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S. Watanabe Yu. V. Prokhorov (Eds.)

Probability Theory and Mathematical Statistics

Fifth Japan-USSR Symposium
Proceedings, 1986

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PREFACE

The Fifth Japan-USSR Symposium on Probability Theory was held at Kyoto University, July 8-14, 1986. Attendance from USSR numbered 25 and from Japan 190. This volume contains the papers presented at the symposium. Records of the meetings, lists of the Organizing Committee and a list of the local editorial committee of the Proceedings are attached at the end of the volume.

As well as the previous four, the fifth one was very fruitful not only in promoting probability and statistics in both countries but also in producing mutual understanding. We express our deep gratitude to all those who have contributed to the success of the symposium and who made efforts in preparing the proceedings. The support made by Mathematical Society of Japan, Kyoto University, and Japan World Exposition ('70) Commemorative Fund is greatly acknowledged.

Professor G. Maruyama deceased three days before the opening. He was chairman of the Japanese organizing committee and made immeasurable contribution to these symposia. In memory of him, Professors H. Tanaka and A.N. Shiryayev delivered two lectures that are contained at the beginning of this volume. We remind ourselves of another sad fact that Professor G.M. Mania passed away without seeing the fifth symposium. He had been chairman of the local organizing committee at Tbilisi for the fourth symposium. These bring great sorrow to all the participants.

S. Watanabe
Yu.V. Prokhorov

CONTENTS

PROFESSOR GISIRO MARUYAMA, IN MEMORIAM

Professor Gisiro Maruyama passed away on July 5, 1986, only three days before the opening of this symposium. His death is mourned over by all mathematicians, but especially at this symposium, which was organized by himself, his death is felt more deeply. The Japan-USSR Symposium on Probability Theory has been held five times in which Maruyama always played a leading part; we have been influenced by him through his work and through his person.

Since everyone in this audience knows Maruyama's work on probability theory, it is hardly necessary to review his work. But, by giving only a few comments on some of the highlights, I would like simply to remind you how important his contributions were and how wide his interests were in probability theory.

Professor Maruyama was born on April 4, 1916. He graduated from Tôhoku Imperial University in 1939, whose department of mathematics was very active in the field of Fourier analysis at that time. There he studied Fourier analysis, but he was also interested in physics and spent a year and a half in the department of physics in Hokkaido University. He began his mathematical work by writing a paper on Fourier series in 1939. But his interest was not limited to Fourier analysis; his main interest was probability theory, especially Wiener's work. Except for a few of his earlier works, most of his works are concerned with probability theory, but we still see that the method of Fourier analysis, which he acquired while he was in Tôhoku University, underlies his whole work as a fundamental tool. He was appointed as a research assistant in Kyushu University in 1941 and was promoted to a professor in 1949. Later he served as a professor in several universities, namely in Ochanomizu University, Kyushu University (for the second time), Tokyo University of Education, the University of Tokyo, the University of Electro-Communications, and finally in Tokyo Denki University where he remained until his death.

In an earlier period Maruyama was much interested in Wiener's generalized harmonic analysis and also in the works of E. Slutsky (1927, 1938), H. Wold (1937) and E. Hopf (1937: Ergodentheorie). Influenced by these works he studied stationary processes and wrote the paper [5] in 1949, for which he received the degree of Doctor of Science. The Japanese version of this paper appeared already in 1947. In this work he showed that the assumption of the Gaussian structure in addition to the stationarity leads to many fruitful results. Among them the following is known as Maruyama's theorem. Let

$$z(t) = x(t) + \sqrt{-1}\, y(t)$$

be a stationary Gaussian process and let F, G and H be the corresponding spectral distribution functions defined by

$$E\{x(t+\tau)x(t)\} = \int_{-\infty}^{\infty} \exp(\sqrt{-1}\,\lambda\tau)\, dF(\lambda)$$

and by similar formulas for $E\{y(t+\tau)y(t)\}$ and $E\{x(t+\tau)y(t)\}$. Then $z(t)$ *is ergodic if and only if* F, G *(and so* H*) are continuous.* He also gave a proof to the result of A. N. Kolmogorov: Sur l'interpolation et extrapolation des suites stationaires, C.R.(1939).

