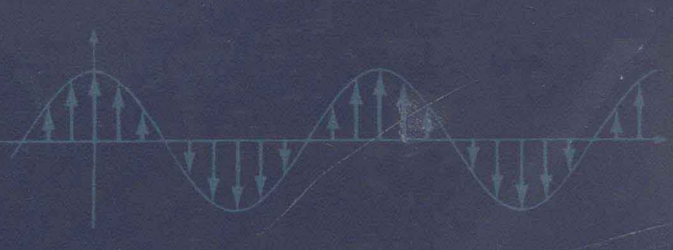
# ELEMENTS OF ENGINEERING ELECTROMAGNETICS

N. Narayana Rao



# ELEMENTS OF ENGINEERING ELECTROMAGNETICS

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PRENTICE-HALL, INC.
*Englewood Cliffs, New Jersey* 07632

*Library of Congress Cataloging in Publication Data*

NARAYANA RAO, NANNAPANENI.
Elements of engineering electromagnetics.

Bibliography: p.
Includes index.
1. Electromagnetic theory. I. Title.
QC670.N3 530.1'41 76-44234
ISBN 0-13-264150-X

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Englewood Cliffs, New Jersey 07632

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10 9 8 7 6 5 4 3 2 1

Printed in the United States of America

PRENTICE-HALL INTERNATIONAL, INC., *London*
PRENTICE-HALL OF AUSTRALIA PTY. LIMITED, *Sydney*
PRENTICE-HALL OF CANADA, LTD., *Toronto*
PRENTICE-HALL OF INDIA PRIVATE LIMITED, *New Delhi*
PRENTICE-HALL OF JAPAN, INC., *Tokyo*
PRENTICE-HALL OF SOUTHEAST ASIA PTE. LTD., *Singapore*
WHITEHALL BOOKS LIMITED, *Wellington, New Zealand*

# PREFACE

Traditionally, the first undergraduate course in engineering electromagnetics has been based upon developing static fields in a historical manner, and culminating in Maxwell's equations, with perhaps a brief discussion of uniform plane waves. This is then followed by one or more courses dealing with transmission lines and wave propagation. Due to the pressure of increasing areas of interest and fewer required courses, there has been in recent years a growing trend in electrical engineering curricula toward limiting the requirement in electromagnetics to a one-semester course or its equivalent. Consequently, and in view of the student's earlier exposure in engineering physics to static fields and Maxwell's equations, it has become increasingly expedient to deviate from the historical approach and to base the first course in electromagnetics upon dynamic fields and their engineering applications.

There are many texts, including one by the author, which fulfill the requirements of the traditional approach. There are also several books devoted to wave propagation and related topics; these, however, rely upon a first course of the traditional type or a variation of it to provide the required background. Thus a need has arisen for a one-semester text in which the basic material is built up on time-varying fields and their engineering applications so as to enhance its utility for the one-semester student of engineering electromagnetics, while enabling the student who will continue to take futher (elective) courses in electromagnetics to learn many of the same field concepts and mathematical tools and techniques provided by the traditional treatment. This book represents an attempt to satisfy this need.

The thread of development of the material is evident from a reading of the table of contents. Some of the salient features of the first nine chapters consist of introducing:

1. the bulk of the material through the use of the Cartesian coordinate system to keep the geometry simple and yet sufficient to learn the physical concepts and mathematical tools, while employing the other coordinate systems where necessary;
2. Maxwell's equations for time-varying fields first in integral form and then in differential form very early in the book;
3. uniform plane wave propagation by obtaining the field solution to the infinite plane current sheet of uniform sinusoidally time-varying density;
4. material media by considering their interaction with uniform plane wave fields;
5. transmission lines by first considering uniform plane waves guided by two parallel, plane perfect conductors and then extending to a line of arbitrary cross section through graphical field mapping;
6. waveguides by considering the superposition of two obliquely propagating uniform plane waves and then placing perfect conductors in appropriate planes so as to satisfy the boundary conditions;
7. antennas by obtaining the complete field solution to the Hertzian dipole through a successive extension of the quasistatic field solution so as to satisfy simultaneously the two Maxwell's curl equations; and
8. Maxwell's equations for static fields as specializations of Maxwell's equations for time-varying fields and then proceeding with the discussion of the more important topics of static and quasistatic fields.

