Reliability and Quality Control

Edited by
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RELIABILITY AND QUALITY CONTROL

edited by

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PREFACE

An international conference on Reliability and Quality Control was held at the University of Missouri-Columbia, Missouri from June 4 - 8, 1984. The purpose of the conference was to review recent developments in those areas and to stimulate interaction among the leading researchers in the world.

This volume consists of refereed invited papers presented at the conference. Here a number of distinguished workers present important results on a broad spectrum of topics in Reliability and Quality Control. The topics covered include acceptance sampling, combining experts' opinions, control charts, distributions with wearout, inference procedures, multivariate distributions, multistate reliability, reliability algorithm, reliability models, repairable systems, software reliability and system reliability. Because of overlap of topics within a paper, instead of classifying according to topics, these papers have been presented in alphabetical order by the first author. The volume will be of interest to mathematical statisticians, probablists, and engineers interested in reliability and quality control.

A list of the titles of the papers presented in the contributed paper sessions, along with the names of the authors, is given at the end of the volume. Many of these papers will be published in a special 'Reliability' issue of the Journal of Statistical Planning and Inference. This issue is currently being edited by Professor J. N. Srivastava and me.

I would like to thank a number of persons who cooperated actively in organizing the conference. R. E. Barlow, B. Hoadley, N. R. Mann, G. McDonald, F. Proschan, J. Rosenblatt and N. D. Singpurwalla were the members of the advisory committee. Dr. M. D. Glick, Dean of Arts and Science, University of Missouri-Columbia, Major B. W. Woodruff of U. S. Air Force Office of Scientific Research and Dr. R. L. Launer of U. S. Army Research Office were kind enough to make some opening remarks. I am grateful to S. J. Amster, H. Ascher, R. E. Barlow, S. Blumenthal, R. L. Chaddha, J. E. Hewett, B. Hoadley, R. A. Johnson, R. L. Launer, R. W. Madsen, N. R. Mann, W. J. Padgett, F. Proschan, N. D. Singpurwalla, R. T. Smythe, J. N. Srivastava and B. W. Woodruff for presiding over different sessions. Thanks are also due to the following for serving as referees of various papers in the volume: S. J. Amster, R. E. Barlow, S. K. Basu, M. C. Bhattacharjee, P. J. Boland, D. Z. Du, R. L. Dykstra, N. Ebrahimi, E. El-Neweihi, J. L. Folks, M. Ghosh, W. S. Griffith, B. Harris, B. Hoadley, J. Kadane, S. N. U. A. Kirmani, J. P. Klein, P. W. Laud, R. W. Madsen, N. Mukhopadhyay, S. E. Rigdon, E. M. Scheuer, J. Sethuraman, T. Seidenfeld, P. L. Speckman, F. W. Spencer and Y. L. Tong.

I am grateful to the contributors to this volume and to Dr. Gerard Wanrooy and other staff members at Elsevier Science Publishers B.V. (North-Holland) for their excellent cooperation. Special thanks are due to the U. S. Air Force Office of Scientific Research, U. S. Army Research Office, National Science Foundation and the University of Missouri for sponsoring the symposium.

