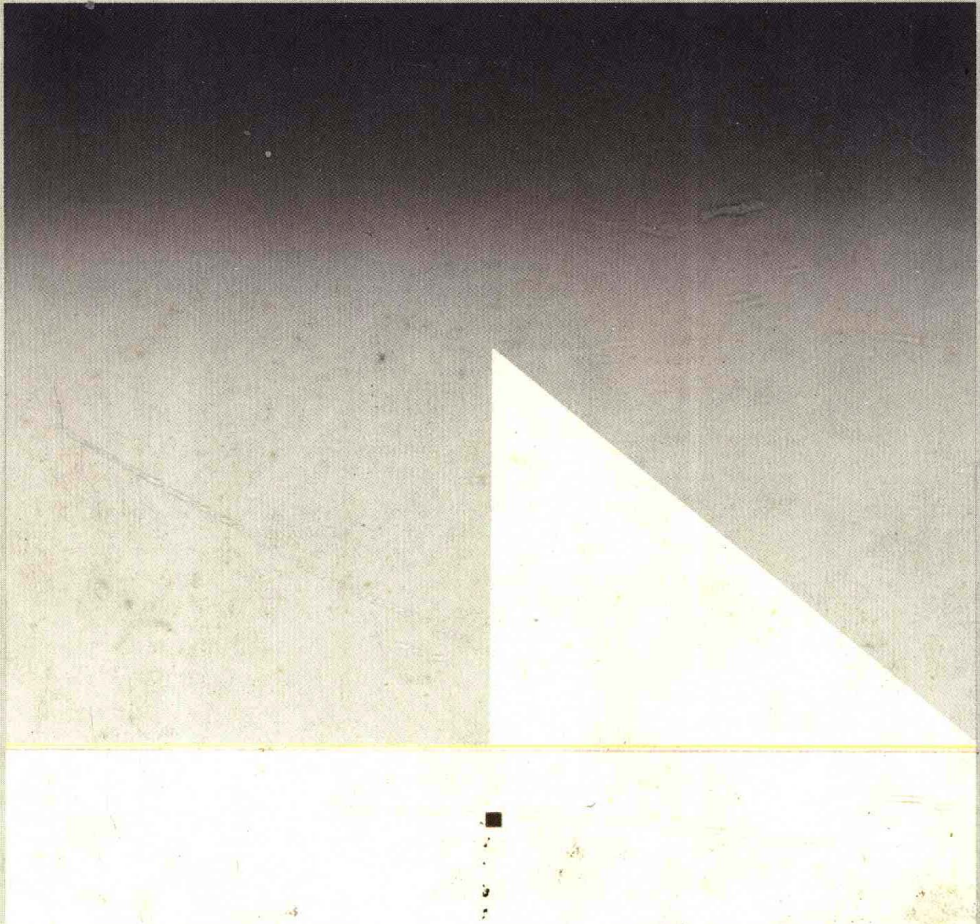


Statistical Thinking for MANAGERS



Ott/Hildebrand

Statistical Thinking for Managers

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To Sally, Curtis, and Kathy,
Pat, Marty, and Jeff,
who all helped

Preface

We believe that there is a need for a thoughtful, critical introductory text about statistics, aimed at students of business, management, and economics. Curiously, as the field of statistics has become more important to managers and as statistics courses have become more central to business school curricula, statistics texts have become more cookbookish, with much emphasis on *how-to* at the expense of *why-to*, *when-to*, and *when-not-to*. We think that restricting the focus to mechanical procedures is a disservice to business and management students.

