Abstracts

Seventh International Symposium on Nonlinear Acoustics

19-21 August 1976

ABSTRACTS OF PAPERS

SUBMITTED TO

SEVENTH INTERNATIONAL SYMPOSIUM

ON NONLINEAR ACOUSTICS

VIRGINIA POLYTECHNIC INSTITUTE AND STATE UNIVERSITY

BLACKSBURG, VIRGINIA, U.S.A.

Edited by

A.H.Nayfeh and J.E.Kaiser

August, 1976

TECHNICAL COMMITTEE

Preface

Nonlinear acoustics is the branch of science that deals with finite-amplitude traveling and standing waves in gases, liquids, and solids, as well as problems relating to cavitation. This includes general mathematical tools, experiments, and applications. It involves waves propagating in homogeneous as well as nonhomogeneous deterministic or stochastic media.

It should be noted that practically all problems arising in nature are nonlinear at the outset and that linearization is an approximating device. Although many useful conclusions about the behavior of physical systems can be drawn from the linearized equations, there are many other behaviors which cannot be explained by using linearization. For example, the occurance and propagation of steep gradient regions (discontinuities and shock waves) cannot be predicted and analyzed from linearized equations. Other phenomena that cannot be explained by using linearized models are generation of harmonic, subharmonic, superharmonic, and combination tones; tuning; saturation phenomena; and the existence of self-relaxation oscillations.

The large growth in the number of works in this area necessitated the organization of the international symposia for the timely exchange of technical information which might appear later in journals. We hope that these symposia will serve to continue the progress in this important branch of science.

A. H. NAYFEH AND J. E. KAISER

Table of Contents

Table of Contents Con't

Table of Contents Con't

Table of Contents Con't

GENERAL ACOUSTICS

V. A. Krasilnikov, Chairman
Moscow State University U.S.S.R.

D. S. Scott, Co-Chairman
University of Toronto, Canada

NONCLASSICAL ACOUSTICS

Czeslaw P. Kentzer

School of Aeronautics and Astronautics
Purdue University, Lafayette, IN 47907

ABSTRACT

The paper considers a statistical approach to sound propagation in situations where, due to the presence of large gradients of properties of the medium, the classical (deterministic) treatment of wave motion is inadequate. A classical approach is valid only if $|\nabla\lambda| \ll 1$, where λ = wavelength. Acoustics of inhomogeneous moving fluids concerns itself usually with the zero-wavelength approximation, $\nabla\lambda = 0$. An extension of the classical approach in the spirit of the WKB (Wentzel-Kramers-Brillouin) approximation, Kentzer (1975a), resulted in a dispersion relation valid for small but not necessarily negligible wavelength. However, situations where $|\nabla\lambda| = O(1)$ are not uncommon in studies of broad spectrum of sound in high speed flows. Thus the motivation behind this paper is to discuss the use of mathematical methods for wave motions not restricted to small wavelengths. Such methods form the basis of quantum mechanics and have been used recently, Kentzer (1975b), to formulate a wave theory of sound in turbulent media.

The objectives of this paper are to formulate a quantum-like stochastic theory of sound of arbitrary intensity under the assumption that the energy of the sound field remains negligible as compared to the thermal energy of the gaseous medium. Of necessity, the theory is statistical in nature.

The arguments of Nee & Kovasznay (1969) show that sound has a negligible effect on the mean flow and on the generation of vorticity. Thus the flow may be assumed irrotational and presumed known. The sound field is then considered to be the oscillatory

motion of the compressible medium relative to the average or mean flow. The averages are defined with respect to a probability distribution (ensemble averages) rather than space or time averages.

Separating the flow into the mean flow and the fluctuations, one separates the governing equations of gasdynamics into those satisfied by the mean flow and those that determine the fluctuations. The latter are nonlinear in the fluctuations. Neglecting the products of the fluctuations would lead to previously obtained results valid for $|\nabla\lambda| < 1$ only. Thus the nonlinearity manifests itself in the present theory in the nonlinear wave interaction terms. Following the approach of Kentzer (1975b), the linear part of the differential equations is used to determine a complete set of orthonormal modes (complex exponentials as basis vectors). The fluctuations are then expanded in terms of the orthonormal modes. In order to account for the nonlinear terms, the frequency of the normal modes is modified by an addition of a random function. The random function is modeled using wave-particle analogy and is determined from the condition that the so modified normal modes satisfy the nonlinear wave interaction terms in the average. This representation separates the effects of sound waves interacting with mean flow gradients from the effects of direct wave interactions through three and four wave resonances.

