An Advanced Course V.M. Starzhinskii of Theoretical Mechanics

for Engineering Students

V.M.Starzhinskii An Advanced Course of Theoretical Mechanics

FOR ENGINEERING STUDENTS

Translated from the Russian by V. M. Volosov, D. Sc.

MIR PUBLISHERS MOSCOW

First published 1982 Revised from the 1980 Russian edition

На английском языке

@ English translation, Mir Publishers, 1982

Издательство «Наука». Главная редакция физикоматематической литературы, 1980



в.м. старжинский теоретическая

Краткий курс по полной программе втузов

МЕХАНИКА

ИЗДАТЕЛЬСТВО «НАУКА» МОСКВА

此为试读,需要完整PDF请访问: www.ertongbook.com

Contents

	Preface to the English Edition	13 15
1	Statics of a Perfectly Rigid Body	19
	Chapter 1. System of Concurrent Forces	19 19
	and Rules of Operations on Free Vectors (20). § 2. Basic Notions of Statics	2 5
	(33). § 3. System of Concurrent Forces	3 6
	Chapter 2. System of Two Parallel Forces. Theory of Couples in the Plane	4 9
	§ 1. System of Two Parallel Forces	49
	§ 2. Theory of Couples in the Plane	54
	Problems	60

Chapter 3. Plane System of Forces	61
§ 1. Reduction of a Plane System of Forces to a Given Centre 1.1. Lemma (61). 1.2. The Poinsot Method. Resultant Force Vector and Resultant Moment (62). 1.3. Case of Reduction of a Plane System of Forces to One Couple (64). 1.4. Varignon's Theorem (64).	61
§ 2. Equilibrium Conditions for a Plane System of Forces 2.1. Three Forms of a System of Equilibrium Equations (66). 2.2. Special Cases of a Plane System of Forces (68). 2.3. Statically Determinate and Statically Indeterminate Systems (71). 2.4. Equilibrium of a System of Bodies under the Action of Forces Lying in One Plane (77). Problems	66 82
Chapter 4. Plane System of Forces: Friction and Trusses	83
§ 1. Friction	83
§ 2. Flat Trusses	89 100
	101
Chapter 5. Arbitrary Force System	101
of Couples in Space	101
Composition of Couples in Space (106). § 2. Reduction of an Arbitrary Force System to a Given Centre 2.1. The Poinsot Method. Resultant Force Vector and Resultant Moment (108). 2.2. Variation of Resultant Moment due to Change of the Reduction Centre. Invariants of a Force System (111). 2.3. Case of Reduction of a Force System to One Couple (113). 2.4. Case of Reduction of a Force System to the Resultant. Varignon's Theo- rem (114). 2.5. Case of Reduction of a Force System to a Wrench.	108
Central Axis (116). § 3. Equilibrium Conditions for an Arbitrary System of Forces 3.1. Vector and Analytical Forms of Equilibrium Conditions (120). 3.2. Special Cases of an Arbitrary System of Forces (122). 3.3. Equilibrium of a Rigid Body with One or Two Fixed Points (123). Problems	120 131
Chapter 6. Centre of Parallel Forces and Centre of Gravity	132
§ 1. Centre of Parallel Forces	132
(132). 1.2. Centre of Parallel Forces (134). § 2. Centre of Gravity	136
Form (145). Problems	143