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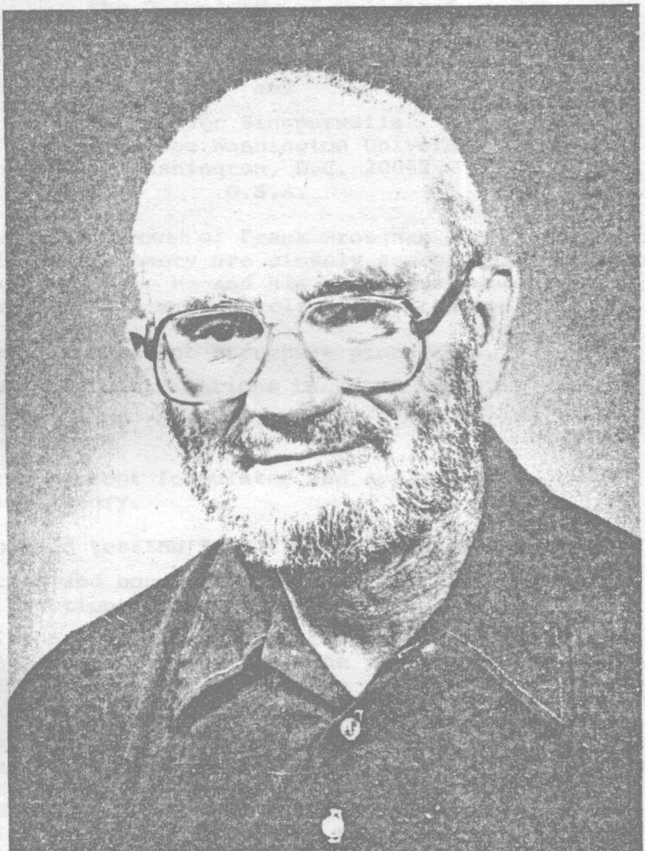

This volume is dedicated to

Frank Proschan

in recognition of his many pioneering contributions

in reliability theory

FRANK PROSCHAN AND RELIABILITY THEORY

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The professional growth of Frank Proschan and the technical growth of reliability theory are closely associated. (See Esary, Proschan, Walkup, 1967). He and his colleagues were responsible for most of the key ideas in modern reliability theory:

Development of coherent structure properties.

Classes of life distributions based on aging.

Symbiotic union of reliability theory and total positivity (TP).

Association concept formulated and applied to reliability theory.

Estimation and testing for life distribution classes.

Inequalities and bounds in reliability theory based on Schur functions.

Many additional key ideas were formulated and developed by Proschan. Viewed as a time series, the ideas occur as a convex function of time; we can hardly wait to see what he contributes posthumously!

One elegant aspect of his approach is that he is not content simply to formulate a new idea of value solely in reliability theory. Rather, he often shows how the new idea, concept, technique, approach, or mathematical theory can be used also in many other areas of statistics and mathematics. For example:

(1) Association has been used in dozens of areas of statistics and mathematics. Hundreds of papers have been written on this concept.

(2) Decreasing in transposition (DT) functions (or arrangement increasing functions as termed by Marshall and Olkin, 1979) have been used to show monotonicity of the power function in a variety of models in selection problems in a single short proof, making obsolete the dozens of separate lengthy proofs showing weaker results in the earlier literature.

(3) His use of TP in reliability theory led to new results in TP theory applicable in many other branches of statistics and mathematics.

As a close colleague of Proschan put it: "Frank is a <u>cream-skinner</u>. He proposes a new idea, method, or theory, writes a few papers exploiting the new approach, gets key results, and while the rest of the crowd of statisticians rushes in behind him, he has disappeared through a back door only to appear in an entirely different area with still another new idea."

In recognition of his accomplishments, he has been awarded:

(1) The ASA Wilks Award.
(2) Distinguished Professor, Florida State University.
(3) Distinguished Alumni Award, George Washington University.
(4) Distinguished Alumni Award, The City University of New York.

What about Frank Proschan, the person? He is not content to teach his students simply statistics, reliability, and other technical knowledge. Rather he shares with them his spiritual principles of living. A young devotee said of him: "Professor Proschan follows a 'cradle-to-grave' approach. He guides you through your dissertation, helps you obtain a desirable position, shares his successful experience in grantsmanship, invites you back to his informal summer institutes to get your research going, and serves as godfather to your first child, but only if you are totally positive in your request."

Frank Proschan is a natural 'stand-up statistician'. He has given many after-dinner talks at meetings, often incorporating a "Dear Abby" session in which he spontaneously solves problems of attendees concerning professional life, not treated in texts. His instant responses are often weird, wild, wonderful and wone-up.

In summary, Frank Proschan is a population consisting of one person, but not degenerate; large variance, large entropy - he's practically unpredictable; totally positive; has Schur instincts in developing new key ideas; coherent in communication; shares information gladly; does not behave like a non-normal deviate; is so concerned about the validity of shock models in application that he personally has experienced them, and similarly is currently undergoing a birth and death process.