Ideas similar to those used in the paper of 1949 are seen in some of his subsequent papers. For example, about 20 years later he again considered stationary processes but he assumed that the finite dimensional distributions are infinitely divisible. He then obtained the following result ([23]): *An infinitely divisible stationary process* $x(t)$ *is mixing if and only if*

(i) $\lim_{t\to\infty} \rho(t) = 0,$

(ii) $\lim_{t\to\infty} Q_{0t}(|xy| > \delta) = 0 \quad (\forall\delta > 0), \qquad \lim_{t\to\infty} \int_{0<x^2+y^2\leq 1} xy\, Q_{0t}(dxdy) = 0,$

where $\rho(t)$ is the correlation function of the Gaussian component of $x(t)$ and $Q_{0t}(dxdy)$ is the 2-point Lévy measure of the Poisson component.

In 1942, K. Itô published his first paper on stochastic differential equations which was written in Japanese. Maruyama immediately recognized the importance of Itô's work. I have heard that Maruyama read Itô's Japanese paper in the military camp where he had been drafted for World War II. At that time Maruyama was the only person who read thoroughly Itô's paper of 1942. Soon he wrote two papers on stochastic differential equations [11],[14].

A simple form of Itô's stochastic differential equation is

$$y(t) = y_0 + \int_0^t a(\tau, y(\tau))d\tau + x(t),$$

where $x(t)$ is a Brownian motion. What Maruyama proved in [11] is that the transition function

$$F(s,x;t,y) = P\{y(t) \leq y \,/\, y(s) = x\}, \quad s < t,$$

of the Markov process $y(t)$ described by the above stochastic differential equation is smooth and satisfies the parabolic equation

$$\{\frac{\partial}{\partial s} + a(s,x)\frac{\partial}{\partial x} + \frac{1}{2}\frac{\partial^2}{\partial x^2}\}F(s,x;t,y) = 0.$$

The proof is purely probabilistic and is based on the following formula, for which [11] may be more appreciated.

$$F(s,y_0;t,x)$$

$$= \int_{-\infty}^{x} E\{\exp[\int_s^t a(\tau,w(\tau))dw(\tau) - \frac{1}{2}\int_s^t a^2(\tau,w(\tau))d\tau]/w(s) = y_0, w(t) = y\}$$

$$\cdot \frac{1}{\sqrt{2\pi(t-s)}}\exp[-(y-y_0)^2/2(t-s)]dy.$$

Here $E\{- - / w(s) = y_0, w(t) = y\}$ is the expectation with respect to the conditional Wiener measure. This formula is now well-known as the Cameron-Martin formula or Girsanov's formula. Before Maruyama was engaged in this work, some mathematicians in analysis in Japan had been interested in a series of papers by Cameron and Martin. I believe, however, he was the first to apply the idea of Cameron-Martin for the study of Markov processes.

In 1938 stochastic difference equations were considered by S. Bernstein (Équations différentielles stochastiques) and then developed by I. I. Gihman. In the paper [14] Maruyama developed Bernstein's idea within the framework of Itô's stochastic differential equations with emphasis on limit theorems; in particular, he formulated an invariance principle in the framework of Markov processes and, as a special case, gave a proof to the Kolmogorov-Smirnov limit theorem concerning empirical distribution functions. Even though he did not formulate the problem as a weak convergence of probability measures on a suitable (Polish) space of trajectories, his work is highly appreciated as one of the earliest limit theorems for stochastic processes. A general and complete formulation of limit theorems of this type was given by Yu. V. Prohorov and A. V. Skorohod in 1956.

In 1956 Maruyama moved again to Kyushu University. I remember that he took a great interest in the work of Prohorov and Skorohod and suggested to his colleagues and students the study of this field. He wrote in Japanese a fine exposition [18] of this field in 1960. I must also say that he always had a great interest in the limit theorems in USSR, especially in the works of S. Bernstein, A. Ya. Khintchine, Yu. V. Linnik and S. H. Sirazdinov. He also recognized the importance of the theory of Markov processes which was rapidly progressing at that time due to the works of W. Feller, J. L. Doob, K. Itô, H. P. McKean and E. B. Dynkin, and encouraged us to work in this direction ([15], [16], [17]).

Since the middle of the 1960's Maruyama was interested again in stationary processes and flows and took a leadership in this field in Japan. In 1966 he showed that *any continuous flow can be realized as a measurable flow on a compact metric space* ([20]). Based on this fundamental result he discussed several topics among which are the following:

(i) Time change of flows.
(ii) Spectral properties of K-flows.
(iii) Natural extension of semigroups to flows.