The final chapter is devoted to seven independent special topics, each based upon one or more of the previous six chapters. It is intended that the instructor will choose one (or more) of these topics for discussion following the corresponding previous chapter(s). Material on cylindrical and spherical coordinate systems is presented as appendices so that it can be studied either immediately following the discussion of the corresponding material on the Cartesian coordinate system or only when necessary.

From considerations of varying degrees of background preparation at different schools, a greater amount of material than can be covered in an average class of three semester-hour credits is included in the book. Since it has been found that nearly eight chapters can be completed during the semester, the first six chapters plus an equivalent of about two chapters from the remaining four is suggested to be typical of coverage. When the background preparation permits an accelerated discussion of the first three chapters, it is possible to cover a greater amount of material. Worked-out examples

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are distributed throughout the text to illustrate and, in some cases, extend the various concepts. Summary of the material and a number of questions are included for each chapter to facilitate review of the chapters. Problems are arranged in the same order as the text material, and answers are provided for the odd-numbered problems.

This text is based primarily on lecture notes for classes taught by the author at the University of Illinois at Urbana–Champaign. The author wishes to express his appreciation to Patricia Sammann for the excellent typing work. Finally, although great care has been exercised, some errors are inevitable. The author earnestly requests readers to inform him of any errors that they may find and to contribute suggestions for improvement.

*Urbana, Illinois* N. NARAYANA RAO

# CONTENTS

# 1. VECTORS AND FIELDS

Electromagnetics deals with the study of electric and magnetic "fields." It is at once apparent that we need to familiarize ourselves with the concept of a "field," and in particular with "electric" and "magnetic" fields. These fields are vector quantities and their behavior is governed by a set of laws known as "Maxwell's equations." The mathematical formulation of Maxwell's equations and their subsequent application in our study of the elements of engineering electromagnetics require that we first learn the basic rules pertinent to mathematical manipulations involving vector quantities. With this goal in mind, we shall devote this chapter to vectors and fields.

We shall first study certain simple rules of vector algebra without the implication of a coordinate system and then introduce the Cartesian coordinate system, which is the coordinate system employed for the most part of our study in this book. After learning the vector algebraic rules, we shall turn our attention to a discussion of scalar and vector fields, static as well as time-varying, by means of some familiar examples. We shall devote particular attention to sinusoidally time-varying fields, scalar as well as vector, and to the phasor technique of dealing with sinusoidally time-varying quantities. With this general introduction to vectors and fields, we shall then devote the remainder of the chapter to an introduction of the electric and magnetic field concepts, from considerations of the experimental laws of Coulomb and Ampere.



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## 1.1 VECTOR ALGEBRA

In the study of elementary physics we come across several quantities such as mass, temperature, velocity, acceleration, force, and charge. Some of these quantities have associated with them not only a magnitude but also a direction in space whereas others are characterized by magnitude only. The former class of quantities are known as "vectors" and the latter class of quantities are known as "scalars." Mass, temperature, and charge are scalars whereas velocity, acceleration, and force are vectors. Other examples are voltage and current for scalars and electric and magnetic fields for vectors.

Vector quantities are represented by boldface roman type symbols, e.g., **A**, in order to distinguish them from scalar quantities which are represented by lightface italic type symbols, e.g., $A$. Graphically, a vector, say **A**, is represented by a straight line with an arrowhead pointing in the direction of **A** and having a length proportional to the magnitude of **A**, denoted $|\mathbf{A}|$ or simply $A$. Figures 1.1(a)–(d) show four vectors drawn to the same scale. If