Dick Barlow: "I have only one small criticism of Frank: everything he has ever done is wrong - his treatment is non-Bayesian."

Nozer Singpurwalla: "His grants and contracts are small - each under half a million. How important can he or his work be?"

Frank Proschan - Rebuttal: "This article is worthless. Dick and Nozer have omitted my most important trait: In spite of my enormous achievements, brilliant insights, elegant proofs, incredible variety of innovations, and professional recognition of my outstanding contributions, I have always remained modest and self-effacing. In fact, I am the most humble person I have ever met!"

CONTENTS

RELIABILITY AND QUALITY CONTROL
A.P. Basu (Editor)

ON THE ASYMPTOTIC BEHAVIOR OF THE MEAN TIME BETWEEN FAILURES FOR REPAIRABLE SYSTEMS

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For failures which occur in accordance with a nonhomogeneous Poisson process, the mean time between failures is constant. In the case of a nonhomogeneous Poisson process, this is no longer true, and it is not clear what concept should take the place of the mean time between failures as a criterion for assessing system reliability. Two possibilities are explored in this paper.

INTRODUCTION

An important problem in reliability is to consider stochastic models which provide for the analysis of interfailure times of a repairable system. Counting processes are very useful in this regard. By a counting process we mean a nondecreasing, nonnegative integer-valued stochastic process X(t) which counts the number of failures occurring in the time interval (0,t]. We also assume that X(0) = 0.

Other characterizations of counting processes are also useful. We will denote the successive failure times of the system by $T_1, T_2, \ldots, T_n, \ldots$, and the times between failures, or interfailure times by $Y_1, Y_2, \ldots, Y_n, \ldots$ where $Y_n = T_n - T_{n-1}$.

The distributional properties of the process will depend largely on the nature of the system and the repair policy. In some situations the distributional properties are stated in terms of the interfailure times, and sometimes the properties are stated in terms of the counting process itself. For example, consider Poisson processes. A common approach in characterizing such a process is to specify a set of axioms which describe the behavior of X(t). An example of such an axiom is "independent increments" which means essentially that the numbers of occurrences in disjoint time intervals are independent. For a discussion of the full set of axioms see e.g. Parzen [1] or Ross [3]. An interesting alternate characterization, which involves the mean function

$$m(t) = E[X(t)] \tag{1}$$

is discussed by Cinlar [3]. A counting process is called <u>regular</u> if m(t) is continuous. It can be shown that a regular process is a Poisson process if and only if it has independent increments and no simultaneous failures.

If m(t) is also differentiable then $\nu(t) = \frac{d}{dt}m(t)$ is called the intensity function of the process. We will assume for the duration of this paper that (1) is differentiable and hence the process is regular.

The best known case of a Poisson process is a homogeneous Poisson process (HPP) in which case the intensity function is constant, say $\nu(t) = \lambda$. It is well known that a HPP has a number of special mathematical properties. For example, the interfailure times are independent and identically distributed exponential random variables. As a result, the mean time between failures (MTBF), $\theta = 1/\lambda$, is constant with respect to both time and the number of failures. This provides a convenient criterion for assessing the reliability of the system, and quite often the reliability specifications of a system are based primarily on this quantity. This is true, for example, when MIL-STD-781C [4] is used.

Much of the recent work on repairable systems has involved Poisson processes with nonconstant intensity functions. Such a process is usually called a nonhomogeneous Poisson process (NHPP). Perhaps the best known example of an NHPP is the "Weibull process" or Weibull Poisson process (WPP) which has intensity function of the form

(2) $$\nu(t) = (\beta/\theta)(t/\theta)^{\beta-1}.$$

Use of the term Weibull is the result of several properties which relate to the Weibull probability distribution. Specifically, (a) $\nu(t)$, as given by (2), is the hazard rate function of a Weibull distribution with shape and scale parameters β and θ respectively, (b) the time to first failure, T_1, is Weibull distributed, and (c) the conditional distribution of T_n given $T_1 = t_1, T_2 = t_2, \ldots, T_{n-1} = t_{n-1}$ is a truncated Weibull distribution with truncation point $t = t_{n-1}$,

(3) $$F_{T_n}(t_n | t_1, t_2, \ldots, t_{n-1}) = 1-\exp[-(t_n/\theta)^{\beta}+(t_{n-1}/\theta)^{\beta}]$$

if $t_n > t_{n-1} > \ldots > t_1$, and zero otherwise.

