, $X_{k|k}^{f(n)}$, is written as

$$X_{k|k}^{f(n)} = \sum_{i=0}^{n} L_i X_{k_i|k_i}^i = \sum_{i=0}^{n} T_i \phi_{k_i}^k X_{k_i|k_i}^i = \sum_{i=0}^{n} T_i X_{k|k_i}^i = T_0 X_{k|k-1}^f + \sum_{i=1}^{n} T_i X_{k|k_i}^i \tag{4.1.1}$$

where $X_{k_0|k_0}^0 = X_{k-1|k-1}^f$, $\phi_{k_0}^k = \phi_{k-1}^k$, and $X_{k|k_0}^0 = X_{k|k-1}^f$. The fused state estimate at time $k-1$ is $X_{k-1|k-1}^f$. In the development, fusion estimates occur at time k and individual state estimates occur at time k_i.

By employing Eqs. (2.1), (2.6), and (2.7), state estimate i when feedback is considered, $X_{k_i|k_i}^i$, can be written as

$$\begin{aligned}
X_{k_i|k_i}^i &= X_{k_i|k-1}^f + K_{k_i}^i (Z_{k_i}^i - H_{k_i}^i X_{k_i|k-1}^f) && (4.1.2)\\
&= (I - K_{k_i}^i H_{k_i}^i) X_{k_i|k-1}^f + K_{k_i}^i H_{k_i}^i X_{k_i} + K_{k_i}^i V_{k_i}^i && (4.1.3)\\
&= (I - K_{k_i}^i H_{k_i}^i)(\phi_{k_i}^k)^{-1} \phi_{k-1}^k X_{k-1|k-1}^f + K_{k_i}^i H_{k_i}^i X_{k_i} + K_{k_i}^i V_{k_i}^i && (4.1.4)\\
&= (I - K_{k_i}^i H_{k_i}^i)(\phi_{k_i}^k)^{-1} X_{k|k-1}^f + K_{k_i}^i H_{k_i}^i (\phi_{k_i}^k)^{-1} (X_k - W_{k_i}^k) + K_{k_i}^i V_{k_i}^i && (4.1.5)\\
&= D_i X_{k|k-1}^f + B_i (X_k - W_{k_i}^k) + K_{k_i}^i V_{k_i}^i && (4.1.6)
\end{aligned}$$

where

$$B_i = K_{k_i}^i H_{k_i}^i (\phi_{k_i}^k)^{-1} \quad \text{and} \quad D_i = (I - K_{k_i}^i H_{k_i}^i)(\phi_{k_i}^k)^{-1} = (\phi_{k_i}^k)^{-1} - B_i \tag{4.1.7}$$

With the fused estimate given by

$$X_{k|k}^{f(n)} = T_0 \phi_{k-1}^k X_{k-1|k-1}^f + \sum_{i=1}^{n} T_i \phi_{k_i}^k X_{k_i|k_i}^i = T_0 X_{k|k-1}^f + \sum_{i=1}^{n} L_i X_{k_i|k_i}^i \tag{4.1.8}$$

substituting Eq. (4.1.6) into Eq. (4.1.8) yields

$$X_{k|k}^{f(n)} = \left(T_0 + \sum_{i=1}^{n} L_i D_i\right) X_{k|k-1}^f + \sum_{i=1}^{n} L_i B_i X_k - \sum_{i=1}^{n} L_i B_i W_{k_i}^k + \sum_{i=1}^{n} L_i K_{k_i}^i V_{k_i}^i \tag{4.1.9}$$

4.2. Development of the Fusion Constraint

The residuals (i.e., estimation errors) for $X_{k|k}^{f(n)}$ and $X_{k|k-1}^f$, $\widetilde{X}_{k|k}^{f(n)}$ and $\widetilde{X}_{k|k-1}^f$ respectively, are defined by

$$\widetilde{X}_{k|k}^{f(n)} = X_{k|k}^{f(n)} - X_k \quad \text{and} \quad \widetilde{X}_{k|k-1}^f = X_{k|k-1}^f - X_k \tag{4.2.1}$$

By using Eqs. (4.1.9) and (4.2.1), $\widetilde{X}_{k|k}^{f(n)}$ can be written as

$$\widetilde{X}_{k|k}^{f(n)} = \left(T_0 + \sum_{i=1}^{n} L_i D_i\right) \widetilde{X}_{k|k-1}^f + \left(T_0 + \sum_{i=1}^{n} L_i (D_i + B_i) - I\right) X_k - \sum_{i=1}^{n} L_i B_i W_{k_i}^k + \sum_{i=1}^{n} L_i K_{k_i}^i V_{k_i}^i \tag{4.2.2}$$