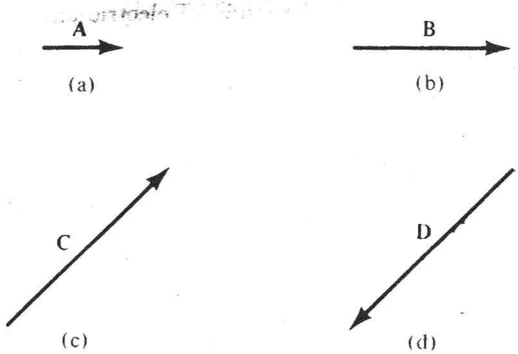


**Figure 1.1.** Graphical representation of vectors.

the top of the page represents north, then vectors **A** and **B** are directed eastward with the magnitude of **B** being twice that of **A**. Vector **C** is directed toward the northeast and has a magnitude three times that of **A**. Vector **D** is directed toward the southwest and has a magnitude equal to that of **C**. Since **C** and **D** are equal in magnitude but opposite in direction, one is the negative of the other.

Since a vector may have in general an arbitrary orientation in three dimensions, we need to define a set of three reference directions at each and every point in space in terms of which we can describe vectors drawn at that point. It is convenient to choose these three reference directions to be mutually orthogonal as, for example, east, north and upward or the three contiguous edges of a rectangular room. Thus let us consider three mutually orthogonal reference directions and direct "unit vectors" along the three directions as shown, for example, in Fig. 1.2(a). A unit vector has magnitude unity. We shall represent a unit vector by the symbol $\mathbf{i}$ and use a subscript to denote its direction. We shall denote the three directions by subscripts 1, 2, and 3. We note that for a fixed orientation of $\mathbf{i}_1$, two combinations are possible for the orientations of $\mathbf{i}_2$ and $\mathbf{i}_3$, as shown in Figs. 1.2(a) and (b). If we take a right-hand screw and turn it from $\mathbf{i}_1$ to $\mathbf{i}_2$ through the 90°-angle, it progresses in the direction of $\mathbf{i}_3$ in Fig. 1.2(a) but opposite to the direction of $\mathbf{i}_3$ in Fig. 1.2(b). Alternatively, a left-hand screw when turned from $\mathbf{i}_1$ to $\mathbf{i}_2$ in Fig. 1.2(b) will progress in the direction of $\mathbf{i}_3$. Hence the set of unit vectors in Fig. 1.2(a) corresponds to a right-handed system whereas the set in Fig. 1.2(b) corresponds to a left-handed system. We shall work consistently with the right-handed system.

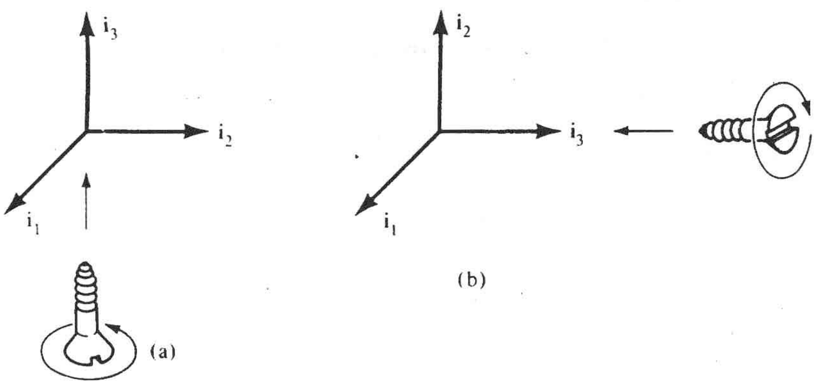


**Figure 1.2.** (a) Set of three orthogonal unit vectors in a right-handed system. (b) Set of three orthogonal unit vectors in a left-handed system.