A complex characteristic function is introduced which, in the case of small amplitude sound in uniform flow, reduces to the Fourier transform of the wave amplitudes. The temporal and spatial derivatives of the characteristic function are simply related to the moments of the frequency and wavenumber with respect to the spectral distribution of the amplitudes, respectively. A partial differential equation for the characteristic function is then derived by taking the moment of the dispersion relation (the characteristic determinant). Since the dispersion relation is quadratic in frequency, one obtains either a single differential equation of second order in time or two equations of first order analogous to or in the general form of the Klein-Gordon and Schrödinger equations, respectively. It is then argued that one should consider two separate and distinct modes of the acoustic field corresponding to plane waves propagating in the direction of and opposite to that of the wavenumber vector thus preserving the radiation condition at the microscopic level. These modes satisfy a convective equation of Schrödinger type. The square of the absolute value of the characteristic function is interpreted as the probability density in space, P, and the spatial gradient of its argument as the average of the wavenumber vector, $\bar{V}$. Transport equations for P and $\bar{V}$ are derived by separating real and imaginary parts of the Schrödinger equation. It is shown that the spatial derivatives of P and $\bar{V}$ determine the moments of the wavenumber with respect to the wavenumber probability distribution function. The knowledge of the moments of the distribution determines the distribution function. Thus the theory is closed and may be used to provide the expectation values (averages with respect to the probability distribution) of spatial and spectral properties of an acoustic field.

Initial and/or boundary value problems may be formulated in terms of P and $\bar{V}$. Only statistical properties of the sound field may be determined from the initial and boundary values. The proposed theory may be found useful in applications where no assumption as to the relative magnitude of the wavelength of sound is appropriate.

Kentzer C. P. (1975a) "Amplification, Attenuation, and Dispersion of Sound in Inhomogeneous Flows", the 89th Meeting of ASA, U. of Texas, Austin, Texas, April 7-11, 1975, available as Report No. 75-1, School of Aeronautics and Astronautics, Purdue U.

Kentzer C. P. (1975b) "Wave Theory of Turbulence in Compressible Media", Report No. 75-2, School of Aeronautics and Astronautics, Purdue U., available also as NACA CR 119143, 1975.

Nee V. W. and L.S.G. Kovasznay (1969) "Simple Phenomenological Theory of Turbulent Shear Flows", Phys. Fluids, Vol. 12, p. 473.

NONLINEAR SOUND NOISE: SHOCK WAVES, ABSORPTION AND BOUNDED BEAMS

E.B.Cherepetskaja, A.S.Chirkin, O.V.Rudenko
Moscow State University

Some results are reported about statistical nonlinear wave theory in homogeneous media. This theory has been developed by the authors for the random Riemann waves. Here more real cases taking into account damped shocks and limited cross-section of sound beams are considered. Some applications of the theory are also discussed.

1. Primary random-modulated waves of finite amplitude propagating through nonlinear medium transform to random discontinuous ones. This process is accompanied by spectrum distortion and wave statistics change at the same time. For the large Reynolds number ($Re \to \infty$) and primary narrow-band Gaussian random waves V, the evolution of the distribution function $W(V; z)$ (here z is the distance) was studied in [1]. Having $W(V; z)$ we can calculate some characteristics of the process taking into account nonlinear absorption of the waves. On fig.1 curves for the average intensity $\overline{V^2}/\sigma^2$ (curve 1) and dispersion $\langle (V^2 - \langle V^2 \rangle)^2 \rangle / 2\sigma^4$ (curve 2) are presented as the function of normalized distance $z = \varepsilon c_0^{-2} \sigma \omega x \sqrt{2}$, where ω is the signal frequency, σ^2 primary intensity, ε is nonlinear parameter. After shock formation, at distances $z > z_{sh} = \sqrt{2}$

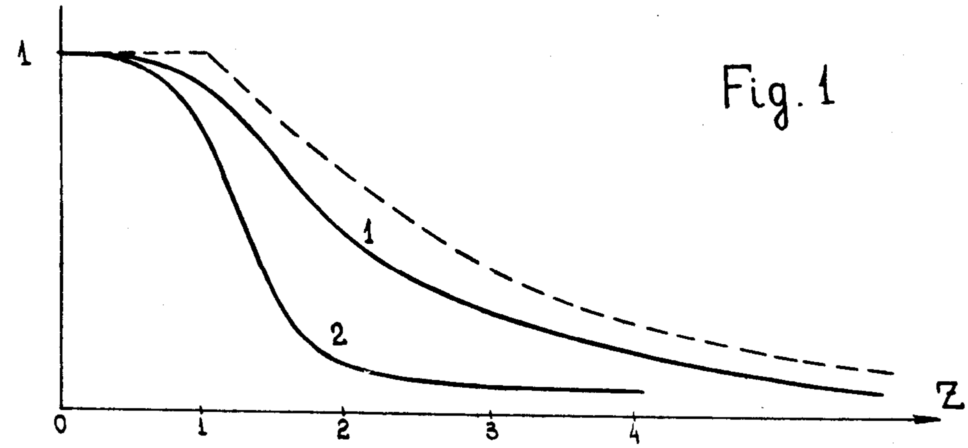

Fig. 1

wave fluctuations are strongly smoothed. On fig. 1 intensity damping for the harmonic signal (dotted line) is also shown. The comparison shows that transmission of regular signal at large distances is more advantageous energetically.