Even more curiously, the enormous growth in the use of hand calculators and of statistical computer program packages has not been reflected in statistics textbooks. These days, since students can buy calculators that will perform multiple regression analyses, there is no reason to spend pages and hours on statistical computations. Rather, with computers and calculators available to do the computational work, a text can focus more on judgment, selection, and interpretation — the human work. We believe that the easy-computation era demands a basic change in the orientation of statistics courses, with less emphasis on the computational aspects of a problem, and greater emphasis on the critical evaluation of the underlying assumptions of the procedures and interpretation of the results. Certainly, one way to understand a statistical procedure is to work an example by hand. Whenever appropriate, we have provided some initial exercises of that kind. But being able to calculate a two-sample t statistic is hardly the most important skill a manager needs to possess. A much more important skill is the ability to assess whether such a statistic is sensible given the problem and the data and, if so, what the result means.

We have paid uncommon attention to the assumptions underlying statistical procedures, the detection of violations of these assumptions, the existence of alternative procedures, and the interpretation of results. We expect our readers to become critical consumers (and sometimes producers) of statistical analyses, alert to dangers of computerized studies untouched by human minds.

Specifically, the discussion of regression analysis in this book is unusually extensive. We believe that regression analysis is the most important topic in an introductory managerial statistics course. Chapter 16, devoted to the process of constructing a multiple regression model, is a step beyond what is offered in other texts. We hope that this chapter will be provocative and useful to students and teachers alike.

Inevitably, we have had to omit some useful ideas, lest the book be accessible only to weightlifters and millionaires. We think that once students understand the spirit of the book, instructors will be able to add supplementary material to taste.

This book can be used in a wide variety of business and managerial statistics courses. Knowledge of calculus is not an absolute prerequisite, nor a handicap. Where calculus is useful, we have used it. These sections of the text are freestanding and can be omitted without disastrous consequences. It is not necessary for students to have access to any particular computer package, because we have illustrated several. In fact, it is not absolutely necessary that students have access to a computer at all, though obviously it's desirable. If necessary, the instructor can provide the relevant output for students to analyze. The ability to use a computer to crunch numbers is a secondary goal; the ability to think about the output is primary.

Instructors should find ample latitude and flexibility in this book. There's plenty of material even for a full-year course. The sequencing of topics is fairly traditional, with two exceptions. Chapter 3 provides an early, descriptive introduction to the crucial topics of regression and analysis of variance. This feature allows students to see some serious applications of statistics early in the course. (It's also handy in MBA courses, where students are often taking concurrent courses that refer to regression.) Bayes' theorem is not introduced in the basic probability chapter, but rather where it's needed, in the decision theory chapters. Some traditional topics, notably grouped data, combinatorial probability, and index numbers, are treated rather briefly because our preference is to spend more effort on critical data analysis. We would be very surprised if an instructor taught straight through this book without additions, deletions, or resequencing. Within a chapter, the more mathematical sections at the end or in the appendix can be omitted without ruining the flow.

The several prepublication drafts of this text were reviewed by some fine critics. Perhaps the most important ones were the students of the Wharton Executive MBA program. We truly appreciate their willingness to put up with bulky, typo-ridden drafts, their insistence on improving what was obscure, and their encouragement to continue. Dr. Patricia Hildebrand and Dr. Brian Joiner produced many of the computer outputs, plus some astute critiques. Dr. Susan Reiland did a very thorough check of the entire manuscript. Many thoughtful critics have commented on our drafts, including: Bruce Bowerman, Miami University (OH); Frank Buffa, Texas A & M; P. L. Claypool, Oklahoma State University; Alan Gleit, University of Massachusetts; F. B. Green, Appalachian State University; James F. Horrell, University of Oklahoma; Dallas E. Johnson, Kansas State University; Abba M. Krieger, University of Pennsylvania; Lynn R. La Motte, University of Houston; William Q. Meeker, Jr., Iowa State University; Alan W. Neebe, University of North Carolina, Chapel Hill; Paul Newbold, University of

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As only co-authors know, it is not possible to identify where the contributions of one ends, and the other starts. In this partnership the authors issue the usual disclaimer that the other one bears responsibility for any inaccuracies.

Illinois; Robert Parsons, Northeastern University; M. Ray Perryman, Baylor University; and Morty Yalovsky, McGill University. We appreciate their advice, even when it went unheeded.