7

	A	pt	ų	b
	٠			p
		,	ø	
á	á			1

Kinemasics	
Introduction to Kinematics	149
1. Mechanical Motion (149). 2. Space and Time (149). 3. Frame of Reference (150). 4. Historical Notes on the Development of Kinematics (151). 5. Differentiation of a Variable Free Vector (151).	
Chapter 7. Kinematics of a Particle	155
§ 1. Methods of Describing the Motion of a Particle	155
§ 2. Velocity of a Particle in Curvilinear Motion 2.1. Velocity Vector of a Particle (159), 2.2. Velocity Vector in Rectangular Cartesian Coordinates (160), 2.3. Algebraic Velocity. Path Length (162).	159
§ 3. Acceleration of a Particle in Curvilinear Motion 3.1. Acceleration Vector of a Particle (164). 3.2. Acceleration Vector in Rectangular Cartesian Coordinates (166). 3.3. Coordinate Axes Connected with a Natural Trihedron (167). 3.4. Tangential and Normal Accelerations of a Particle (168). 3.5. Rectilinear Motion of a Particle (175).	164
Problems	177
Chapter 8. Simplest Forms of Motion of a Rigid Body	179
§ 1. Translatory Motion of a Rigid Body	179
§ 2. Rotation of a Rigid Body about a Fixed Axis	181
Problems	190
Chapter 9. General Case of Motion of a Free Rigid Body. Motion of a Rigid Body about a Fixed Point	192
§ 1. General Case of Motion of a Free Rigid Body	192
§ 2. Motion of a Rigid Body about a Fixed Point 2.1. Instantaneous Axis of Rotation and Instantaneous Angular Velocity of a Body (196). 2.2. Velocities of the Particles of a Rigid Body Moving about a Fixed Point. Euler's Formulas (197). 2.3. Accelerations of the Particles of a Rigid Body Moving about a Fixed Point (198). 2.4. Accelerations of the Particles of a Free Rigid Body (200).	196
Problems	201

	Chapter 10. Plane Motion of a Rigid Body	201
	§ 1. Velocity of Plane Motion	202
	§ 2. Acceleration of Plane Motion	210216
	Chapter 11. Compound Motion of a Particle	217
	§ 1. Velocity of Compound Motion of a Particle	217
	ical Proof of the Theorem on Composition of Velocities (221). § 2. Acceleration of Compound Motion of a Particle 2.1. The Coriolis Theorem on Composition of Accelerations (223). 2.2. Vector of Coriolis Acceleration of a Particle (224). 2.3. Expression for the Acceleration of a Particle in Polar Coordinates (230).	223
	Problems	231
	Chapter 12. Compound Motion of a Rigid Body	232
	§ 1. Composition of Simplest Motions 1.1. Composition of Two Translatory Motions (233). 1.2. Composi- tion of Rotations about Two Intersecting Asses (233). 1.3. Ex-	233
	ample of Helical Motion of a Rigid Body (237). § 2. Composition of Rotations about Two Parallel Axes 2.1. Case of Rotations of Same Sense (239). 2.2. Case of Rotations of Opposite Sense (240). 2.3. Couple of Rotations (242).	2 39
)	Dynamics	
	Introduction to Dynamics	245
	Chapter 13. Motion of a Free Particle	247
	§ 1. Basic Laws of Classical Mechanics	247
	§ 2. Differential Equations of Motion of a Free Particle 2.1. Equations of Motion in Cartesian Coordinates (252). 2.2. Natural Equations of Motion (254). 2.3. First Basic Problem of Particle Dynamics (254). 2.4. Second Basic Problem of Particle	252
	Dynamics (256). § 3. Integration of Differential Equations of Motion of a Particle for Simplest Cases of Rectilinear Motion 3.1. Case when Force Depends Solely on Time (259). 3.2. Case when Force Depends Solely on Velocity (261). 3.3. Case when Force Depends Solely on the Position of the Moving Particle (263).	259
	Problems	265