The WPP or, more generally, the NHPP can be used as a model for the situation where a "minimal repair" is made whenever the system fails. In other words, the reliability of the system is essentially unchanged by failure and repair, although the system, while in operation, may be deteriorating with the passage of time.

Another approach is needed if the policy is to either replace the system or to repair it to "like new" condition whenever it fails. This situation corresponds to a renewal process. In this case, the distributional properties are imposed directly on the interfailure times. Specifically, the assumption is that the interfailure times are independent identically distributed random variables. Of course, the HPP is a special case of a renewal process, where there is no deterioration between failures. More generally, we could have a renewal process where there is deterioration between failures, but the system is renewed each time it fails. An example of this would be a Weibull renewal process (WRP) where the interfailure times are Weibull distributed with an increasing hazard rate.

General renewal processes retain the property of HPP's that the MTBF is constant with respect to both time and the number of failures.

However, NHPP's do not share this property, and it is not clear what concept should take the place of the MTBF in this case. Several possible generalizations will be explored in the next section.

MTBF and GENERALIZATIONS

We will discuss the MTBF and several alternatives, some of which are also discussed by Thompson [5].

The MTBF of a general NHPP with mean function m(t) is

$$(4) \qquad E(T_n - T_{n-1}) = \int_0^\infty m^{-1}(z)\left[\frac{z^{n-1}}{\Gamma(n)} - \frac{z^{n-2}}{\Gamma(n-1)}\right]e^{-z}dz$$

where $m^{-1}(z)$ is the inverse function of m(t) and $\Gamma(\alpha)$ is the gamma function defined by $\Gamma(\alpha) = \int_0^\infty z^{\alpha-1}e^{-z}dz$ (see Appendix).

For a WPP we have that $m^{-1}(z) = \theta z^{1/\beta}$ so that

$$(5) \qquad E(T_n - T_{n-1}) = \theta\left[\frac{\Gamma(n+1/\beta)}{\Gamma(n)} - \frac{\Gamma(n-1+1/\beta)}{\Gamma(n-1)}\right]$$

This has the disadvantage of depending on the number of failures which have occurred, and is it also a rather complicated function of n, θ and β. If we use asymptotic approximations for the gamma function, such as those provided by Abramowitz and Stegun [6], Chapter 6, it is possible to show that as $n \to \infty$, for the WPP

$$(6) \qquad E(T_n - T_{n-1}) \sim (\theta/\beta)n^{1/\beta - 1} = 1/\nu(m^{-1}(n)),$$

which still depends on n, but is somewhat simpler. The relation ~ has the usual meaning that in the limit the ratio of the expressions on either side of ~ approaches 1.

Another candidate for MTBF is $E(T_n - T_{n-1} | T_{n-1} = t)$, which, for an NHPP with mean function m(t), has the form

$$(7) \qquad E(T_n - T_{n-1} | T_{n-1} = t) = e^{m(t)}\int_{m(t)}^\infty m^{-1}(u)e^{-u}du - t$$

(see Appendix).

For a WPP we have that (7) becomes

$$(8) \qquad E(T_n - T_{n-1} | T_{n-1} = t) = e^{m(t)}\int_{m(t)}^\infty \theta u^{1/\beta}e^{-u}du - t$$

which can also be written as

$$(9) \qquad E(T_n - T_{n-1} | T_{n-1} = t) = \Theta e^{m(t)}\Gamma(1+1/\beta, m(t)) - t$$

where $\Gamma(\alpha, z) = \int_z^\infty u^{\alpha-1}e^{-u}du$ is the incomplete gamma function.

