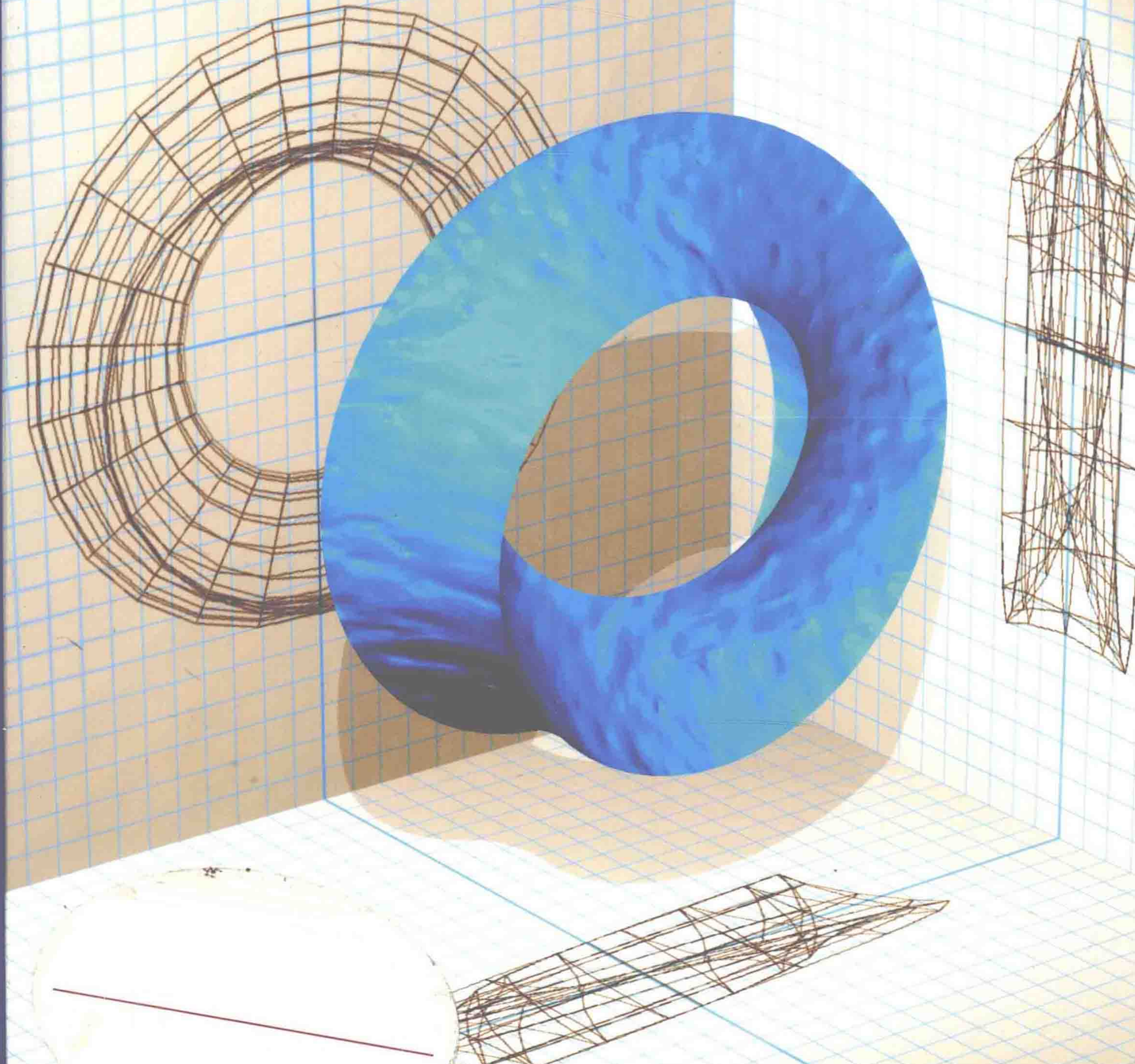
Calculus

SIXTH EDITION



La... Hostetler ▪ Edwards

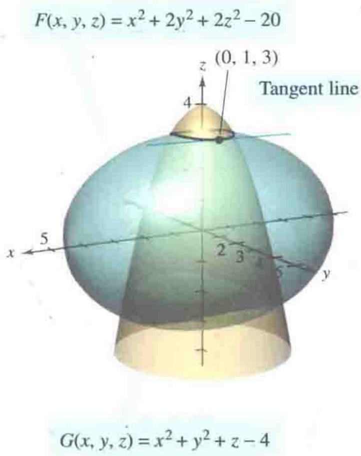
"A picture is worth a thousand words." This is an odd saying for authors to use, but it has appeared often in our text because we believe that seeing the *images* of mathematics goes hand-in-hand with understanding the *concepts* of mathematics.

Some think of the "graphical-analytic" approach as new. In reality it was precisely this approach that catalyzed the rapid development of calculus in the late 1600s and early 1700s.

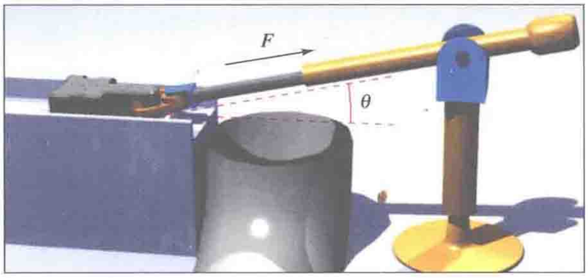
In keeping with this history, we have always included thousands of graphs in the text—and they have always been as accurate as current technology allowed. Now, using innovative technology, this edition is the first to bring "virtual reality" to the 3-D art package.

We hope you enjoy the images. They took many months to generate and the results are dazzling! Some examples are shown below.

Ron Larson
Bob Hostetler
Bruce Edwards



Using adjustable translucency, we are now able to show interiors and intersections of surfaces and solids in ways that were not possible before.



Every 3-D graphic in the book has a new realism—even the real-life graphics. For instance, in the engineering graphic above, notice how the lighting and shadows give the sense that you are looking at a real object.

As we were working on each piece, we spent a great deal of time deciding which angle presented the best view. Recognizing that students would also benefit from the experience of rotating the surface to better understand its shape, we decided to provide instructors with rotatable versions of the surfaces on disk to demonstrate in class. For instance, above are shown three of many possible views of a surface in the text.

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Sixth Edition

Calculus
with Analytic Geometry

Roland E. Larson
Robert P. Hostetler
The Pennsylvania State University
The Behrend College

Bruce H. Edwards
University of Florida

with the assistance of
David E. Heyd
The Pennsylvania State University
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Houghton Mifflin Company Boston New York

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We have included examples and exercises that use real-life data as well as technology output from a variety of software. This would not have been possible without the help of many people and organizations. Our wholehearted thanks goes to all for their time and effort.

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A word from the authors . . .

Welcome to *Calculus with Analytic Geometry,* Sixth Edition! We are excited about the Sixth Edition and hope you will be too after you hear about why we wrote it, what's new about it, and how it will carry you and your calculus students into the twenty-first century!

Reform in Mathematics Education

As you know, the current reform movement in math education started about 15 years ago and has involved all levels of mathematics education from kindergarten through college. There may be some who say reform was not needed. However, the vast majority of math educators agree that reform was essential. In fact, the new math of the 1960s, 1970s, and early 1980s was a national disaster. It was far too abstract and far too removed from the real-life applications that were the actual foundation of mathematics. The result was evident to everyone—math phobia, falling test scores, high drop-out rates, and a general sense that students were not learning to be creative problem solvers.

So what were the proposed solutions? There were many—more real-life connections, more incorporation of technology, curriculum revisions, and the development of alternative forms of teaching, assessment, and learning.

Where This Text Stands with Respect to Reform

"The text has definitely benefited from reform. More than that, over the years our calculus text has actually led the way in developing many innovative learning techniques."

Where does the Sixth Edition stand with respect to reform? This is one of the most common questions we are asked. Our answer: The text has definitely benefited from reform. More than that, over the years our calculus text has actually led the way in developing many innovative learning techniques. From its first edition, the text stressed the importance of graphical learning—much more than other texts in use in the late 1970s and early 1980s. This text was one of the first to incorporate computer-generated art—both two-dimensional and three-dimensional—to aid in the visualization of complex mathematical concepts.

We have always paid careful attention to the presentation—using precise mathematical language, innovative full-color designs for emphasis and clarity, and a level of exposition that appeals to students—to create an effective teaching and learning tool. Although difficult to quantify, this feature has been praised by thousands of students and their instructors over the past 20 years. With each edition, we have continued to incorporate the best strategies for teaching calculus, using the pedagogy we have developed as a result of our teaching experiences, as well as many suggestions from thoughtful users.

This Sixth Edition might best be described as fitting midway between texts that define themselves as traditional and those that are considered reform texts. Our approach is like that of a traditional text in that we firmly believe in the importance of carefully developed theory, correct statements of theorems, inclusion of proofs, and mastery of traditional calculus skills. We have found no evidence that it is somehow possible to apply calculus in real-life situations without first being able to understand and "do" calculus.

On the other hand, we wholeheartedly embrace many of the features of calculus reform. For instance, this edition features additional opportunities to use technology, an increased emphasis on real-life applications and modeling data, new motivational features, many more conceptual exercises and multi-part exercises, new explorations in many sections, and a myriad of student and teacher aids.