Chapter 14. Rectilinear Oscillation of a Particle	266
§ 1. Harmonic Oscillation	266
§ 2. Damped Oscillation	272
Properties of Damped Oscillation (274). § 3. Forced Oscillation 3.1. Forced Oscillation without Resistance (277). 3.2. Resonance (280). 3.3. Effect of Resistance on Forced Oscillation (282).	277 286
Problems	200
Chapter 15. General Principles of Dynamics of Absolute Motion of a Particle	287
§ 1. Principle of Momentum for a Particle	287
§ 2. Principle of Angular Momentum for a Particle	291
 2.3. Corollaries (293). § 3. Principle of Energy for a Particle 3.1. Work and Power of a Force (294). 3.2. Principle of Energy (297). 3.3. Potential Force Field. Force Function (298). 3.4. Conservation of the Mechanical Energy of a Particle in Its Motion in 	294
a Potential Force Field (300). Problems	302
Chapter 16. Motion of a Constrained Particle. Relative Motion of a Particle	302
§ 1. Motion of a Constrained Particle	303
Particle along a Curve (306). 1.3. Simple Pendulum (309). § 2. Relative Motion of a Particle	310
2.2. Special Cases (312). Problems	315
Chapter 17. Analytic Statics	316
§ 1. Principle of Virtual Work	316
(317). 1.3. Principle of Virtual Work (319). § 2. Equilibrium Conditions for a System of Particles in Generalized Coordinates	323
Generalized Coordinates (326). Problems	332
Chapter 18. General Equations of Dynamics. Lagrange's Equations	334
§ 1. General Equation of Dynamics	334

eral Equation of Dynamics (the D'Alembert-Lagrange Principle) (334).	
§ 2. Differential Equations of Motion of a System of Particles in Generalized Coordinates (Lagrange's Equations) 2.1. Statement of the Problem (338). 2.2. Derivation of Lagrange's Equations (339). 2.3. Lagrange's Equations for the Case of a Potential Force Field (342). 2.4. Conclusion (343). Problems	338 347
Chapter 19. General Principles of Dynamics of a System of Particles	348
§ 1. Principle of Momentum for a System of Particles. Principle of the Motion of the Centre of Mass	349
Centre of Mass of a System (352). 1.4. First Integrals (353). § 2. Principle of Angular Momentum for a System of Particles 2.1. Definitions (355). 2.2. Principle of Angular Momentum for a System of Particles about a Fixed Axis (356). 2.3. First In-	355
tegrals (358). § 3. Principle of Energy for a System of Particles 3.1. Differential Form of the Principle of Energy for a System of Particles (359). 3.2. Integral Form of the Principle of Energy for a System of Particles (361). 3.3. Remark on the Work of Internal Forces (362). 3.4. Energy Integral (364). 3.5. Remarks on the Application of the General Principles of Dynamics of a System of Particles (365).	359
§ 4. General Principles of Motion of a System of Particles Relative to the Centre of Mass	366 370
Chapter 20. D'Alembert's Principle. Stability of Equilibrium and Small Oscillation	371
§ 1. D'Alembert's Principle	371
bert's Principle for a System of Particles (379). § 2. Stability of Equilibrium and Small Oscillation 2.1. Statement of the Problem (378). 2.2. Lagrange's Theorem on Stability of Equilibrium (379). 2.3. Small Free Oscillation of a Mechanical System with One Degree of Freedom about Its Stable Equilibrium Position (381).	378
Problems	383
Chapter 21. Elements of Dynamics of a Rigid Body	384
§ 1. Moment of Inertia of a Rigid Body with Respect to an Axis 1.1. Definitions (384). 1.2. The Huygens-Steiner Parallel Axis Theorem (385). 1.3. Moments of Inertia of Simplest Bodies (387). § 2. Rotation of a Rigid Body about a Fixed Axis 2.1. Angular Momentum and Kinetic Energy of a Body in Rota-	389

