An Advanced Course of Theoretical Mechanics

V.M. Starzhinskii

for Engineering Students

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FOR ENGINEERING STUDENTS

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ТЕОРЕТИЧЕСКАЯ МЕХАНИКА

Краткий курс по полной программе втузов

ИЗДАТЕЛЬСТВО «НАУКА»
МОСКВА

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Preface to the English Edition

The present textbook comprises a course in theoretical mechanics together with some elements of analytical mechanics.

The first part of the book is devoted to statics of a rigid body. The study of the equilibrium of a body under the action of an arbitrary system of forces is preceded by a separate presentation of the case of a plane system of forces. For the latter system the equilibrium involving friction forces and the determination of internal forces in the bars of trusses are considered in detail (Chap. 4).

In the second part of the book (Kinematics) the major role is played by Chap. 9 (General Case of Motion of a Free Rigid Body). This chapter is based on Euler's theorem on instantaneous motion characterized by the velocity field of the particles of a body at each given instant. However, since this general approach may seem too complicated, in Chap. 10 where the plane motion is studied, besides references to Euler's theorem, the proofs independent of that theorem are also given.

In the third part of the book (Dynamics) Chap. 19 (General Principles of Dynamics of a System of Particles) is the central one. In this chapter we employ the teaching methods of the university courses by N. E. Joukowski, S. A. Chaplygin and N. G. Chetayev. Their approach based on the D'Alembert-Lagrange general equation of dynamics makes it possible not to introduce the reactions of ideal constraints and provides a general method of solving problems: the determination of virtual displacements and the first integrals corresponding to them. However, in the Appendix a more traditional derivation of the general principles is also given.

The book includes some optional material: stability of equilibrium and small vibrations (Chap. 20), advanced topics in dynamics of a rigid body (Chap. 22), the theory of impact and the theory of motion of a body with variable mass (Chap. 23), the motion of a particle in a field of central force and the trajectories of the Earth's artificial satellites (Chap. 24), and a detailed representation of mechanics of a flexible inextensible thread (Chap. 25) concluding the book.

The book contains more than 200 problems; for 120 of them detailed solutions and for the rest hints and answers are given. This enables the reader to use the book both for studying the theory and for problem-solving practice.

The textbook is intended for engineering students and can be used for studying under the guidance of a teacher as well as for self-instruction.

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Introduction

1. Subject of Theoretical Mechanics. *Theoretical mechanics* is a science treating of the simplest form of motion of substance, namely the general laws of mechanical motion and of equilibrium of material bodies or their parts.

In the broad sense motion of substance is understood as any change of the state of a material body or bodies in thermal, chemical, electromagnetic, intratomic and other processes. Theoretical mechanics is confined to the simplest form of motion—mechanical motion.

By *mechanical motion* is meant variation with time of the position of material bodies relative to one another. Since the state of equilibrium is a special case of mechanical motion, theoretical mechanics also includes the study of equilibrium of bodies.

The observation of various natural phenomena shows that not all properties of the bodies involved in the phenomenon in question equally affect the course of the phenomenon or its final result. For instance, experiments show that the forces with which a beam having two supports acts on them are essentially dependent on the position of the supports and are practically independent of the deflection of the beam (provided that the deflection is small). Therefore when determining these forces we may conditionally replace the real beam by a nondeformable (perfectly rigid) one. In the investigation of other phenomena analogous arguments lead us to the notions of models of bodies such as a material point (a particle), a point charge, etc. An attempt to solve even the simplest problems without introducing such simplified models would fail. However, one should bear in mind that in nature there are no perfectly rigid bodies, particles, point charges, etc. and that all these are abstractions which enable us to consider theoretically the phenomena in question and to solve the required problems.

The present course is devoted to the study of classical mechanics based on the laws which were first stated exactly by G. Galileo (1564-1642) and I. Newton (1642-1727). At the end of the 19th century and the beginning of the 20th century it was found that the laws of classical mechanics do not apply to motion of microparticles and to bodies moving with velocities close to that of light. At the beginning of the 20th century relativistic mechanics was created; relativistic mechanics is based on the relativity theory of A. Einstein (1879-1955). The relativity theory establishes the general laws concerning the relationship between space, time, mass and energy. Relativistic mechanics is applicable to the study of motion with velocity close to the speed of light and indicates the limits within which the laws of classical mechanics remain valid. This however does not diminish the role of classical mechanics as a practical method for studying motion of macroscopic bodies whose velocities are small compared with that of light, that is the motion usually dealt with in engineering.

2. Methods of Theoretical Mechanics. Theoretical mechanics, like other natural sciences, widely uses the method of abstraction. The application of this method and the generalization of the results of human experience, technological practice and experiment made it possible to establish some general laws playing the role of axioms. All the further propositions of classical mechanics can be derived from these axioms using logical argument and mathematical calculations. Since theoretical mechanics mostly deals with quantitative relationships it is clear that mathematical analysis must play a very important role in it. However, although the course of theoretical mechanics is saturated with mathematics and contains few references to experimental studies, this does not at all mean that theoretical mechanics can do without experimental verification of its laws and conjectures. On the contrary, as in all other branches of knowledge, the final proof of the propositions of theoretical mechanics lies in experiment and practice. The history of the development of science and, particularly, of theoretical mechanics confirms that only experiment and practice can decide whether or not a hypothesis or a theory is correct.

3. Historical Notes. Theoretical mechanics is closely connected with practice and is one of the most ancient sciences. Although the earliest manuscripts on mechanics known to us belong to the 4th century B.C. the remnants of ancient structures show that even much earlier the ancients were familiar with some elements of mechanics.

The beginning of the development of mechanics was primarily connected with statics—the science treating of the equilibrium of material bodies. As early as the 3rd century B.C. the scientific basis of statics was founded, mainly in the works of the great Greek scientist Archimedes (*circa* 287-212 B.C.). He elaborated the exact solution of the problem of equilibrium of the lever, introduced the concept of the centre of gravity, discovered the well-known law of hydrostatics named after him, etc.