A vector of magnitude different from unity along any of the reference directions can be represented in terms of the unit vector along that direction. Thus $4\mathbf{i}_1$ represents a vector of magnitude 4 units in the direction of $\mathbf{i}_1$, $6\mathbf{i}_2$ represents a vector of magnitude 6 units in the direction of $\mathbf{i}_2$, and $-2\mathbf{i}_3$ represents a vector of magnitude 2 units in the direction opposite to that of $\mathbf{i}_3$, as shown in Fig. 1.3. Two vectors are added by placing the beginning of the second vector at the tip of the first vector and then drawing the sum vector from the beginning of the first vector to the tip of the second vector. Thus to add $4\mathbf{i}_1$ and $6\mathbf{i}_2$, we simply slide $6\mathbf{i}_2$ without changing its direction until its beginning coincides with the tip of $4\mathbf{i}_1$ and then draw the vector $4\mathbf{i}_1 + 6\mathbf{i}_2$ from the beginning of $4\mathbf{i}_1$ to the tip of $6\mathbf{i}_2$, as shown in Fig. 1.3. By adding $-2\mathbf{i}_3$ to this vector $4\mathbf{i}_1 + 6\mathbf{i}_2$ in a similar manner, we obtain the vector

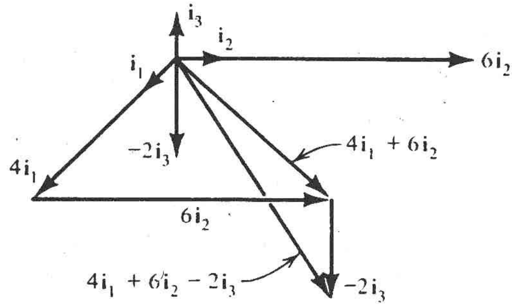


**Figure 1.3.** Graphical addition of vectors.

$(4\mathbf{i}_1 + 6\mathbf{i}_2 - 2\mathbf{i}_3)$, as shown in Fig. 1.3. We note that the magnitude of $(4\mathbf{i}_1 + 6\mathbf{i}_2)$ is $\sqrt{4^2 + 6^2}$ or 7.211 and that the magnitude of $(4\mathbf{i}_1 + 6\mathbf{i}_2 - 2\mathbf{i}_3)$ is $\sqrt{4^2 + 6^2 + 2^2}$ or 7.483. Conversely to the foregoing discussion, a vector **A** at a given point is simply the superposition of three vectors $A_1\mathbf{i}_1$, $A_2\mathbf{i}_2$, and $A_3\mathbf{i}_3$ which are the projections of **A** onto the reference directions at that point. $A_1$, $A_2$, and $A_3$ are known as the components of **A** along the 1, 2, and 3 directions, respectively. Thus

$$\mathbf{A} = A_1\mathbf{i}_1 + A_2\mathbf{i}_2 + A_3\mathbf{i}_3 \tag{1.1}$$

We now consider three vectors **A**, **B**, and **C** given by

$$\mathbf{A} = A_1\mathbf{i}_1 + A_2\mathbf{i}_2 + A_3\mathbf{i}_3 \tag{1.2a}$$

$$\mathbf{B} = B_1\mathbf{i}_1 + B_2\mathbf{i}_2 + B_3\mathbf{i}_3 \tag{1.2b}$$

$$\mathbf{C} = C_1\mathbf{i}_1 + C_2\mathbf{i}_2 + C_3\mathbf{i}_3 \tag{1.2c}$$

at a point and discuss several algebraic operations involving vectors as follows.

VECTOR ADDITION AND SUBTRACTION: Since a given pair of like components of two vectors are parallel, addition of two vectors consists simply of adding the three pairs of like components of the vectors. Thus

$$\begin{aligned}\mathbf{A} + \mathbf{B} &= (A_1\mathbf{i}_1 + A_2\mathbf{i}_2 + A_3\mathbf{i}_3) + (B_1\mathbf{i}_1 + B_2\mathbf{i}_2 + B_3\mathbf{i}_3)\\ &= (A_1 + B_1)\mathbf{i}_1 + (A_2 + B_2)\mathbf{i}_2 + (A_3 + B_3)\mathbf{i}_3\end{aligned} \tag{1.3}$$

Vector subtraction is a special case of addition. Thus

$$\begin{aligned}\mathbf{B} - \mathbf{C} = \mathbf{B} + (-\mathbf{C}) &= (B_1\mathbf{i}_1 + B_2\mathbf{i}_2 + B_3\mathbf{i}_3) + (-C_1\mathbf{i}_1 - C_2\mathbf{i}_2 - C_3\mathbf{i}_3)\\ &= (B_1 - C_1)\mathbf{i}_1 + (B_2 - C_2)\mathbf{i}_2 + (B_3 - C_3)\mathbf{i}_3\end{aligned} \tag{1.4}$$