2. For the correlation function $R(\theta, x)$ of the process wich is described by Burgers equation we abtained the equation for arbitrary values of Reynolds number. At distances $x > x_0 = (\delta\omega^2)^{-1}$ (here $\delta\omega^2$ is sound absorbtion coefficient) this equation has the simple form

$$\frac{\partial F}{\partial x} - \frac{\alpha^2}{\delta} F \frac{\partial F}{\partial \Theta} = 2\delta \frac{\partial^2 F}{\partial \Theta^2} \; ; \quad F(\Theta, x) = \int_0^\Theta R(\tau, x)\, d\tau \qquad (1)$$

It follows from (1) that one conservation law $F(\infty, x) = M_0$ takes place as well as stationary form of correlation function $R(\Theta, x \to \infty)$ (analogous to corresponding solutions for regular waves). One can establish from (1) some peculiarties of the noise behaviour at distances

$x > x_0$.

3. We have also considered effects of the space modulation of bounded beams in nonlinear medium. Essentially we made the generalization of the results [2] on the case of simultaneous time and space modulations. In the case of the primary beam having only the space modulation which discribed by random Gaussian function $f(r)$, for normalized harmonic intensities one can obtained

$$\tilde{I}_n = \frac{(n\varkappa)^{2(n-1)}}{\pi^{1/2}\, 2^{n-1}} \frac{\Gamma(n+\frac{1}{2})}{\Gamma^2(n+1)} \cdot {}_2F_2\left[n+\frac{1}{2},\, n+\frac{1}{2};\; n+1,\, 2n+1;\; -(n\varkappa)^2 \right] \qquad (2)$$

where $\varkappa = \varepsilon\omega\rho_0' x / c_0\rho_0$, $\tilde{I}_n = I_n(n\omega)/\rho_0'^2$; $n = 1, 2, \ldots$, $\Gamma(n)$ is gamma function, ${}_2F_2$ is hypergeometric function.

It follows from (2) that in the example considered the space modulation influence differs from influence of the random temporal modulation (conf. [2]). The harmonic intensities of the space modulation signal are less than for the plane waves.

References

1 O.V.Rudenko, A.S.Chirkin. Dokladi AN USSR, 1975, 225, 520.

2 O.V.Rudenko, A.S.Chirkin. Dokladi AN USSR, 1974, 214, 1045.

ASYMPTOTIC SOLUTIONS OF NONLINEAR WAVE EQUATIONS

R. W. Lardner, Simon Fraser University

The method of averaging and the two-time method are techniques which were developed originally for the purpose of providing approximate solutions to ordinary differential equations containing small nonlinear terms, especially as related to problems of nonlinear vibration. The equivalence of these two methods for certain classes of ordinary differential equations was established by Morrison [1], a result which is significant because of the rigorous foundation of the averaging method by means of a theorem of Bogoliubov's [2], which therefore indirectly provides a justification of the two-time procedure.

In recent years, both of these methods have been used by a number of authors to find solutions of nonlinear partial differential equations, of which work we refer to two examples. Keller and Kogelman [3] used the two-variable technique to investigate the Klein-Gordon equation with Van der Pol type of nonlinearity and Lardner [4] has investigated the formation of plane shock-waves in a nonlinear viscoelastic medium using the method of averaging. In both of these papers the solution function is expanded in terms of the spatial eigenfunctions of the linearized equation, and the partial differential equation replaced effectively by an infinite system of ordinary differential equations.

It has been pointed out by Nayfeh [5] that a direct use of a two-variable expansion, without expansion in terms of spatial eigenfunctions, can in certain cases enable the solution of a partial differential equation to be obtained more straightforwardly than through any of the above-mentioned methods. A similar direct two-variable expansion has also been used by Chikwendu and Kevorkian [6]. The purpose of the present paper is to investigate this type of method in the context of a wide class of hyperbolic partial differential equations, and in particular to compare it with the method of averaging. We shall show that in the lowest approximation, the two methods are formally equivalent for the whole class of equations considered and that for a certain sub-class of nonlinear wave equations, the direct two-time method offers considerable computational advantage.

We shall consider a partial differential equation for the function $u(x,t)$ of the following type:

$$\rho(x)u_{tt} - [k(x)u_x]_x + q(x)u = \varepsilon E(x,t,u,u_x,u_t, \ldots, \varepsilon). \qquad (1)$$

Here subscripts of x and t denote the corresponding partial derivatives, $\rho(x) > 0$, $k(x) > 0$ and $q(x)$ are given functions of x, ε is a small parameter and E a general, but suitably differentiable, function of the indicated arguments. The dots indicate that E may depend on higher-order derivatives of u than those shown explicitly, except that derivatives of order no higher than the second should occur. Eqn. (1) is assumed to hold for $t > 0$, with the associated initial conditions that $u(x,0)$ and $u_t(x,0)$ are prescribed. It will be supposed that the variable x is restricted to the interval $(0,\ell)$ with u satisfying a pair of linear boundary conditions which we write in the symbolic form $B_1\{u\} = B_2\{u\} = 0$, where

$$B_i\{u\} = \alpha_i u(0,t) + \beta_i u_x(0,t) + \gamma_i u(\ell,t) + \delta_i u_x(\ell,t) \qquad (i = 1,2)$$

for certain given values of the eight constants $\{\alpha_1, \ldots, \delta_2\}$.