tional Motion (389). 2.2. Differential Equation of Rotation of a Rigid Body about a Fixed Axis (390). 2.3. Compound Pendulum (391). 2.4. Work and Power of a System of Forces Applied to a Rigid Body Rotating about a Fixed Axis (392). 2.5. Principle of Energy in Rotational Motion (394). § 3. Plane Motion of a Rigid Body	39 40 40
Chapter 22. Dynamics of a Rigid Body (Continued)	40
§ 1. Moments and Products of Inertia (404)	40
Inertia (409). § 2. Pressure Acting on the Axis of a Rotating Rigid Body 2.1. Resultant Force Vector and Resultant Moment of Inertial Forces (411). 2.2. Dynamic Reactions (413). 2.3. Balancing Inertial Forces (414). § 3. Application of Lagrange's Equations to Dynamics of a Rigid	41
Body (Examples)	416
Chapter 23. Fundamentals of the Theory of Impact and of Dynamics of a Particle with Variable Mass	422
§ 1. Fundamentals of the Theory of Impact	42:
Centre of Percussion (432). § 2. Fundamentals of Dynamics of a Particle with Variable Mass 2.1. Meshchersky's Equation (433). 2.2. Tsiolkovsky's First Problem (435). 2.3. Tsiolkovsky's Second Problem (436). Problems	433 433
Chapter 24, Motion of a Particle in a Field of Central Force § 1. Binet's Formulas	438 438 442
ficial Satellites. 2.1. Derivation of Newton's Law of Gravitation from Kepler's Laws (442), 2.2. The Kepler-Newton Problem (443), 2.3. Trajectories of the Earth's Artificial Satellites (444).	442
Chapter 25. Fundamentals of Mechanics of a Thread . § 1. Equilibrium of an Ideal Inextensible Thread in a Stationary Force Field	446 447
of a Thread (447). 1.2. Equilibrium Equations in Terms of the	

12 Contents

Projections on Natural Axes (448). 1.3. Equilibrium Equations of a Thread in Terms of the Projections on Cartesian Coordinate Axes (448). 1.4. Equilibrium Equations of a Thread on a Smooth Surface in Terms of the Projections on Local Coordinate Axes (451). 1.5. Equilibrium Equations of a Thread on a Smooth Surface in Terms of the Projections on the Axes of a Rectangular Cartesian Coordinate System (451). 1.6. Equilibrium of a Thread	
on a Rough Cylindrical Surface. Euler's Formula (452). § 2. Fundamentals of Dynamics of an Ideal Inextensible Thread 2.1. Vector Equation of Motion of a Thread (455). 2.2. The Case of Stationary Motion (455). 2.3. Motion and Tension of a Thread Sliding along a Fixed Rough Plane Curve. The A. P. Minakov Generalization of Euler's Formula (457).	455
Appendix. Simplified Derivation of Basic Principles of Dynamics of a System of Particles for Absolute Motion	460
 (A) Principle of Momentum for a System of Particles and Principle of Motion of the Centre of Mass	460 462 464
Bibliography	467 468

Preface to the English Edition

The present textbook comprises a course in theoretical mechanics

together with some elements of analytical mechanics.

The first part of the book is devoted to statics of a rigid body. The study of the equilibrium of a body under the action of an arbitrary system of forces is preceded by a separate presentation of the case of a plane system of forces. For the latter system the equilibrium involving friction forces and the determination of internal forces in the bars of trusses are considered in detail (Chap. 4).

In the second part of the book (Kinematics) the major role is played by Chap. 9 (General Case of Motion of a Free Rigid Body). This chapter is based on Euler's theorem on instantaneous motion characterized by the velocity field of the particles of a body at each given instant. However, since this general approach may seem too complicated, in Chap. 10 where the plane motion is studied, besides references to Euler's theorem, the proofs independent of that theorem are also given.

In the third part of the book (Dynamics) Chap. 19 (General Principles of Dynamics of a System of Particles) is the central one. In this chapter we employ the teaching methods of the university courses by N. E. Joukowski, S. A. Chaplygin and N. G. Chetayev. Their approach based on the D'Alembert-Lagrange general equation of dynamics makes it possible not to introduce the reactions of ideal constraints and provides a general method of solving problems: the determination of virtual displacements and the first integrals corresponding to them. However, in the Appendix a more traditional derivation of the general principles is also given.

The book includes some optional material: stability of equilibrium and small vibrations (Chap. 20), advanced topics in dynamics of a rigid body (Chap. 22), the theory of impact and the theory of motion of a body with variable mass (Chap. 23), the motion of a particle in a field of central force and the trajectories of the Earth's artificial satellites (Chap. 24), and a detailed representation of mechanics of a flexible inextensible thread (Chap. 25) concluding the book.