MULTIPLICATION AND DIVISION BY A SCALAR: Multiplication of a vector **A** by a scalar $m$ is the same as repeated addition of the vector. Thus

$$m\mathbf{A} = m(A_1\mathbf{i}_1 + A_2\mathbf{i}_2 + A_3\mathbf{i}_3) = mA_1\mathbf{i}_1 + mA_2\mathbf{i}_2 + mA_3\mathbf{i}_3 \tag{1.5}$$

Division by a scalar is a special case of multiplication by a scalar. Thus

$$\frac{\mathbf{B}}{n} = \frac{1}{n}(\mathbf{B}) = \frac{B_1}{n}\mathbf{i}_1 + \frac{B_2}{n}\mathbf{i}_2 + \frac{B_3}{n}\mathbf{i}_3 \tag{1.6}$$

MAGNITUDE OF A VECTOR: From the construction of Fig. 1.3 and the associated discussion, we have

$$|\mathbf{A}| = |A_1\mathbf{i}_1 + A_2\mathbf{i}_2 + A_3\mathbf{i}_3| = \sqrt{A_1^2 + A_2^2 + A_3^2} \tag{1.7}$$

UNIT VECTOR ALONG A: The unit vector $\mathbf{i}_A$ has a magnitude equal to unity but its direction is the same as that of **A**. Hence

$$\begin{aligned}\mathbf{i}_A &= \frac{\mathbf{A}}{|A|} = \frac{A_1\mathbf{i}_1 + A_2\mathbf{i}_2 + A_3\mathbf{i}_3}{\sqrt{A_1^2 + A_2^2 + A_3^2}} \\ &= \frac{A_1}{\sqrt{A_1^2 + A_2^2 + A_3^2}}\mathbf{i}_1 + \frac{A_2}{\sqrt{A_1^2 + A_2^2 + A_3^2}}\mathbf{i}_2 + \frac{A_3}{\sqrt{A_1^2 + A_2^2 + A_3^2}}\mathbf{i}_3\end{aligned} \tag{1.8}$$

SCALAR OR DOT PRODUCT OF TWO VECTORS: The scalar or dot product of two vectors **A** and **B** is a scalar quantity equal to the product of the magnitudes of **A** and **B** and the cosine of the angle between **A** and **B**. It is represented by a dot between **A** and **B**. Thus if $\alpha$ is the angle between **A** and **B**, then

$$\mathbf{A} \cdot \mathbf{B} = |\mathbf{A}||\mathbf{B}| \cos \alpha = AB \cos \alpha \tag{1.9}$$

For the unit vectors $\mathbf{i}_1, \mathbf{i}_2, \mathbf{i}_3$, we have

$$\mathbf{i}_1 \cdot \mathbf{i}_1 = 1 \qquad \mathbf{i}_1 \cdot \mathbf{i}_2 = 0 \qquad \mathbf{i}_1 \cdot \mathbf{i}_3 = 0 \tag{1.10a}$$

$$\mathbf{i}_2 \cdot \mathbf{i}_1 = 0 \qquad \mathbf{i}_2 \cdot \mathbf{i}_2 = 1 \qquad \mathbf{i}_2 \cdot \mathbf{i}_3 = 0 \tag{1.10b}$$

$$\mathbf{i}_3 \cdot \mathbf{i}_1 = 0 \qquad \mathbf{i}_3 \cdot \mathbf{i}_2 = 0 \qquad \mathbf{i}_3 \cdot \mathbf{i}_3 = 1 \tag{1.10c}$$

By noting that $\mathbf{A} \cdot \mathbf{B} = A(B \cos \alpha) = B(A \cos \alpha)$, we observe that the dot product operation consists of multiplying the magnitude of one vector by the scalar obtained by projecting the second vector onto the first vector as shown in Figs. 1.4(a) and (b). The dot product operation is commutative since

$$\mathbf{B} \cdot \mathbf{A} = BA \cos \alpha = AB \cos \alpha = \mathbf{A} \cdot \mathbf{B} \tag{1.11}$$