We believe in giving teachers options to teach calculus the way they want. Because of this, you will find a wide variety of approaches and features in the text. By choosing the options that best fit you as a teacher, you can customize the book in dozens of different ways: lecture or discovery approaches, pencil and paper skills or technology, formal or informal, theoretical or intuitive, analytic or graphical, mathematics-centered or applied, classroom presentation or distance learning—they are all here as options for you.

What We Changed in the Sixth Edition

"In the Sixth Edition, we continue to lead the way in incorporating the best aspects of reform in a meaningful yet easy-to-use manner."

In the Sixth Edition, we continue to lead the way in incorporating the best aspects of reform in a meaningful yet easy-to-use manner. Here are some of the most significant new features.

New *Explorations* For instructors and students who benefit from a frequent or occasional "discovery" mode, we have incorporated this option into the Sixth Edition. By discovering concepts that are new to them, students get a taste of what it is like to be a real mathematician. Also, some students find it easier to remember concepts they have "discovered." Truly effective explorations are difficult to create. The challenge is finding a good balance between what is given and what is expected to be discovered. Our Explorations represent a combination of suggestions by users and the results of our own classroom experience.

"As all calculus teachers know, one of the difficulties in teaching calculus is that we are often answering questions that students have not yet asked. The motivators address this dilemma by presenting real-life situations with exploratory questions."

New *Motivating the Chapter* Each chapter now begins with a full-page chapter motivator. As all calculus teachers know, one of the difficulties in teaching calculus is that we are often answering questions that students have not yet asked. The motivators address this dilemma by presenting real-life situations with exploratory questions. As students attempt to use the techniques of their current skill set to answer the questions, they will learn to appreciate the power and efficiency of the new calculus techniques presented in the chapter. We spent a lot of time researching effective settings and developing meaningful questions, and we found that the ones selected for this text sparked interest among the students in our classes.

New Lab Manuals The real-life application that is introduced in each *Motivating the Chapter* forms the basis for the extended technology lab projects in the supplementary Lab Manuals. In each *Motivating the Chapter*, students are asked to solve a problem using the techniques of their current skill set. As the chapter progresses, students can be assigned projects from the Lab Manuals that ask them to take a new look at these problems using concepts learned in that chapter as well as technology. The Lab Manuals come in five versions (each with data disks) designed for use with *Maple, Mathematica, Derive, Mathcad,* and the *TI-92* graphing calculator.

New Art Program Visualization is a problem-solving skill that is critical for the understanding of complex calculus theory and concepts. To help students develop this skill, the Sixth Edition features a completely new art program that was created using state-of-the-art computer technology for accuracy, clarity, and realism. The effect of the new three-dimensional art is particularly striking. Using computer graphics software, we adjusted the color and transparency, view, light sources, and shadows on the solids and surfaces until we found the optimal combination of features to show true perspective. The result, we believe, is the most accurate representation of three-dimensional calculus surfaces ever! For instance, the image on the book's cover was produced with this computer graphics system and the illusion of three-dimensionality is amazing.

"... the most accurate representation of three-dimensional calculus surfaces ever!"

Revised Exercise Sets All of the exercise sets of the previous edition were considered for revision, and many new exercises were added to the Sixth Edition. We focused on adding problems that were technology-oriented, thought-provoking, conceptual, creative, real, and engaging. There are many more opportunities for writing, for individual and group projects, and for solving problems with graphical, numerical, and analytical approaches. We expanded on the wide variety of applications, which were already distinctive for their relevance and originality. Also new to the Sixth Edition are many modeling data exercises that ask students to find and interpret mathematical models from the real-life data that are given. To the many who helped us in our search for such exercises, we offer our thanks.

"We focused on adding problems that were technology-oriented, thought-provoking, conceptual, creative, real, and engaging."

Revised *Interactive Calculus* The Fifth Edition of this text was available in an interactive CD-ROM format that was innovative and well received. Now, four years later, we have continued to push the limits of what technology can do as a medium for teaching calculus. Among the new and enhanced features of the CD-ROM version of the Sixth Edition are: all of the content of the revised text, more active mathematics, editable two-dimensional graphs, dazzling three-dimensional graphs, additional explorations and simulations, and a syllabus builder for instructors.

Table of Contents Some of the text was revised to enhance the flexibility of the presentation and to reflect our perspective on reform. We rewrote Chapter P, moving much of the precalculus review to Appendix A. What remains of precalculus in Chapter P was restructured to introduce students to new ways of thinking about math, including graphical, numerical, and analytical approaches; modeling; problem solving; and data analysis. Instructors who skipped this chapter in previous editions might want to reconsider that decision.

Chapter 1 now begins with a new section, "A Preview of Calculus," which introduces students to the distinctions between precalculus and calculus, emphasing the necessity of calculus in our dynamic, everyday surroundings. One section on differential equations was moved from Chapter 15 to Chapter 5 to allow students to make more use of this material, and to better prepare them for their courses in other disciplines such as physics and chemistry. In Chapter 6, we repositioned the section on moments, centers of mass, and centroids so that it precedes the section on fluid pressure and fluid force, to address changing teaching styles. We reduced the coverage of conic sections in Chapter 9, because this material is mostly a review for students at this level. This edition discusses conics in the same chapter that covers parametric equations and the polar coordinate system. Finally, the section on rotation and the general second-degree equation was moved to Appendix E, leaving it optional for those instructors who wish to cover it.

What We Didn't Change

"Although we carefully and thoroughly revised the text …, we* didn't *change many of the things that our colleagues and the 1,500,000 students who have used the book have told us worked for them."

Although we carefully and thoroughly revised the text by enhancing the usefulness of some features and topics and adding others, we *didn't* change many of the things that our colleagues and the 1,500,000 students who have used this book have told us worked for them. We still offer comprehensive coverage of the material required by students in a three-semester calculus course, including carefully stated theory and proofs. Additionally, as we do with all our books, we painstakingly formatted every page of the Sixth Edition to achieve a clear presentation of the material. Finally, the text was carefully written in a style that is mathematically precise, as well as engaging, direct, and readable.

We hope you will enjoy the Sixth Edition. We are proud to have it as our calculus entry for the next century!

Ron Larson

Roland E. Larson

Robert P. Hostetler

Robert P. Hostetler

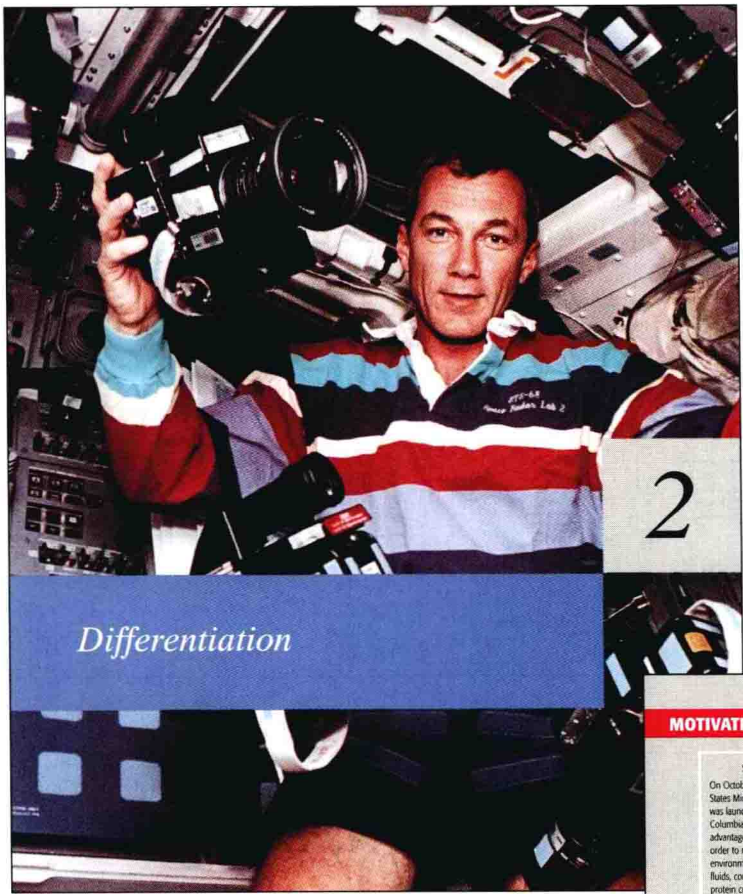
Bruce H. Edwards

Bruce H. Edwards

Features

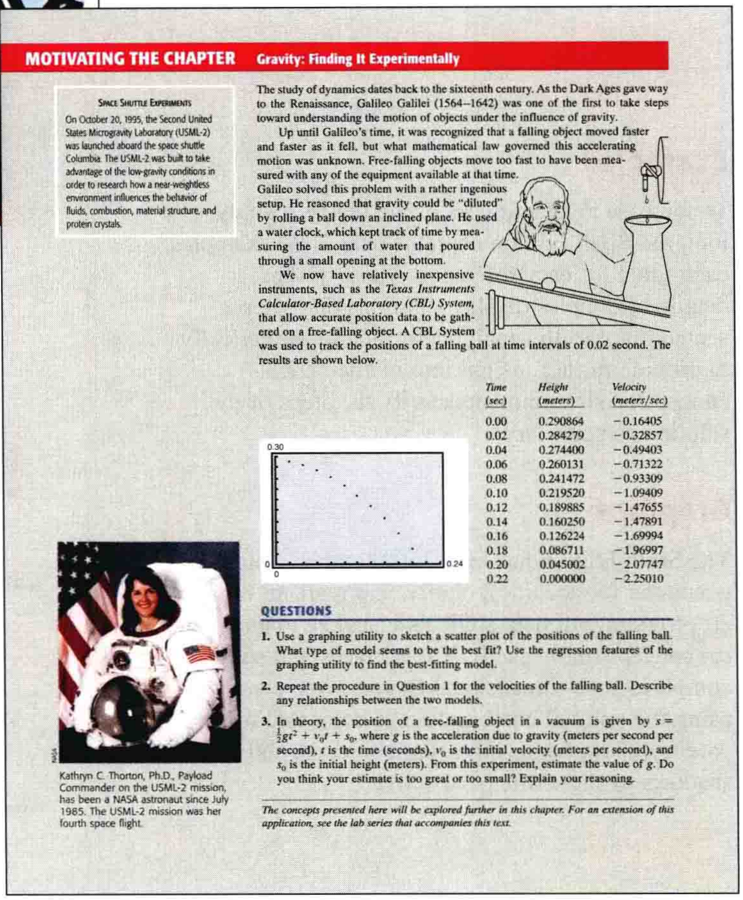
Chapter Openers

Each chapter opens with a photograph that corresponds to the mathematical application in *Motivating the Chapter.*



Motivating the Chapter

Each *Motivating the Chapter* explores the concepts to be covered in the chapter using a real-world setting. Following a short introduction, open-ended questions guide students through an introduction to the main themes of the chapter.