14 Preface to the English Edition

The book contains more than 200 problems; for 120 of them detailed solutions and for the rest hints and answers are given. This enables the reader to use the book both for studying the theory and for problem-solving practice.

The textbook is intended for engineering students and can be used for studying under the guidance of a teacher as well as for self-instruction.

V. M. Starzhinskii

Introduction

1. Subject of Theoretical Mechanics. Theoretical mechanics is a science treating of the simplest form of motion of substance, namely the general laws of mechanical motion and of equilibrium of material bodies or their parts.

In the broad sense motion of substance is understood as any change of the state of a material body or bodies in thermal, chemical, electromagnetic, intratomic and other processes. Theoretical mechanics is confined to the simplest form of motion—mechanical motion.

By mechanical motion is meant variation with time of the position of material bodies relative to one another. Since the state of equilibrium is a special case of mechanical motion, theoretical mechanics also includes the study of equilibrium of bodies.

The observation of various natural phenomena shows that not all properties of the bodies involved in the phenomenon in question equally affect the course of the phenomenon or its final result. For instance, experiments show that the forces with which a beam having two supports acts on them are essentially dependent on the position of the supports and are practically independent of the deflection of the beam (provided that the deflection is small). Therefore when determining these forces we may conditionally replace the real beam by a nondeformable (perfectly rigid) one. In the investigation of other phenomena analogous arguments lead us to the notions of models of bodies such as a material point (a particle), a point charge, etc. An attempt to solve even the simplest problems without introducing such simplified models would fail. However, one should bear in mind that in nature there are no perfectly rigid bodies. particles, point charges, etc. and that all these are abstractions which enable us to consider theoretically the phenomena in question and to solve the required problems.

The present course is devoted to the study of classical mechanics based on the laws which were first stated exactly by G. Galileo (1564-1642) and I. Newton (1642-1727). At the end of the 19th century and the beginning of the 20th century it was found that the

laws of classical mechanics do not apply to motion of microparticles and to bodies moving with velocities close to that of light. At the beginning of the 20th century relativistic mechanics was created; relativistic mechanics is based on the relativity theory of A. Einstein (1879-1955). The relativity theory establishes the general laws concerning the relationship between space, time, mass and energy. Relativistic mechanics is applicable to the study of motion with velocity close to the speed of light and indicates the limits within which the laws of classical mechanics remain valid. This however does not diminish the role of classical mechanics as a practical method for studying motion of macroscopic bodies whose velocities are small compared with that of light, that is the motion usually dealt with in engineering.

2. Methods of Theoretical Mechanics. Theoretical mechanics, like other natural sciences, widely uses the method of abstraction. The application of this method and the generalization of the results of human experience, technological practice and experiment made it possible to establish some general laws playing the role of axioms. All the further propositions of classical mechanics can be derived from these axioms using logical argument and mathematical calculations. Since theoretical mechanics mostly deals with quantitative relationships it is clear that mathematical analysis must play a very important role in it. However, although the course of theoretical mechanics is saturated with mathematics and contains few references to experimental studies, this does not at all mean that theoretical mechanics can do without experimental verification of its laws and conjectures. On the contrary, as in all other branches of knowledge, the final proof of the propositions of theoretical mechanics lies in experiment and practice. The history of the development of science and, particularly, of theoretical mechanics confirms that only experiment and practice can decide whether or not a hypothesis or a theory is correct.

3. Historical Notes. Theoretical mechanics is closely connected with practice and is one of the most ancient sciences. Although the earliest manuscripts on mechanics known to us belong to the 4th century B.C. the remnants of ancient structures show that even much earlier the ancients were familiar with some elements of mechanics.

The beginning of the development of mechanics was primarily connected with statics—the science treating of the equilibrium of material bodies. As early as the 3rd century B.C. the scientific basis of statics was founded, mainly in the works of the great Greek scientist Archimedes (circa 287-212 B.C.). He elaborated the exact solution of the problem of equilibrium of the lever, introduced the concept of the centre of gravity, discovered the well-known law of hydrostatics named after him, etc.