Since $E[W_{k_i}^k] = E[V_{k_i}^i] = E[\widetilde{X}_{k|k}^{f(n)}] = E[\widetilde{X}_{k|k-1}^f] = 0$ for an unbiased estimate of $X_{k|k}^{f(n)}$, the following must be true in Eq. (4.2.2)

$$T_0 + \sum_{i=1}^{n} L_i (D_i + B_i) = T_0 + \sum_{i=1}^{n} L_i (\phi_{k_i}^k)^{-1} = T_0 + \sum_{i=1}^{n} T_i \phi_{k_i}^k (\phi_{k_i}^k)^{-1} = \sum_{i=0}^{n} T_i = I \tag{4.2.3}$$

By employing Eq. (4.2.3), Eq. (4.2.2) can be written as

$$\widetilde{X}^{f(n)}_{k|k} = \Big(I - \sum_{i=1}^{n} L_i B_i\Big)\widetilde{X}^{f}_{k|k-1} - \sum_{i=1}^{n} L_i B_i W^k_{k_i} + \sum_{i=1}^{n} L_i K^i_{k_i} V^i_{k_i} \tag{4.2.4}$$

4.3. Formulation of the Fused Error Covariance

The error covariance associated with $X^{f(n)}_{k|k}$, $P^{f(n)}_{k|k}$, is given by

$$P^{f(n)}_{k|k} = E\Big[\widetilde{X}^{f(n)}_{k|k}(\widetilde{X}^{f(n)}_{k|k})^T\Big] \tag{4.3.1}$$

Employing Eqs. (2.2), (2.3), and (4.2.4), and the fact that

$$E\Big[\widetilde{X}^{f}_{k|k-1}(\widetilde{X}^{f}_{k|k-1})^T\Big] = P^f_{k|k-1} \qquad \text{and} \qquad E\Big[V^i_{k_i}A^T\Big] = E\Big[A(V^i_{k_i})^T\Big] = 0 \tag{4.3.2}$$

where A is a matrix, the fused error covariance $P^{f(n)}_{k|k}$ can be written as

$$\begin{aligned} P^{f(n)}_{k|k} = {} & C_0 P^f_{k|k-1} C_0^T - C_0 \sum_{j=1}^{n} E\Big[\widetilde{X}^{f}_{k|k-1}(W^k_{k_j})^T\Big](L_j B_j)^T - \sum_{i=1}^{n} L_i B_i E\Big[W^k_{k_i}(\widetilde{X}^{f}_{k|k-1})^T\Big] C_0^T \\ & + \sum_{i=1}^{n} L_i B_i Q^k_{k_i}(L_i B_i)^T + \sum_{i=1}^{n} L_i K^i_{k_i} R^i_{k_i}(L_i K^i_{k_i})^T \end{aligned} \tag{4.3.3}$$

where

$$C_0 = I - \sum_{i=1}^{n} L_i B_i \tag{4.3.4}$$

By using Eqs. (2.1), (2.3), and (4.3.4), and the fact that

$$E\Big[\widetilde{X}^{f}_{k|k-1}(W^k_{k_i})^T\Big] = E\Big[W^k_{k_i}(\widetilde{X}^{f}_{k|k-1})^T\Big] = -Q^k_{k_i} \tag{4.3.5}$$

Eq. (4.3.3) can be rewritten as

$$P^{f(n)}_{k|k} = C_0 P^f_{k|k-1} C_0^T + \sum_{i=1}^{n}\Big(C_0 Q^k_{k_i}(L_i B_i)^T + L_i B_i Q^k_{k_i} C_0^T + L_i B_i Q^k_{k_i}(L_i B_i)^T + L_i K^i_{k_i} R^i_{k_i}(L_i K^i_{k_i})^T\Big) \tag{4.3.6}$$

$$\begin{aligned} = {} & P^f_{k|k-1} - \sum_{i=1}^{n} L_i B_i (P^f_{k|k-1} - Q^k_{k_i}) - \sum_{i=1}^{n}(P^f_{k|k-1} - Q^k_{k_i})(L_i B_i)^T + \sum_{i=1}^{n} L_i B_i Q^k_{k_i}(L_i B_i)^T \\ & + \sum_{i=1}^{n} L_i K^i_{k_i} R^i_{k_i}(L_i K^i_{k_i})^T + \sum_{i=1}^{n}\sum_{j=1}^{n} L_i B_i (P^f_{k|k-1} - Q^k_{k_i} - Q^k_{k_j})(L_j B_j)^T \end{aligned} \tag{4.3.7}$$