MOTIVATING THE CHAPTER **Gravity: Finding It Experimentally**

SPACE SHUTTLE EXPERIMENTS

On October 20, 1995, the Second United States Microgravity Laboratory (USML-2) was launched aboard the space shuttle Columbia. The USML-2 was built to take advantage of the low-gravity conditions in order to research how a near-weightless environment influences the behavior of fluids, combustion, material structure, and protein crystals.

The study of dynamics dates back to the sixteenth century. As the Dark Ages gave way to the Renaissance, Galileo Galilei (1564–1642) was one of the first to take steps toward understanding the motion of objects under the influence of gravity.

Up until Galileo's time, it was recognized that a falling object moved faster and faster as it fell, but what mathematical law governed this accelerating motion was unknown. Free-falling objects move too fast to have been measured with any of the equipment available at that time. Galileo solved this problem with a rather ingenious setup. He reasoned that gravity could be "diluted" by rolling a ball down an inclined plane. He used a water clock, which kept track of time by measuring the amount of water that poured through a small opening at the bottom.

We now have relatively inexpensive instruments, such as the *Texas Instruments Calculator-Based Laboratory (CBL) System,* that allow accurate position data to be gathered on a free-falling object. A CBL System was used to track the positions of a falling ball at time intervals of 0.02 second. The results are shown below.

Time (sec)	*Height (meters)*	*Velocity (meters/sec)*
0.00	0.290864	−0.16405
0.02	0.284279	−0.32857
0.04	0.274400	−0.49403
0.06	0.260131	−0.71322
0.08	0.241472	−0.93309
0.10	0.219520	−1.09409
0.12	0.189885	−1.47655
0.14	0.160250	−1.47891
0.16	0.126224	−1.69994
0.18	0.086711	−1.96997
0.20	0.045002	−2.07747
0.22	0.000000	−2.25010

Kathryn C. Thorton, Ph.D., Payload Commander on the USML-2 mission, has been a NASA astronaut since July 1985. The USML-2 mission was her fourth space flight.

QUESTIONS

1. Use a graphing utility to sketch a scatter plot of the positions of the falling ball. What type of model seems to be the best fit? Use the regression features of the graphing utility to find the best-fitting model.
2. Repeat the procedure in Question 1 for the velocities of the falling ball. Describe any relationships between the two models.
3. In theory, the position of a free-falling object in a vacuum is given by $s = \frac{1}{2}gt^2 + v_0t + s_0$, where g is the acceleration due to gravity (meters per second per second), t is the time (seconds), v_0 is the initial velocity (meters per second), and s_0 is the initial height (meters). From this experiment, estimate the value of g. Do you think your estimate is too great or too small? Explain your reasoning.

The concepts presented here will be explored further in this chapter. For an extension of this application, see the lab series that accompanies this text.

SECTION 6.5 Work

Work Done by a Constant Force • Work Done by a Variable Force

Work Done by a Constant Force

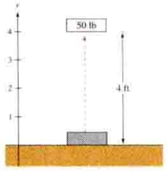
The concept of work is important to scientists and engineers for determining the energy needed to perform various jobs. For instance, it is useful to know the amount of work done when a crane lifts a steel girder, when a spring is compressed, when a rocket is propelled into the air, or when a truck pulls a load along a highway.

In general, we say that **work** is done by a force when it moves an object. If the force applied to the object is *constant*, we have the following definition of work.

Definition of Work Done by a Constant Force

If an object is moved a distance D in the direction of an applied constant force F, then the **work** W done by the force is defined as $W = FD$.

There are many types of forces—centrifugal, electromotive, and gravitational, to name a few. A **force** can be thought of as a *push* or a *pull*; a force changes the state of rest or state of motion of a body. For gravitational forces on the earth, it is common to use units of measure corresponding to the weight of an object.



The work done in lifting a 50-pound object 4 feet is 200 foot-pounds.
Figure 6.49

EXAMPLE 1 Lifting an Object

Determine the work done in lifting a 50-pound object 4 feet.

Solution The magnitude of the required force F is the weight of the object, as shown in Figure 6.49. Thus, the work done in lifting the object 4 feet is

$$\begin{aligned} W &= FD && \text{Work = (force)(distance)} \\ &= 50(4) && \text{Force = 50 pounds, distance = 4 feet} \\ &= 200 \text{ foot-pounds.} \end{aligned}$$

In the U.S. measurement system, work is typically expressed in foot-pounds (ft · lb), inch-pounds, or foot-tons. In the centimeter-gram-second (C-G-S) system, the basic unit of force is the **dyne**—the force required to produce an acceleration of 1 centimeter per second per second on a mass of 1 gram. In this system, work is typically expressed in dyne-centimeters (ergs) or newton-meters (joules), where 1 joule $= 10^7$ ergs.

EXPLORATION

How Much Work? In Example 1, it required 200 foot-pounds of work to lift the 50-pound object 4 feet, vertically off the ground. Suppose that once you lifted the object, you held it and walked a horizontal distance of 4 feet. Would this require an additional 200 foot-pounds of work? Explain your reasoning.

Section Topics

Each section begins with a list of subsection topics. This outline will help instructors with class planning and help students with studying and synthesizing the material in the section.

Explorations

Before students are exposed to selected topics, exploratory projects allow them to discover concepts on their own, making them more likely to remember the results. These optional boxed features can be omitted, if the instructor so desires, with no loss of continuity in the coverage of material.

Examples

To increase the usefulness of the text as a study tool, the Sixth Edition contains over 1000 examples, each titled for easy reference. Many of these detailed examples display solutions that are presented graphically, analytically, and/or numerically to provide further insight into mathematical concepts. Side comments clarify the steps of the solution as necessary.

Graphics

The Sixth Edition has over 3500 figures. Computer-generated for accuracy, clarity, and realism, this new art program will help students visualize mathematical concepts more easily. The surfaces and solids of complex, three-dimensional figures were created using the optimal combination of color and transparency, view, perspective, light sources, and shadows to show true perspective.

EXAMPLE 1 Approximating the Volume of a Solid

Approximate the volume of the solid lying between the paraboloid

$$f(x, y) = 1 - \frac{1}{2}x^2 - \frac{1}{2}y^2$$

and the square region R given by $0 \le x \le 1$, $0 \le y \le 1$. Use a partition made up of squares whose edges have a length of $\frac{1}{4}$.

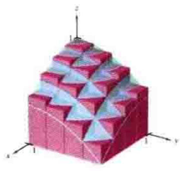
Solution Begin by forming the specified partition of R. For this partition, it is convenient to choose the centers of the subregions as the points at which to evaluate $f(x, y)$.

$$\begin{matrix} \left(\frac{1}{8}, \frac{1}{8}\right) & \left(\frac{1}{8}, \frac{3}{8}\right) & \left(\frac{1}{8}, \frac{5}{8}\right) & \left(\frac{1}{8}, \frac{7}{8}\right) \\ \left(\frac{3}{8}, \frac{1}{8}\right) & \left(\frac{3}{8}, \frac{3}{8}\right) & \left(\frac{3}{8}, \frac{5}{8}\right) & \left(\frac{3}{8}, \frac{7}{8}\right) \\ \left(\frac{5}{8}, \frac{1}{8}\right) & \left(\frac{5}{8}, \frac{3}{8}\right) & \left(\frac{5}{8}, \frac{5}{8}\right) & \left(\frac{5}{8}, \frac{7}{8}\right) \\ \left(\frac{7}{8}, \frac{1}{8}\right) & \left(\frac{7}{8}, \frac{3}{8}\right) & \left(\frac{7}{8}, \frac{5}{8}\right) & \left(\frac{7}{8}, \frac{7}{8}\right) \end{matrix}$$

Because the area of each square is $\Delta x_i \Delta y_i = \frac{1}{16}$, you can approximate the volume by the sum

$$\sum_{i=1}^{16} f(x_i, y_i)\Delta x_i \Delta y_i = \sum_{i=1}^{16}\left(1 - \frac{1}{2}x_i^2 - \frac{1}{2}y_i^2\right)\left(\frac{1}{16}\right) \approx 0.672.$$

This approximation is shown graphically in Figure 13.12. The exact volume of the solid is $\frac{2}{3}$ (see Example 2). You can obtain a better approximation by using a finer partition. For example, with a partition of squares with sides of length $\frac{1}{10}$, the approximation is 0.668.