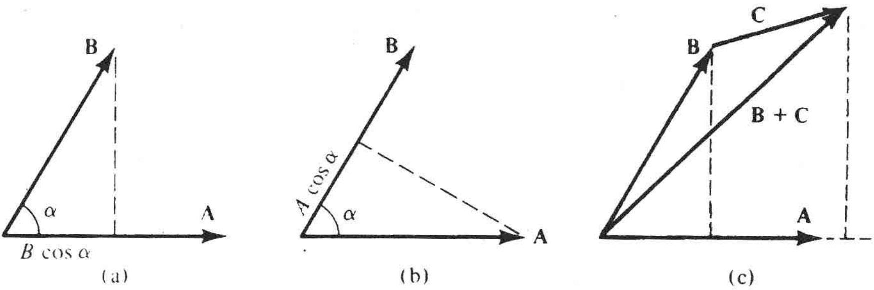


**Figure 1.4.** (a) and (b) For showing that the dot product of two vectors **A** and **B** is the product of the magnitude of one vector and the projection of the second vector onto the first vector. (c) For proving the distributive property of the dot product operation.

The distributive property also holds for the dot product as can be seen from the construction of Fig. 1.4(c), which illustrates that the projection of **B** + **C** onto **A** is equal to the sum of the projections of **B** and **C** onto **A**. Thus

$$\mathbf{A} \cdot (\mathbf{B} + \mathbf{C}) = \mathbf{A} \cdot \mathbf{B} + \mathbf{A} \cdot \mathbf{C} \tag{1.12}$$

Using this property, and the relationships (1.10a)–(1.10c), we have

$$\begin{aligned}
\mathbf{A} \cdot \mathbf{B} &= (A_1\mathbf{i}_1 + A_2\mathbf{i}_2 + A_3\mathbf{i}_3) \cdot (B_1\mathbf{i}_1 + B_2\mathbf{i}_2 + B_3\mathbf{i}_3) \\
&= A_1\mathbf{i}_1 \cdot B_1\mathbf{i}_1 + A_1\mathbf{i}_1 \cdot B_2\mathbf{i}_2 + A_1\mathbf{i}_1 \cdot B_3\mathbf{i}_3 \\
&\quad + A_2\mathbf{i}_2 \cdot B_1\mathbf{i}_1 + A_2\mathbf{i}_2 \cdot B_2\mathbf{i}_2 + A_2\mathbf{i}_2 \cdot B_3\mathbf{i}_3 \\
&\quad + A_3\mathbf{i}_3 \cdot B_1\mathbf{i}_1 + A_3\mathbf{i}_3 \cdot B_2\mathbf{i}_2 + A_3\mathbf{i}_3 \cdot B_3\mathbf{i}_3 \\
&= A_1B_1 + A_2B_2 + A_3B_3
\end{aligned} \tag{1.13}$$

Thus the dot product of two vectors is the sum of the products of the like components of the two vectors.

VECTOR OR CROSS PRODUCT OF TWO VECTORS: The vector or cross product of two vectors **A** and **B** is a vector quantity whose magnitude is equal to the product of the magnitudes of **A** and **B** and the sine of the angle $\alpha$ between **A** and **B** and whose direction is the direction of advance of a right-hand screw as it is turned from **A** to **B** through the angle $\alpha$, as shown in Fig. 1.5. It is represented by a cross between **A** and **B**. Thus if $\mathbf{i}_N$ is the unit vector in the direction of advance of the right-hand screw, then

$$\mathbf{A} \times \mathbf{B} = |\mathbf{A}||\mathbf{B}| \sin \alpha \, \mathbf{i}_N = AB \sin \alpha \, \mathbf{i}_N \tag{1.14}$$