By defining the following quantities

$$Y_{ij} = B_i(P^f_{k|k-1} - Q^k_{k_i} - Q^k_{k_j} + \delta_{ij} Q^k_{k_i}) B_j^T + \delta_{ij} K^i_{k_i} R^i_{k_i} (K^i_{k_i})^T \qquad \text{and} \qquad U_i = B_i(P^f_{k|k-1} - Q^k_{k_i}) \tag{4.3.8}$$

Eq. (4.3.7) can be expressed as

$$P^{f(n)}_{k|k} = P^f_{k|k-1} - \sum_{i=1}^{n}\Big(L_i U_i + (L_i U_i)^T\Big) + \sum_{i=1}^{n}\sum_{j=1}^{n} L_i Y_{ij} L_j^T \tag{4.3.9}$$

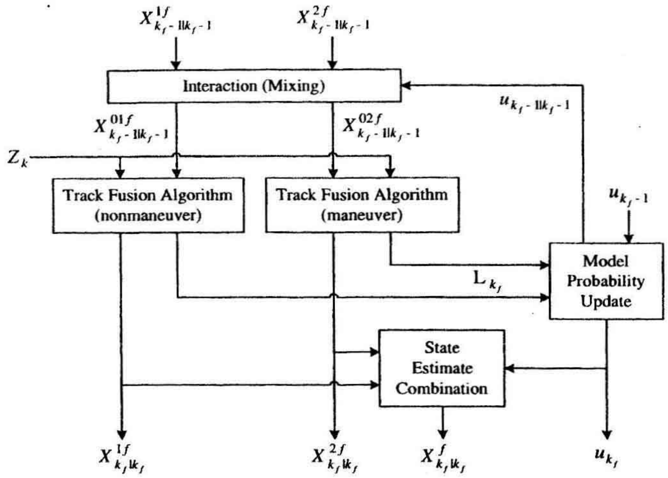

Figure 5.1 IMM-OATFA for Two Models

4.4. Solution for the Fusion Estimates

When taking the derivative of $tr\left(P_{k|k}^{f(n)}\right)$, as defined in Eq. (4.3.9), with respect to each element of $\{L_i\}_{i=1}^{n}$, only the terms for $k = i, j$ need to be considered. The derivative of Eq. (4.3.9) with respect to L_k is given by

$$\frac{\partial\, tr\left(P_{k|k}^{f(n)}\right)}{\partial L_k} = \sum_{i=1}^{n} L_i Y_{ki}^T + \sum_{i=1}^{n} L_i Y_{ik} - U_k^T - U_k^T = 2\left(\sum_{i=1}^{n} L_i Y_{ik} - U_k^T\right) \tag{4.4.1}$$

The values of L_i can be computed using the n generalized equations given by

$$\sum_{i=1}^{n} L_i Y_{ij} = U_j^T, \qquad \text{for} \quad j = 1, \ldots, n \tag{4.4.2}$$

5. IMM-OATFA

The IMM-OATFA is presented in this section. The IMM-OATFA combines the IMM algorithm with the OATFA to exploit the inherent benefits of both algorithms while simultaneously minimizing their deficiencies [4-6]. A flow diagram of an IMM-OATFA with two models is given in Fig. 5.1, where $X_{k_f|k_f}^{f}$ is the output fused state estimate based on both models, $X_{k_f|k_f}^{jf}$ is the fused state estimate for time k_f based on model j, $X_{k_f-1|k_f-1}^{0jf}$ is the mixed fused estimate for each OATFA, L_{k_f} is the vector of model likelihoods, and u_{k_f} is the vector of model probabilities. The IMM-OATFA operates in a manner similar to the conventional IMM algorithm.

Step 1: Mixing of Fused State Estimates

The filtering process starts with *a priori* fused state estimates $X_{k_f-1|k_f-1}^{jf}$, fused state error covariances $P_{k_f-1|k_f-1}^{jf}$, and the associated probabilities $\mu_{k_f-1}^{j}$ for each model. The mixed fused state estimate for model j at time k_f, $M_{k_f}^{j}$, is computed as

$$X_{k_f-1|k_f-1}^{0jf} = \sum_{i=1}^{N} \mu_{k_f-1|k_f-1}^{i|j} X_{k_f-1|k_f-1}^{if} \tag{5.1}$$