Surface: $f(x, y) = 1 - \frac{1}{2}x^2 - \frac{1}{2}y^2$

Figure 13.12

TECHNOLOGY Some three-dimensional graphing utilities are capable of sketching figures such as that shown in Figure 13.12. For instance, the sketch shown in Figure 13.13 was drawn with a computer program. In this sketch, note that each of the rectangular prisms lies within the solid region.

In Example 1, note that by using finer partitions, you can obtain better approximations of the volume. This observation suggests that you could obtain the exact volume by taking a limit. That is

$$\text{Volume} = \lim_{\|\Delta\| \to 0} \sum_{i=1}^{n} f(x_i, y_i)\Delta x_i \Delta y_i.$$

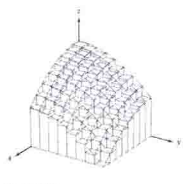


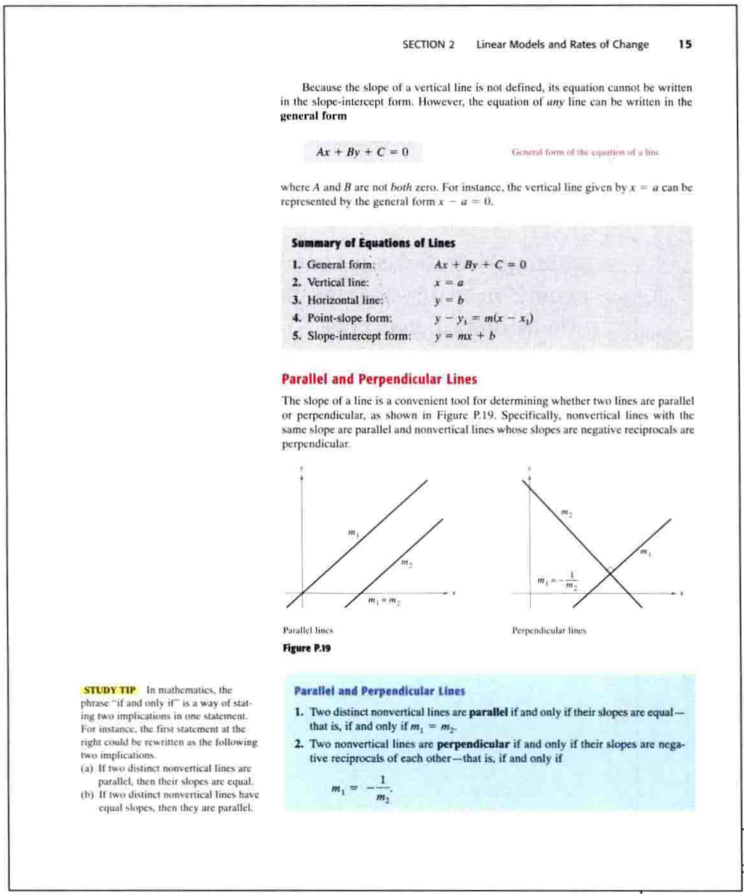
Figure 13.13

The precise meaning of this limit is that the limit is equal to L if for every $\varepsilon > 0$ there exists a $\delta > 0$ such that

$$\left| L - \sum_{i=1}^{n} f(x_i, y_i)\Delta x_i \Delta y_i \right| < \varepsilon$$

for all partitions Δ of the plane region R (that satisfy $\|\Delta\| < \delta$) and for all possible choices of x_i and y_i in the ith region.

Using the limit of a Riemann sum to define volume is a special case of using the limit to define a **double integral.** The general case, however, does not require that the function be positive or continuous.



SECTION 2 Linear Models and Rates of Change 15

Because the slope of a vertical line is not defined, its equation cannot be written in the slope-intercept form. However, the equation of *any* line can be written in the **general form**

$Ax + By + C = 0$ General form of the equation of a line

where A and B are not *both* zero. For instance, the vertical line given by $x = a$ can be represented by the general form $x - a = 0$.

Summary of Equations of Lines

1. General form: $Ax + By + C = 0$
2. Vertical line: $x = a$
3. Horizontal line: $y = b$
4. Point-slope form: $y - y_1 = m(x - x_1)$
5. Slope-intercept form: $y = mx + b$

Parallel and Perpendicular Lines

The slope of a line is a convenient tool for determining whether two lines are parallel or perpendicular, as shown in Figure P.19. Specifically, nonvertical lines with the same slope are parallel and nonvertical lines whose slopes are negative reciprocals are perpendicular.

Parallel lines Perpendicular lines

Figure P.19

STUDY TIP In mathematics, the phrase "if and only if" is a way of stating two implications in one statement. For instance, the first statement at the right could be rewritten as the following two implications.
(a) If two distinct nonvertical lines are parallel, then their slopes are equal.
(b) If two distinct nonvertical lines have equal slopes, then they are parallel.

Parallel and Perpendicular Lines

1. Two distinct nonvertical lines are **parallel** if and only if their slopes are equal—that is, if and only if $m_1 = m_2$.
2. Two nonvertical lines are **perpendicular** if and only if their slopes are negative reciprocals of each other—that is, if and only if

$$m_1 = -\frac{1}{m_2}.$$

Summaries

Many sections have summaries that identify core ideas and procedures. In some instances, an entire section summarizes the preceding topics.

Definitions and Theorems

All definitions and theorems are highlighted for emphasis and easy reference.

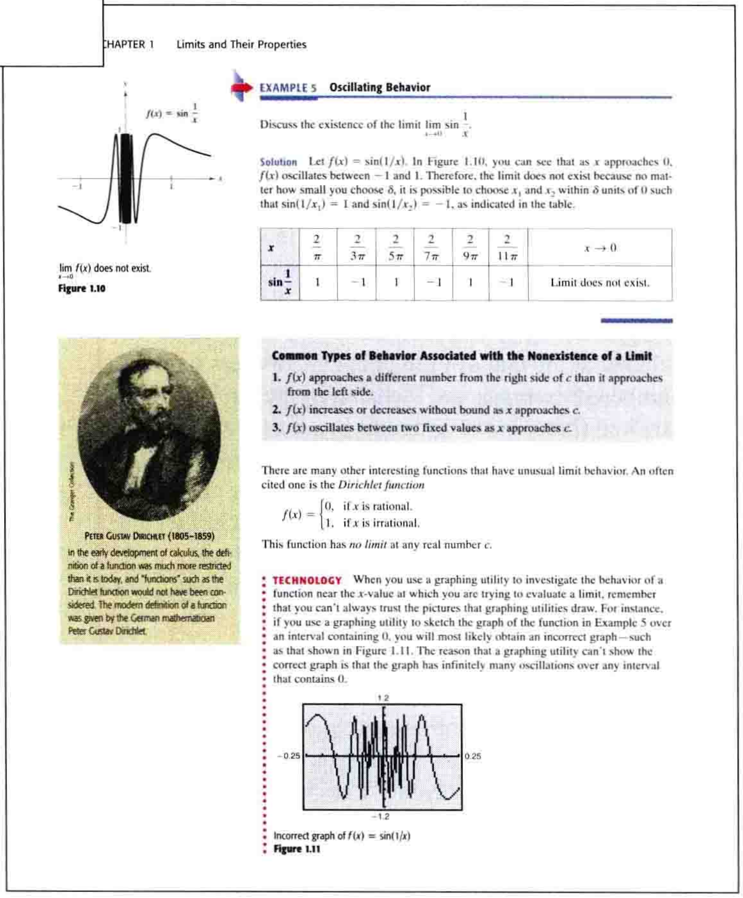
FEATURES

Technology

Students are encouraged to use a graphing utility or computer algebra system as a tool for exploration, discovery, and problem solving. Many opportunities to execute complicated computations, to visualize theoretical concepts, to discover alternative approaches, and to verify the results of other solution methods using technology are presented. However, students are not required to have access to a graphing utility to use this text effectively. In addition to describing the benefits of using technology, the text also pays special attention to its possible misuse or misinterpretation.

Historical Notes

Historical notes are integrated throughout the text to help students understand that calculus has a past.



CHAPTER 1 Limits and Their Properties

$f(x) = \sin\frac{1}{x}$

$\lim_{x\to 0} f(x)$ does not exist.

Figure 1.10

EXAMPLE 5 Oscillating Behavior

Discuss the existence of the limit $\lim_{x\to 0} \sin\frac{1}{x}$.

Solution Let $f(x) = \sin(1/x)$. In Figure 1.10, you can see that as x approaches 0, $f(x)$ oscillates between -1 and 1. Therefore, the limit does not exist because no matter how small you choose δ, it is possible to choose x_1 and x_2 within δ units of 0 such that $\sin(1/x_1) = 1$ and $\sin(1/x_2) = -1$, as indicated in the table.

x	$\frac{2}{\pi}$	$\frac{2}{3\pi}$	$\frac{2}{5\pi}$	$\frac{2}{7\pi}$	$\frac{2}{9\pi}$	$\frac{2}{11\pi}$	$x \to 0$
$\sin\frac{1}{x}$	1	-1	1	-1	1	-1	Limit does not exist.

Common Types of Behavior Associated with the Nonexistence of a Limit

1. $f(x)$ approaches a different number from the right side of c than it approaches from the left side.
2. $f(x)$ increases or decreases without bound as x approaches c.
3. $f(x)$ oscillates between two fixed values as x approaches c.