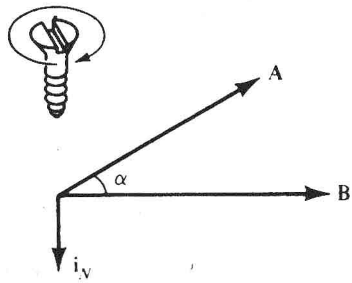


**Figure 1.5.** The cross product operation $\mathbf{A} \times \mathbf{B}$.

For the unit vectors $\mathbf{i}_1$, $\mathbf{i}_2$, $\mathbf{i}_3$, we have

$$\mathbf{i}_1 \times \mathbf{i}_1 = 0 \qquad \mathbf{i}_1 \times \mathbf{i}_2 = \mathbf{i}_3 \qquad \mathbf{i}_1 \times \mathbf{i}_3 = -\mathbf{i}_2 \tag{1.15a}$$

$$\mathbf{i}_2 \times \mathbf{i}_1 = -\mathbf{i}_3 \qquad \mathbf{i}_2 \times \mathbf{i}_2 = 0 \qquad \mathbf{i}_2 \times \mathbf{i}_3 = \mathbf{i}_1 \tag{1.15b}$$

$$\mathbf{i}_3 \times \mathbf{i}_1 = \mathbf{i}_2 \qquad \mathbf{i}_3 \times \mathbf{i}_2 = -\mathbf{i}_1 \qquad \mathbf{i}_3 \times \mathbf{i}_3 = 0 \tag{1.15c}$$

Note that the cross product of identical vectors is zero. If we arrange the unit vectors in the manner $\mathbf{i}_1\mathbf{i}_2\mathbf{i}_3\mathbf{i}_1\mathbf{i}_2$ and then go forward, the cross product of any two successive unit vectors is equal to the following unit vector, but if we go backward, the cross product of any two successive unit vectors is the negative of the following unit vector.

The cross product operation is not commutative since

$$\mathbf{B} \times \mathbf{A} = |\mathbf{B}||\mathbf{A}| \sin\alpha\,(-\mathbf{i}_N) = -AB\sin\alpha\,\mathbf{i}_N = -\mathbf{A} \times \mathbf{B} \tag{1.16}$$

The distributive property holds for the cross product (we shall prove this later in this section) so that

$$\mathbf{A} \times (\mathbf{B} + \mathbf{C}) = \mathbf{A} \times \mathbf{B} + \mathbf{A} \times \mathbf{C} \tag{1.17}$$

Using this property and the relationships (1.15a)–(1.15c), we obtain

$$\begin{aligned}
\mathbf{A} \times \mathbf{B} &= (A_1\mathbf{i}_1 + A_2\mathbf{i}_2 + A_3\mathbf{i}_3) \times (B_1\mathbf{i}_1 + B_2\mathbf{i}_2 + B_3\mathbf{i}_3) \\
&= A_1\mathbf{i}_1 \times B_1\mathbf{i}_1 + A_1\mathbf{i}_1 \times B_2\mathbf{i}_2 + A_1\mathbf{i}_1 \times B_3\mathbf{i}_3 \\
&\quad + A_2\mathbf{i}_2 \times B_1\mathbf{i}_1 + A_2\mathbf{i}_2 \times B_2\mathbf{i}_2 + A_2\mathbf{i}_2 \times B_3\mathbf{i}_3 \\
&\quad + A_3\mathbf{i}_3 \times B_1\mathbf{i}_1 + A_3\mathbf{i}_3 \times B_2\mathbf{i}_2 + A_3\mathbf{i}_3 \times B_3\mathbf{i}_3 \\
&= A_1B_2\mathbf{i}_3 - A_1B_3\mathbf{i}_2 - A_2B_1\mathbf{i}_3 + A_2B_3\mathbf{i}_1 \\
&\quad + A_3B_1\mathbf{i}_2 - A_3B_2\mathbf{i}_1 \\
&= (A_2B_3 - A_3B_2)\mathbf{i}_1 + (A_3B_1 - A_1B_3)\mathbf{i}_2 \\
&\quad + (A_1B_2 - A_2B_1)\mathbf{i}_3
\end{aligned} \tag{1.18}$$

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