The conditional MTBF (7) for an NHPP has the advantage that it depends on time t but not on the number of failures n which have occurred. For more general counting processes (7) may also depend on n.

A related concept is the mean waiting time to failure from a fixed time t. The expression $W_t = T_{X(t)+1} - t$ is the waiting time from an arbitrary fixed time t until the next failure.

For more general counting processes, $E(W_t)$ and (7) are different, but for a regular NHPP they turn out to be the same, and for the duration of this paper we will use the simple notation $E(W_t)$ instead of (7).

One other possibility which we will consider is the reciprocal of the intensity function, $1/\nu(t)$. In the case of an HPP this would be the same as the MTBF, but for a NHPP it does not agree exactly with any of the notions which we have discussed. The function $1/\nu(t)$ is referred to by Duane [7] as the instantaneous system mean time between failures at cumulative test time t. It is also referred to by Crow [8] as the achieved mean time between failures of the system.

The latter terminology results from the application of the WPP as a model for a system under development. Presumably, the system is improving under a development program until the desired reliability goals are achieved, say at time t_0. After development is ceased it is assumed that such a system will behave as a HPP with fixed intensity $\lambda = \nu(t_0)$ or achieved MTBF $\Theta = 1/\nu(t_0)$. In other words the system is WPP with intensity $\nu(t) = (\beta/\theta)(t/\theta)^{\beta-1}$ if $0 < t \leq t_0$, and HPP with intensity $\lambda = \nu(t_0)$ if $t > t_0$. It is assumed in this application that $\beta < 1$ since, for a developmental system, the intensity of failure is assumed to be decreasing until t_0. This approach to modeling and tracking reliability growth is discussed in MIL-HDBK-189 [9].

A result which suggests that the simpler $1/\nu(t)$ might be used instead of $E(W_t)$, at least after the system has been in operation for a while, involves an asymptotic result. It can be shown that in certain cases $E(W_t) \sim 1/\nu(t)$ as $t \to \infty$.

Perhaps the most important case is the WPP, where, by applying asymptotic approximations for the incomplete gamma function in (9), we obtain

$$E(W_t) = 1/\nu(t) + o(1/\nu(t)) \tag{10}$$

where $o(1/\nu(t))$ is an error term which is negligible relative to $1/\nu(t)$ (see Appendix). Of course, this implies that $E(W_t) \sim 1/\nu(t)$. Notice that a similar relationship holds for the exact MTBF and $1/\nu(m^{-1}(n))$ as demonstrated by (6).

It is possible to obtain a more explicit relationship between $E(W_t)$ and $1/\nu(t)$ if $\beta = 1/2$. In particular, $1/\nu(t) = 2\theta\, m(t)$ and $E(W_t) = 2\theta[1+m(t)]$ so that $E(W_t) = 2\theta + 1/\nu(t)$ in this special case.

The property of asymptotic equivalence is not possessed exclusively by WPP's. For example, an NHPP is said to have a log-linear intensity if it has intensity function of the form

(11) $$\nu(t) = (\beta/\theta)\exp(t/\theta).$$

This terminology is based on the fact that $\ln \nu(t) = \ln(\beta/\theta)+t/\theta$ is a linear function of t. This process could serve as a model for a repairable system with extremely rapid deterioration while in operation, since the failure intensity is increasing at an exponential rate with time.

It is possible to show, by means of the appropriate asymptotic approximations that (10) is also valid for this model, and consequently $E(W_t) \sim 1/\nu(t)$ as $t \to \infty$ (see Appendix).

General conditions on the mean function or intensity function of an NHPP, which will imply (10) are not known at this time, but it is not hard to show that some conditions are required. To see this, consider the NHPP with mean function of the form

(12) $$m(t) = \beta \ln(1+t/\theta)$$

in which case the intensity function has the form $\nu(t)=(\beta/\theta)/(1+t/\theta)$, which is decreasing with time. Notice that the distribution of time to first failure is a special form of the Pareto distribution. This would serve as a model for a system with a very slowly increasing mean number of failures and a very slowly decreasing intensity function.