The Granger Collection

PETER GUSTAV DIRICHLET (1805–1859)

In the early development of calculus, the definition of a function was much more restricted than it is today, and "functions" such as the Dirichlet function would not have been considered. The modern definition of a function was given by the German mathematician Peter Gustav Dirichlet.

There are many other interesting functions that have unusual limit behavior. An often cited one is the *Dirichlet function*

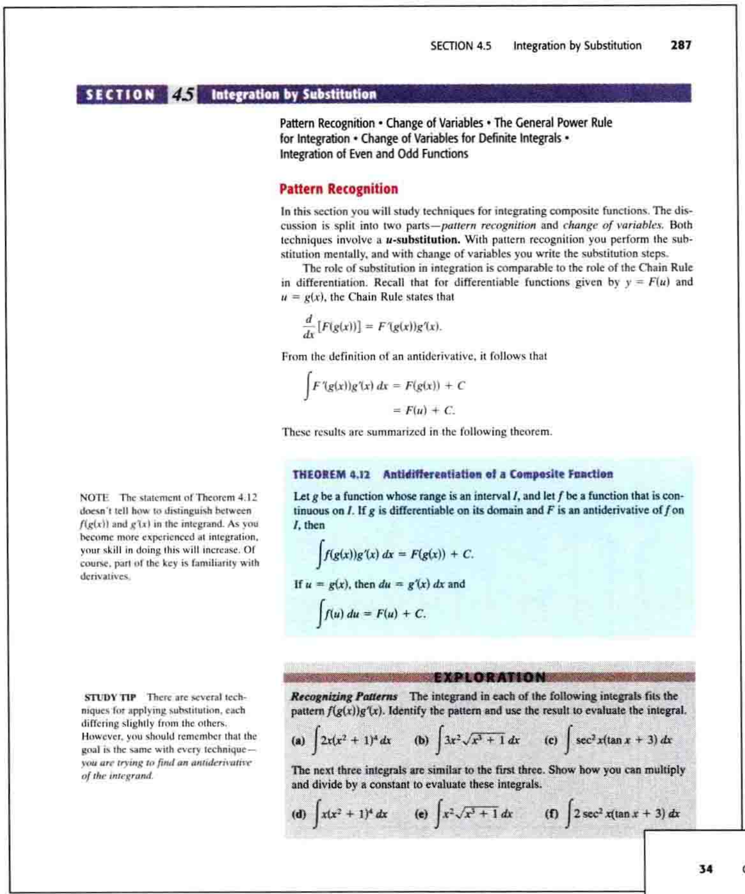
$$f(x) = \begin{cases} 0, & \text{if } x \text{ is rational.} \\ 1, & \text{if } x \text{ is irrational.} \end{cases}$$

This function has *no limit* at any real number c.

TECHNOLOGY When you use a graphing utility to investigate the behavior of a function near the x-value at which you are trying to evaluate a limit, remember that you can't always trust the pictures that graphing utilities draw. For instance, if you use a graphing utility to sketch the graph of the function in Example 5 over an interval containing 0, you will most likely obtain an incorrect graph—such as that shown in Figure 1.11. The reason that a graphing utility can't show the correct graph is that the graph has infinitely many oscillations over any interval that contains 0.

Incorrect graph of $f(x) = \sin(1/x)$

Figure 1.11



SECTION 4.5 Integration by Substitution 287

SECTION 4.5 Integration by Substitution

Pattern Recognition • Change of Variables • The General Power Rule for Integration • Change of Variables for Definite Integrals • Integration of Even and Odd Functions

Pattern Recognition

In this section you will study techniques for integrating composite functions. The discussion is split into two parts—*pattern recognition* and *change of variables.* Both techniques involve a **u-substitution.** With pattern recognition you perform the substitution mentally, and with change of variables you write the substitution steps.

The role of substitution in integration is comparable to the role of the Chain Rule in differentiation. Recall that for differentiable functions given by $y = F(u)$ and $u = g(x)$, the Chain Rule states that

$$\frac{d}{dx}[F(g(x))] = F'(g(x))g'(x).$$

From the definition of an antiderivative, it follows that

$$\int F'(g(x))g'(x)\,dx = F(g(x)) + C$$
$$= F(u) + C.$$

These results are summarized in the following theorem.

NOTE The statement of Theorem 4.12 doesn't tell how to distinguish between $f(g(x))$ and $g'(x)$ in the integrand. As you become more experienced at integration, your skill in doing this will increase. Of course, part of the key is familiarity with derivatives.

THEOREM 4.12 Antidifferentiation of a Composite Function

Let g be a function whose range is an interval I, and let f be a function that is continuous on I. If g is differentiable on its domain and F is an antiderivative of f on I, then

$$\int f(g(x))g'(x)\,dx = F(g(x)) + C.$$

If $u = g(x)$, then $du = g'(x)\,dx$ and

$$\int f(u)\,du = F(u) + C.$$

STUDY TIP There are several techniques for applying substitution, each differing slightly from the others. However, you should remember that the goal is the same with every technique—*you are trying to find an antiderivative of the integrand.*

EXPLORATION

Recognizing Patterns The integrand in each of the following integrals fits the pattern $f(g(x))g'(x)$. Identify the pattern and use the result to evaluate the integral.

(a) $\int 2x(x^2 + 1)^4\,dx$ (b) $\int 3x^2\sqrt{x^3 + 1}\,dx$ (c) $\int \sec^2 x(\tan x + 3)\,dx$

The next three integrals are similar to the first three. Show how you can multiply and divide by a constant to evaluate these integrals.

(d) $\int x(x^2 + 1)^4\,dx$ (e) $\int x^2\sqrt{x^3 + 1}\,dx$ (f) $\int 2\sec^2 x(\tan x + 3)\,dx$

Notes

Many instructional notes accompany definitions, theorems, and examples to give additional insight or describe generalizations.

Study Tips

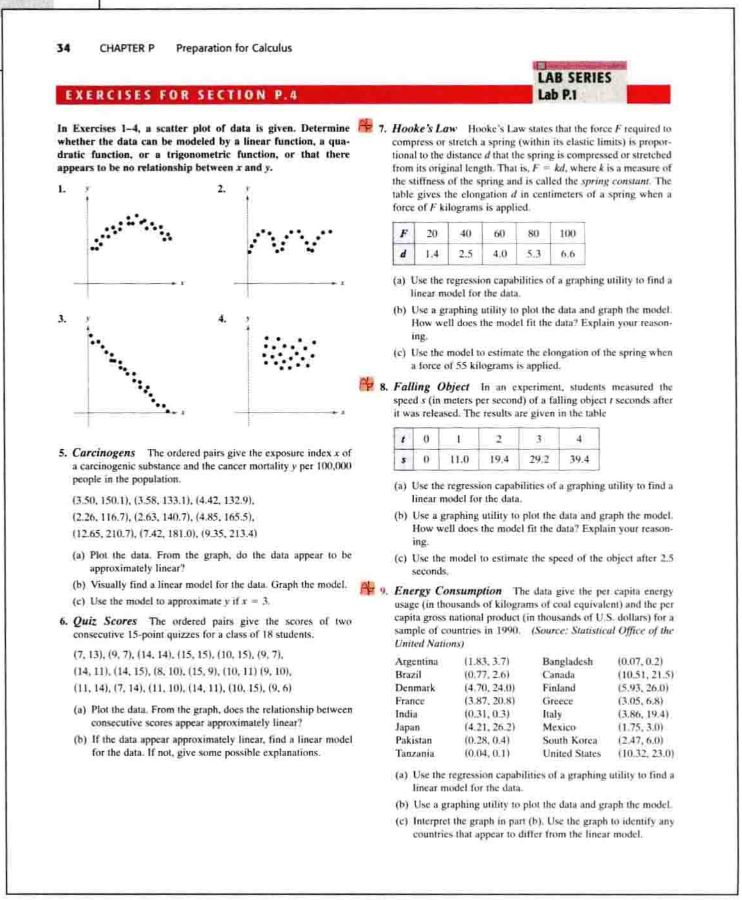
Throughout the text, *Study Tips* help students avoid common errors, address special cases, and expand on theoretical concepts.

Exercises

The text contains nearly 10,000 exercises. Each exercise set is graded, progressing from skill-development problems to more challenging problems involving applications and proofs. The wide variety of exercises includes many real, technology-oriented, thought-provoking, and engaging problems. Review exercises are included at the end of each chapter. Answers to all odd-numbered exercises are included in the back of the text. Red (blue, in the appendix) exercise numbers indicate selected exercises that can be found in the *Study and Solutions Guide.* To help instructors make homework assignments, many of the exercises in the text are labeled to indicate the area of application (e.g., *Break-Even Point*) or the type of exercise (e.g., *Writing* or *Approximation*).

Lab Series

The real-life application introduced in each *Motivating the Chapter* forms the basis for extended lab projects in the supplementary Lab Manuals using *Maple, Mathematica, Derive, Mathcad,* and the *TI-92* graphing calculator. The labs are referenced in the text where it seemed most appropriate to assign them.



34 CHAPTER P Preparation for Calculus

EXERCISES FOR SECTION P.4

LAB SERIES
Lab P.1

In Exercises 1–4, a scatter plot of data is given. Determine whether the data can be modeled by a linear function, a quadratic function, or a trigonometric function, or that there appears to be no relationship between x and y.

1.

2.

3.

4.

5. ***Carcinogens*** The ordered pairs give the exposure index x of a carcinogenic substance and the cancer mortality y per 100,000 people in the population.