Authors don't produce books all by themselves. Kathryn A. Szabat, Stephen J. Ruberg, and Elizabeth Morgenthien all labored long and well to produce and check solutions to the exercises. We stand in grateful awe of the ability of Carol Sexton (particularly), Margo Holloway, and Lucy Zuckerman to translate two forms of hieroglyphics into typed text. Mary LeQuesne of Duxbury Press provided useful suggestions. Helen Walden was Duxbury's magician in charge of assembling all the pieces of the finished book. The editorial double-play team of Jerry Lyons to Pat Fitzgerald to Jerry Lyons nurtured the project from start to finish.

It is a total cliché for authors to thank their families for their patience, support, and understanding through the long and sometimes painful process of producing a book. Only those who have been there can understand the sincerity behind the cliché.

Cincinnati, Ohio
Penn Wynne, Pennsylvania
December 1982

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1

Statistics: making sense of data

numerical

Statistics, as a subject, is the study of making sense of data. Almost every manager—corporate president, cabinet member, hospital director, third assistant to the associate vice comptroller—must deal with data. For statistical purposes, the word data means any collection of **numerical** values, along with an explicit or implicit definition of how these values were measured. This book is an introduction to statistical methods: methods for gathering data, for summarizing data in a coherent way, for making predictions and forecasts from data, and for making decisions based on data.

1.1

Gathering data

Part of the business of statistics is to indicate good and bad ways to gather data. If a manager wants to conduct market research on a new product, evaluate the occupancy rate of hospital beds, experiment with the effects of different compensation plans, audit the accounts receivable of a chain store, or survey the opinions of businessmen about a proposed set of government regulations, the first requirement is intelligent data-gathering.

Statistical sampling theory and the theory of experimental design provide useful guides to good methods of data collection. Usually, statistical sampling is more or less passive; the aim is to gather (or survey) data on existing conditions, attitudes, or behavior. For example, a manager of a large manufacturing

plant might be interested in attitudes of production supervisors toward an incentive (bonus) plan. To do this the manager might survey their opinions. Experimental studies tend to be more active; the person conducting such a study deliberately varies certain conditions, such as the noise level in a manufacturing plant, and observes the results, gain or loss in productivity. Often studies combine elements of sampling and experimentation, as in a market research study in which individuals indicate their purchasing habits and also indicate preferences among several formulations of a product.

Since most managers will not be closely involved in the data-gathering process, we will assume in the early sections of the text that the data have been gathered in an intelligent manner. This is not done to negate the importance of the data-gathering process; rather, it allows us to focus initially on the important topics of statistics related to summarizing data, analyzing data for meaningful trends, and analyzing data to develop predictions (or forecasts) of future events. Some of the material in the latter sections of the text, particularly Chapter 20 on statistical sampling methods, deals directly with data-gathering methods. Other ideas for collecting data are scattered throughout the book.

1.2

Summarizing data

Once the data are gathered, they need to be summarized before any meaningful interpretations can be made. Imagine facing a computer printout of the current balances of every MasterCard holder, or of the occupancy status of every bed in a large hospital for the last five years, and being asked to describe the data. It's almost impossible to describe such detailed data; instead the raw data must be summarized to be understandable. **The first step in trying to summarize data is to graph the data.** Draw some intelligent pictures and try to make some sense of the data.

Next focus on the average or typical value in the data set and some measure of the range or spread of the data. For example, in trying to summarize data on starting salaries for college graduates with a chemical engineering degree, it would be important to know that the starting salaries ranged from a low of \$10,500 to a high of \$35,000 and that the average starting salary was \$21,200. This focus on the typical or average is necessary and probably inevitable in making sense of data.

In some situations though, the average or typical values may be irrelevant. For example, we would prefer not to live next to a flood-control system that was designed to handle the average rainfall in a given place. Here we would look for protection against large values, which are greatly different from the average or typical value. One of the more important aspects of summarizing data is to discover if there are wildly extreme cases, ones that differ greatly from the average or typical value.