It is possible to determine the exact relationship between $E(W_t)$ and $1/\nu(t)$ for this model. Using straightforward integration of (7) with m(t) as given by (12), we obtain

(13) $$E(W_t) = \begin{cases} \infty & \text{if } \beta \le 1 \\ (\frac{\beta}{\beta-1})\cdot 1/\nu(t) & \text{if } \beta > 1. \end{cases}$$

Thus, for $\beta \le 1$, $E(W_t)$ is infinite, and for $\beta > 1$ it is proportional to $1/\nu(t)$ with proportionality constant $\beta/(\beta-1) > 1$. For this model $E(W_t)$ and $1/\nu(t)$ are clearly <u>not</u> asymptotically equivalent.

We can conclude that NHPP's do not retain the convenient property of HPP's that the MTBF is constant with respect to both time and the number of failures. The MTBF will, in general depend on the number of failures, and $E(W_t)$ will depend on time, but not on the number of failures. It also appears that the criterion for reliability assessment could reasonably be based on $1/\nu(t)$, rather than on the more complicated $E(W_t)$.

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APPENDIX

Most of the results in Section 2 are based on the fact that if T_1, $T_2,\ldots,T_n,\ldots$ are successive occurrence times of a regular NHPP then $Z_1,Z_2,\ldots,Z_n,\ldots$ defined by $Z_n = m(T_n)$ are distributed as occurrence times of a HPP with $\lambda = 1$.

It follows, for strictly increasing $m(t)$, that

$$\text{(A1)} \qquad E(T_n) = \int_0^\infty m^{-1}(z)\,\frac{z^{n-1}}{\Gamma(n)}\,e^{-z}dz.$$

In particular, (4) is obtained from (A1).

Equation (6) is obtained by refinements of Stirling's approximation of $\Gamma(\alpha)$ for large α.

Equation (7) is based on the fact that $E(T_{n-1}|T_{n-1}=t) = t$ and

$$\begin{aligned} E(T_n|T_{n-1}=t) &= E[m^{-1}(Z_n)\,|\,m^{-1}(Z_{n-1})=t] \\ &= E[m^{-1}(Z_n-Z_{n-1})+Z_{n-1})\,|\,m^{-1}(Z_{n-1}) = t] \\ &= E[m^{-1}(Z_n-Z_{n-1} + m(t))] \\ &= \int_0^\infty m^{-1}(y+m(t))\,e^{-y}dy \\ &= e^{m(t)}\int_{m(t)}^\infty m^{-1}(u)\,e^{-u}du. \end{aligned}$$

Approximation (10) is obtained from (9) using the approximation

$$\text{(A2)} \qquad \Gamma(\alpha,z) = z^{\alpha-1}e^{-z}[1 + \frac{\alpha-1}{z} + O(\frac{1}{z^2})]$$

as $z \to \infty$. It follows that, for the WPP model,

$$\begin{aligned} E(W_t) &= t + \frac{1}{\nu(t)} + O(\frac{1}{m(t)\nu(t)}) - t \\ &= \frac{1}{\nu(t)} + o(\frac{1}{\nu(t)}) \end{aligned}$$

since $m(t) \to \infty$ as $t \to \infty$.

The log linear case is based on (7) with $m^{-1}(u) = \Theta\ln(1+u/\beta)$. Following substitutions $z = u + \beta$ and $m(t) + \beta = \Theta\nu(t)$ we obtain

$$\begin{aligned} E(W_t) &= \Theta e^{m(t)+\beta}\int_{m(t)+\beta}^\infty \ln(z/\beta)\,e^{-z}dz - t \\ &= \Theta e^{\Theta\nu(t)}\,[\int_{\theta\nu(t)}^\infty \ln(z)e^{-z}dz - \ln(\beta)e^{-\theta\nu(t)}] - t. \end{aligned}$$

The integral term can be analyzed using the relationship

$$\int_x^\infty \ln(z)e^{-z}dz = \ln(x)e^{-x} + \Gamma(0,x).$$