(3.50, 150.1), (3.58, 133.1), (4.42, 132.9),
(2.26, 116.7), (2.63, 140.7), (4.85, 165.5),
(12.65, 210.7), (7.42, 181.0), (9.35, 213.4)

(a) Plot the data. From the graph, do the data appear to be approximately linear?
(b) Visually find a linear model for the data. Graph the model.
(c) Use the model to approximate y if $x = 3$.

6. ***Quiz Scores*** The ordered pairs give the scores of two consecutive 15-point quizzes for a class of 18 students.

(7, 13), (9, 7), (14, 14), (15, 15), (10, 15), (9, 7),
(14, 11), (14, 15), (8, 10), (15, 9), (10, 11) (9, 10),
(11, 14), (7, 14), (11, 10), (14, 11), (10, 15), (9, 6)

(a) Plot the data. From the graph, does the relationship between consecutive scores appear approximately linear?
(b) If the data appear approximately linear, find a linear model for the data. If not, give some possible explanations.

7. ***Hooke's Law*** Hooke's Law states that the force F required to compress or stretch a spring (within its elastic limits) is proportional to the distance d that the spring is compressed or stretched from its original length. That is, $F = kd$, where k is a measure of the stiffness of the spring and is called the *spring constant.* The table gives the elongation d in centimeters of a spring when a force of F kilograms is applied.

F	20	40	60	80	100
d	1.4	2.5	4.0	5.3	6.6

(a) Use the regression capabilities of a graphing utility to find a linear model for the data.
(b) Use a graphing utility to plot the data and graph the model. How well does the model fit the data? Explain your reasoning.
(c) Use the model to estimate the elongation of the spring when a force of 55 kilograms is applied.

8. ***Falling Object*** In an experiment, students measured the speed s (in meters per second) of a falling object t seconds after it was released. The results are given in the table

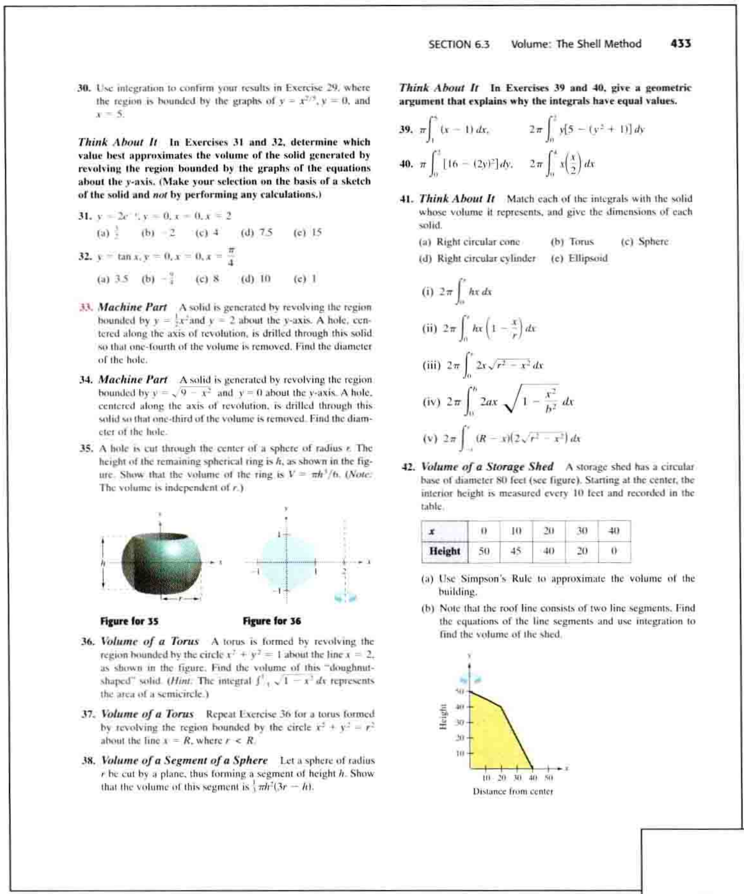
t	0	1	2	3	4
s	0	11.0	19.4	29.2	39.4

(a) Use the regression capabilities of a graphing utility to find a linear model for the data.
(b) Use a graphing utility to plot the data and graph the model. How well does the model fit the data? Explain your reasoning.
(c) Use the model to estimate the speed of the object after 2.5 seconds.

9. ***Energy Consumption*** The data give the per capita energy usage (in thousands of kilograms of coal equivalent) and the per capita gross national product (in thousands of U.S. dollars) for a sample of countries in 1990. *(Source: Statistical Office of the United Nations)*

Argentina	(1.83, 3.7)	Bangladesh	(0.07, 0.2)
Brazil	(0.77, 2.6)	Canada	(10.51, 21.5)
Denmark	(4.70, 24.0)	Finland	(5.93, 26.0)
France	(3.87, 20.8)	Greece	(3.05, 6.8)
India	(0.31, 0.3)	Italy	(3.86, 19.4)
Japan	(4.21, 26.2)	Mexico	(1.75, 3.0)
Pakistan	(0.28, 0.4)	South Korea	(2.47, 6.0)
Tanzania	(0.04, 0.1)	United States	(10.32, 23.0)

(a) Use the regression capabilities of a graphing utility to find a linear model for the data.
(b) Use a graphing utility to plot the data and graph the model.
(c) Interpret the graph in part (b). Use the graph to identify any countries that appear to differ from the linear model.



30. Use integration to confirm your results in Exercise 29, where the region is bounded by the graphs of $y = x^{2/5}$, $y = 0$, and $x = 5$.

Think About It **In Exercises 31 and 32, determine which value best approximates the volume of the solid generated by revolving the region bounded by the graphs of the equations about the *y*-axis. (Make your selection on the basis of a sketch of the solid and *not* by performing any calculations.)**

31. $y = 2e^{-x}$, $y = 0$, $x = 0$, $x = 2$
(a) $\frac{3}{2}$ (b) -2 (c) 4 (d) 7.5 (e) 15

32. $y = \tan x$, $y = 0$, $x = 0$, $x = \frac{\pi}{4}$
(a) 3.5 (b) $-\frac{9}{4}$ (c) 8 (d) 10 (e) 1

33. ***Machine Part*** A solid is generated by revolving the region bounded by $y = \frac{1}{2}x^2$ and $y = 2$ about the *y*-axis. A hole, centered along the axis of revolution, is drilled through this solid so that one-fourth of the volume is removed. Find the diameter of the hole.

34. ***Machine Part*** A solid is generated by revolving the region bounded by $y = \sqrt{9 - x^2}$ and $y = 0$ about the *y*-axis. A hole, centered along the axis of revolution, is drilled through this solid so that one-third of the volume is removed. Find the diameter of the hole.

35. A hole is cut through the center of a sphere of radius r. The height of the remaining spherical ring is h, as shown in the figure. Show that the volume of the ring is $V = \pi h^3/6$. (*Note:* The volume is independent of r.)

Figure for 35 **Figure for 36**

36. ***Volume of a Torus*** A torus is formed by revolving the region bounded by the circle $x^2 + y^2 = 1$ about the line $x = 2$, as shown in the figure. Find the volume of this "doughnut-shaped" solid. (*Hint:* The integral $\int_{-1}^{1} \sqrt{1 - x^2}\,dx$ represents the area of a semicircle.)

37. ***Volume of a Torus*** Repeat Exercise 36 for a torus formed by revolving the region bounded by the circle $x^2 + y^2 = r^2$ about the line $x = R$, where $r < R$.

38. ***Volume of a Segment of a Sphere*** Let a sphere of radius r be cut by a plane, thus forming a segment of height h. Show that the volume of this segment is $\frac{1}{3}\pi h^2(3r - h)$.

Think About It **In Exercises 39 and 40, give a geometric argument that explains why the integrals have equal values.**

39. $\pi\int_1^5 (x - 1)\,dx$, $\quad 2\pi\int_0^2 y[5 - (y^2 + 1)]\,dy$

40. $\pi\int_0^2 [16 - (2y)^2]\,dy$, $\quad 2\pi\int_0^4 x\left(\frac{x}{2}\right)dx$

41. ***Think About It*** Match each of the integrals with the solid whose volume it represents, and give the dimensions of each solid.
(a) Right circular cone (b) Torus (c) Sphere
(d) Right circular cylinder (e) Ellipsoid

(i) $2\pi\int_0^r hx\,dx$
(ii) $2\pi\int_0^r hx\left(1 - \frac{x}{r}\right)dx$
(iii) $2\pi\int_0^r 2x\sqrt{r^2 - x^2}\,dx$
(iv) $2\pi\int_0^b 2ax\sqrt{1 - \frac{x^2}{b^2}}\,dx$
(v) $2\pi\int_{-r}^{r} (R - x)(2\sqrt{r^2 - x^2})\,dx$

42. ***Volume of a Storage Shed*** A storage shed has a circular base of diameter 80 feet (see figure). Starting at the center, the interior height is measured every 10 feet and recorded in the table.

x	0	10	20	30	40
Height	50	45	40	20	0

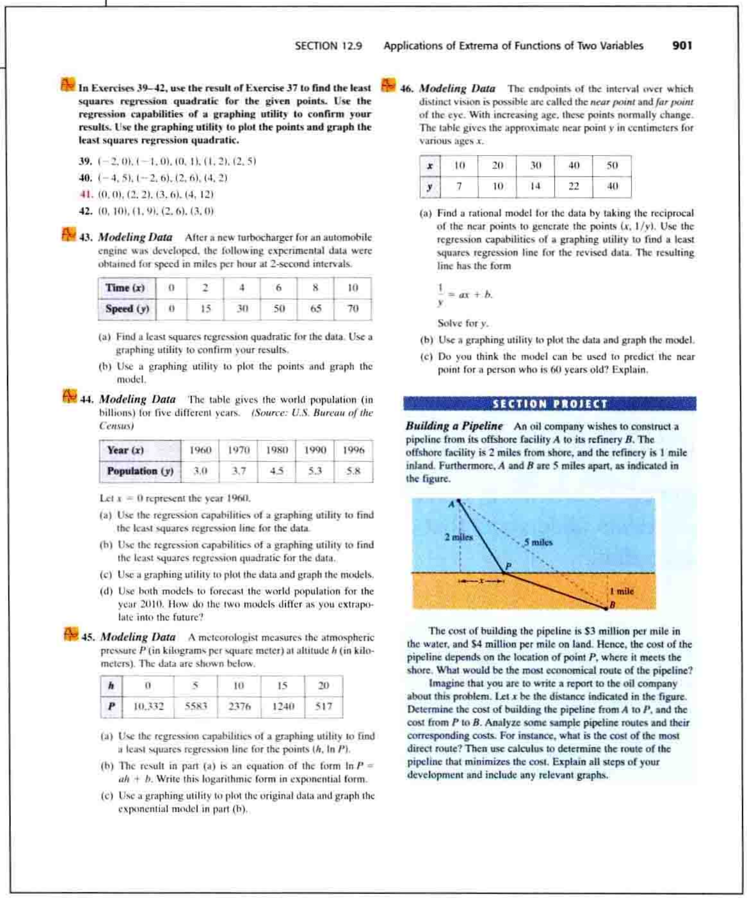
(a) Use Simpson's Rule to approximate the volume of the building.
(b) Note that the roof line consists of two line segments. Find the equations of the line segments and use integration to find the volume of the shed.

Think About It

These exercises are thought-provoking, conceptual problems that help students grasp the underlying theories.

Applications

A wide variety of relevant application problems are included to demonstrate clearly the real-world usage of mathematics.



In Exercises 39–42, use the result of Exercise 37 to find the least squares regression quadratic for the given points. Use the regression capabilities of a graphing utility to confirm your results. Use the graphing utility to plot the points and graph the least squares regression quadratic.

39. $(-2, 0), (-1, 0), (0, 1), (1, 2), (2, 5)$
40. $(-4, 5), (-2, 6), (2, 6), (4, 2)$
41. $(0, 0), (2, 2), (3, 6), (4, 12)$
42. $(0, 10), (1, 9), (2, 6), (3, 0)$

43. ***Modeling Data*** After a new turbocharger for an automobile engine was developed, the following experimental data were obtained for speed in miles per hour at 2-second intervals.

Time (x)	0	2	4	6	8	10
Speed (y)	0	15	30	50	65	70

(a) Find a least squares regression quadratic for the data. Use a graphing utility to confirm your results.
(b) Use a graphing utility to plot the points and graph the model.

44. ***Modeling Data*** The table gives the world population (in billions) for five different years. (*Source: U.S. Bureau of the Census*)

Year (x)	1960	1970	1980	1990	1996
Population (y)	3.0	3.7	4.5	5.3	5.8

Let $x = 0$ represent the year 1960.
(a) Use the regression capabilities of a graphing utility to find the least squares regression line for the data.
(b) Use the regression capabilities of a graphing utility to find the least squares regression quadratic for the data.
(c) Use a graphing utility to plot the data and graph the models.
(d) Use both models to forecast the world population for the year 2010. How do the two models differ as you extrapolate into the future?

45. ***Modeling Data*** A meteorologist measures the atmospheric pressure P (in kilograms per square meter) at altitude h (in kilometers). The data are shown below.

h	0	5	10	15	20
P	10,332	5583	2376	1240	517

(a) Use the regression capabilities of a graphing utility to find a least squares regression line for the points $(h, \ln P)$.
(b) The result in part (a) is an equation of the form $\ln P = ah + b$. Write this logarithmic form in exponential form.
(c) Use a graphing utility to plot the original data and graph the exponential model in part (b).

46. ***Modeling Data*** The endpoints of the interval over which distinct vision is possible are called the *near point* and *far point* of the eye. With increasing age, these points normally change. The table gives the approximate near point y in centimeters for various ages x.

x	10	20	30	40	50
y	7	10	14	22	40

(a) Find a rational model for the data by taking the reciprocal of the near points to generate the points $(x, 1/y)$. Use the regression capabilities of a graphing utility to find a least squares regression line for the revised data. The resulting line has the form

$$\frac{1}{y} = ax + b.$$

Solve for y.
(b) Use a graphing utility to plot the data and graph the model.
(c) Do you think the model can be used to predict the near point for a person who is 60 years old? Explain.

SECTION PROJECT

Building a Pipeline An oil company wishes to construct a pipeline from its offshore facility A to its refinery B. The offshore facility is 2 miles from shore, and the refinery is 1 mile inland. Furthermore, A and B are 5 miles apart, as indicated in the figure.

The cost of building the pipeline is \$3 million per mile in the water, and \$4 million per mile on land. Hence, the cost of the pipeline depends on the location of point P, where it meets the shore. What would be the most economical route of the pipeline?

Imagine that you are to write a report to the oil company about this problem. Let x be the distance indicated in the figure. Determine the cost of building the pipeline from A to P, and the cost from P to B. Analyze some sample pipeline routes and their corresponding costs. For instance, what is the cost of the most direct route? Then use calculus to determine the route of the pipeline that minimizes the cost. Explain all steps of your development and include any relevant graphs.

Modeling Data

These new multipart questions ask students to find and interpret mathematical models from real-life data. Often the questions are enhanced by the use of a graphing utility.

Section Projects

New to the Sixth Edition, *Section Projects* appear at the end of selected exercise sets. These extended applications can be assigned to individual students or to groups of students in a peer-assisted learning environment.

FEATURES

REVIEW EXERCISES 815

LAB SERIES
Lab 11.2

REVIEW EXERCISES FOR CHAPTER 11

In Exercises 1–4, (a) find the domain of r and (b) determine the values (if any) of t for which the function is continuous.

1. $\mathbf{r}(t) = t\mathbf{i} + \csc t\mathbf{k}$
2. $\mathbf{r}(t) = \sqrt{t}\mathbf{i} + \frac{1}{t-4}\mathbf{j} + \mathbf{k}$
3. $\mathbf{r}(t) = \ln t\mathbf{i} + t\mathbf{j} + t\mathbf{k}$
4. $\mathbf{r}(t) = (2t+1)\mathbf{i} + t^2\mathbf{j} + t\mathbf{k}$

In Exercises 5 and 6, evaluate (if possible) the vector-valued function at the indicated value of t.

5. $\mathbf{r}(t) = (2t+1)\mathbf{i} + t^2\mathbf{j} - \frac{1}{3}t^3\mathbf{k}$
   (a) $\mathbf{r}(0)$ (b) $\mathbf{r}(-2)$ (c) $\mathbf{r}(c-1)$ (d) $\mathbf{r}(1+\Delta t) - \mathbf{r}(1)$
6. $\mathbf{r}(t) = 3\cos t\mathbf{i} + (1 - \sin t)\mathbf{j} - t\mathbf{k}$
   (a) $\mathbf{r}(0)$ (b) $\mathbf{r}(\frac{\pi}{2})$ (c) $\mathbf{r}(s-\pi)$ (d) $\mathbf{r}(\pi+\Delta t) - \mathbf{r}(\pi)$

Think About It **In Exercises 7 and 8, find $\mathbf{r}(t) \cdot \mathbf{u}(t)$. Is the result a vector-valued function? Explain.**

7. $\mathbf{r}(t) = \tan t\mathbf{i} - \sec t\mathbf{j} + (1-t)\mathbf{k}$
   $\mathbf{u}(t) = 2\cos t\mathbf{i} + 3\sin t\cos t\mathbf{j} + 1\mathbf{k}$
8. $\mathbf{r}(t) = \langle e^t, e^{-t}, 2\rangle$, $\mathbf{u}(t) = \langle 4e^{-t}, 2, t\rangle$

In Exercises 9 and 10, sketch the plane curve represented by the vector-valued function.

9. $\mathbf{r}(t) = \langle \cos t, 2\sin^2 t\rangle$
10. $\mathbf{r}(t) = \langle t, t/(t-1)\rangle$

In Exercises 11–14, use a computer algebra system to graph the space curve represented by the vector-valued function.

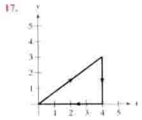
11. $\mathbf{r}(t) = \mathbf{i} + t\mathbf{j} + t^2\mathbf{k}$
12. $\mathbf{r}(t) = 2t\mathbf{i} + t\mathbf{j} + t^2\mathbf{k}$
13. $\mathbf{r}(t) = \langle 1, \sin t, 1\rangle$
14. $\mathbf{r}(t) = \langle 2\cos t, t, 2\sin t\rangle$

In Exercises 15 and 16, use a graphing utility to graph the space curve represented by the vector-valued function.

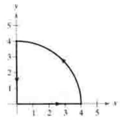
15. $\mathbf{r}(t) = \langle t, \ln t, \frac{1}{2}t^2\rangle$
16. $\mathbf{r}(t) = \langle \frac{1}{2}t, \sqrt{t}, \frac{1}{4}t^3\rangle$

In Exercises 17 and 18, find vector-valued functions forming the boundaries of the region in the figure.

17.



18.



19. A particle moves on a straight-line path that passes through the points $(-2, -3, 8)$ and $(5, 1, -2)$. Find a vector-valued function for the path. (There are many correct answers.)

20. The outer edge of a spiral staircase is in the shape of a helix of radius 2 meters. The staircase has a height of 2 meters and is three-fourths of one complete revolution from bottom to top. Find a vector-valued function for the helix. (There are many correct answers.)

In Exercises 21 and 22, sketch the space curve represented by the intersection of the surfaces. Use the parameter $x = t$ to find a vector-valued function for the space curve.

21. $z = x^2 + y^2$, $x + y = 0$
22. $x^2 + z^2 = 4$, $x - y = 0$

In Exercises 23 and 24, find the limit.

23. $\lim_{t\to 2} (t^2\mathbf{i} + \sqrt{4-t^2}\mathbf{j} + \mathbf{k})$
24. $\lim_{t\to 0} \left(\frac{\sin 2t}{t}\mathbf{i} + e^{-t}\mathbf{j} + e^t\mathbf{k}\right)$

In Exercises 25 and 26, find the following.

(a) $\mathbf{r}'(t)$ (b) $\mathbf{r}''(t)$ (c) $D_t[\mathbf{r}(t) \cdot \mathbf{u}(t)]$
(d) $D_t[\mathbf{u}(t) - 2\mathbf{r}(t)]$ (e) $D_t[\|\mathbf{r}(t)\|]$, $t > 0$ (f) $D_t[\mathbf{r}(t) \times \mathbf{u}(t)]$

25. $\mathbf{r}(t) = 3t\mathbf{i} + (t-1)\mathbf{j}$, $\mathbf{u}(t) = t\mathbf{i} + t^2\mathbf{j} + \frac{2}{3}t^3\mathbf{k}$
26. $\mathbf{r}(t) = \sin t\mathbf{i} + \cos t\mathbf{j} + t\mathbf{k}$, $\mathbf{u}(t) = \sin t\mathbf{i} + \cos t\mathbf{j} + \frac{1}{t}\mathbf{k}$

In Exercises 27 and 28, find a set of parametric equations for the line tangent to the space curve at the indicated point.

27. $\mathbf{r}(t) = 2\cos t\mathbf{i} + 2\sin t\mathbf{j} + t\mathbf{k}$, $t = \frac{3\pi}{4}$
28. $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + \frac{2}{3}t^3\mathbf{k}$, $t = 2$

29. ***Writing*** The x- and y-components of the derivative of the vector-valued function $\mathbf{u}$ are positive at $t = t_0$, and the z-component is negative. Describe the behavior of $\mathbf{u}$ at $t = t_0$.

30. ***Writing*** The x-component of the derivative of the vector-valued function $\mathbf{u}$ is 0 for t in the domain of the function. What does this information imply about the graph of $\mathbf{u}$?

Linear Approximation **In Exercises 31 and 32, find a set of parametric equations for the tangent line to the graph of the vector-valued function at $t = t_0$. Use the equations for the line to approximate $\mathbf{r}(t_0 + 0.1)$.**

31. $\mathbf{r}(t) = \ln(t-3)\mathbf{i} + t^2\mathbf{j} + \frac{1}{2}t\mathbf{k}$, $t_0 = 4$
32. $\mathbf{r}(t) = 3\cosh t\mathbf{i} + \sinh t\mathbf{j} - 2t\mathbf{k}$, $t_0 = 0$

Writing

To develop students' reasoning skills and make them comfortable discussing mathematical concepts, the text now contains many *Writing* exercises.

Graphing Utilities

Many exercises in the text can be solved using technology; however, the symbol identifies all exercises in which students are specifically instructed to use a graphing utility or a computer algebra system.

True or False?

To help students understand the logical structure of calculus, a set of "true or false" questions is included toward the ends of many exercise sets. These questions help students focus on concepts, common errors, and the correct statements of definitions and theorems.

Journal References

References to articles in readily available journals help students understand that calculus is a current, dynamic field.

CHAPTER 3 Applications of Differentiation

48. ***Inventory Cost*** A retailer has determined that the cost C of ordering and storing x units of a certain product is

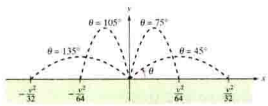
$$C = 2x + \frac{300{,}000}{x}, \quad 1 \le x \le 300.$$

The delivery truck can bring at most 300 units per order. Find the order size that will minimize cost. Could the cost be decreased if the truck were replaced with one that could bring at most 400 units? Explain.

49. ***Lawn Sprinkler*** A lawn sprinkler is constructed in such a way that $d\theta/dt$ is constant, where θ ranges between 45° and 135° (see figure). The distance the water travels horizontally is

$$x = \frac{v^2 \sin 2\theta}{32}, \quad \frac{\pi}{4} \le \theta \le \frac{3\pi}{4}$$

where v is the speed of the water. Find dx/dt and explain why this lawn sprinkler does not water evenly. What part of the lawn receives the most water?



Water sprinkler: $45° \le \theta \le 135°$

FOR FURTHER INFORMATION For more information on the "calculus of lawn sprinklers," see the article "Design of an Oscillating Sprinkler" by Bart Braden in the January 1985 issue of *Mathematics Magazine*.

50. ***Modeling Data*** The defense outlays as percents of the gross domestic product for the years 1976 through 1995 are as follows. *(Source: U.S. Office of Management and Budget)*

1976: (5.3%); 1977: (5.1%); 1978: (4.8%); 1979: (4.8%);
1980: (5.1%); 1981: (5.3%); 1982: (5.9%); 1983: (6.3%);
1984: (6.2%); 1985: (6.4%); 1986: (6.5%); 1987: (6.3%);
1988: (6.0%); 1989: (5.9%); 1990: (5.5%); 1991: (4.8%);
1992: (5.0%); 1993: (4.7%); 1994: (4.2%); 1995: (3.9%);

(a) Use the regression capabilities of a graphing utility to find a model of the form $y = at^4 + bt^3 + ct^2 + dt + e$ for the data. (Let t represent the time in years, with $t = 0$ corresponding to 1980.)
(b) Use a graphing utility to plot the data and graph the model.
(c) Locate the absolute extrema of the model on the interval $[-4, 15]$.

51. ***Honeycomb*** The surface area of a cell in a honeycomb is

$$S = 6hs + \frac{3s^2}{2}\left(\frac{\sqrt{3} - \cos\theta}{\sin\theta}\right)$$

where h and s are positive constants and θ is the angle at which the upper faces meet the altitude of the cell. Find the angle θ $(\pi/6 \le \theta \le \pi/2)$ that minimizes the surface area S.

FOR FURTHER INFORMATION For more information on the geometric structure of a honeycomb cell, see the article "The Design of Honeycombs" by Anthony L. Peressini in UMAP Module 502, published by COMAP, Inc., Suite 210, 57 Bedford Street, Lexington, MA.

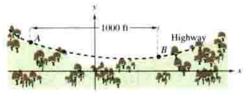
52. ***Highway Design*** In order to build a highway it is necessary to fill a section of a valley where the grades (slopes) of the sides are 6% and 9% (see figure). The top of the filled region will have the shape of a parabolic arc that is tangent to the two slopes at the points A and B. The horizontal distance between the points A and B is 1000 feet.

(a) Find a quadratic function $y = ax^2 + bx + c$, $-500 \le x \le 500$, that describes the top of the filled region.
(b) Complete the table giving the depths d of the fill at the specified values of x.

x	−500	−400	−300	−200	−100
d					

x	0	100	200	300	400	500
d						

(c) What will be the lowest point on the completed highway? Will it be directly over the point where the two hillsides come together?



True or False? **In Exercises 53–56, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.**

53. The maximum of a function that is continuous on a closed interval can occur at two different values in the interval.
54. If a function is continuous on a closed interval, then it must have a minimum on the interval.
55. If $x = c$ is a critical number of the function f, then it is also a critical number of the function $g(x) = f(x) + k$, where k is a constant.
56. If $x = c$ is a critical number of the function f, then it is also a critical number of the function $g(x) = f(x - k)$, where k is a constant.

57. Find all critical numbers of the greatest integer function $f(x) = [\![x]\!]$.

Supplements

Calculus with Analytic Geometry, Sixth Edition, by Larson, Hostetler, and Edwards, is accompanied by a comprehensive supplements package with ancillaries for students, for instructors, and for classroom resource. Most items are keyed directly to the book.

Printed Resources

For the Instructor

Instructor's Resource Guide by Ann R. Kraus, The Pennsylvania State University, The Behrend College

- Notes to the instructor
- Chapter summaries
- Ready-made chapter tests and finals
- Gateway tests
- Teaching strategies
- Sample syllabi
- Suggested solutions for *Exploration, Technology,* and *Motivating the Chapter* features
- Solutions to *Section Projects*

Complete Solutions Guide, Volumes I, II, and III, by Bruce H. Edwards, University of Florida

- Detailed solutions to all text exercises

Test Item File by Ann R. Kraus, The Pennsylvania State University, The Behrend College

- Printed test bank
- Approximately 3000 test items
- Multiple-choice and open-ended questions coded by level of difficulty
- Technology required to solve some test questions
- Also available as test-generating software

Graphing Calculator Demonstration Problems by August J. Zarcone and Russell Lundstrom, College of DuPage

- Classroom demonstration problems using the *TI-85* graphing calculator

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