Outside mathematics Maruyama was also active in his efforts to reflect aspirations of scientists in the governmental policy on science; he was a member of the Science Council of Japan for nine years from 1969 to 1978.

For the past several years he was engaged in the work on Wiener functionals and probability limit theorems. His ideas, some of which seem to have been in his mind ever since his work of 1949, are to consider a stationary Gaussian process as a basic field on which various limit theorems for subordinated stationary processes are discussed. Recently he concentrated on writing papers on this subject and, astonishingly, he actually wrote up three long papers [30], [31], [32]. The third paper [32] which became his last one was completed only last April.

It is obviously impossible to give a complete description of his work here. We have a plan to publish a volume of his selected papers in which all the (English) papers I mentioned here will be included.

Until quite recently everyone here expected to listen to him deliver his lecture "Gaussian limit theorems for Wiener functionals" ([33]) in this hall, but now we can learn his results only through reading the three papers [30], [31], [32]. The author is no longer with us, but his works remain with us. To the end he really loved mathematics and dedicated his life to it.

H. Tanaka

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[33] Gaussian limit theorems for Wiener functionals, in this volume.

SOME WORDS IN MEMORY OF PROFESSOR G. MARUYAMA

On the day of arrival of the Soviet delegation in Japan, July 5, 1986, to attend the 5-th Japan-Soviet Symposium on Probability Theory we learned with regret about the decease of Professor G. Maruyama.

Professor G. Maruyama was well-known to Soviet probabilists both as a distinguished scientist and a permanent co-chairman of our Symposia. All those who were immediately involved in the organization of the symposia are aware of his distinguished role in the realization of the concept of the regular joint Soviet-Japan meeting on probability theory and in the concrete organization of all the Soviet-Japan and Japan-Soviet Symposia.

In giving a tribute of gratitude and deep respect both for the personality and for the scientific achievement of Professor G. Maruyama one wants to share with the participants of the Symposium a notion of in what way the works of G.Maruyama exerted a large influence on the several lines of research in the theory of probability, especially, in our country.

In the famous paper of G. Maruyama, "The harmonic analysis of stationary stochastic processes", published in 1949 (the Japanese version was published in 1947) in addition to such major results as the spectral representation

$$(1) \qquad \xi_t = \int_{-\infty}^{\infty} e^{it\lambda} X(d\lambda)$$

for stationary stochastic processes $\xi_t = \xi_0(T^t\omega)$, $-\infty < t < \infty$, were also included necessary and sufficient conditions for validity, in Gaussian case, of the properties

$$(M) = \textit{Mixing}, \quad \textit{i.e.} \quad P(A \cap T^{-s}B) - P(A)P(B) \longrightarrow 0 \quad s \to \infty,$$

$$(E) = \textit{Ergodicity}, \quad \textit{i.e.} \quad \frac{1}{t}\int_0^t [P(A \cap T^{-s}B) - P(A)P(B)]ds \longrightarrow 0, \quad t \to \infty,$$

$(MW) = \textit{Mixing in the wide sense}, \quad \textit{i.e.}$

$$\frac{1}{t}\int_0^t [P(A \cap T^{-s}B) - P(A)P(B)]^2 ds \longrightarrow 0, \quad t \to \infty,$$

If $E\xi_t = 0$ and $R_\xi(\tau) \equiv E\xi_{t+\tau}\xi_t = \int_{-\infty}^{\infty} e^{i\tau\lambda} F_\xi(d\lambda)$ then the results of Maruyama can be formulated as

$$(M) \Longleftrightarrow R_\xi(\tau) \longrightarrow 0, \quad \tau \to \infty;$$

$$(MW) \Longleftrightarrow (E) \Longleftrightarrow \frac{1}{t}\int_0^t |R_\xi(s)|^2 ds \longrightarrow 0, \quad t \to s$$

$$\Longleftrightarrow F_\xi(\lambda) \textit{ continuous in } \lambda$$

where $\quad F_\xi(\lambda) = F_\xi(-\infty, \lambda]$.

In 1957, A. N. Kolmogorov in the context of nonlinear analysis of stochastic processes, posed the problem of extension of Maruyama's necessary and sufficient conditions for properties (M), (MW) and (E) to the case of non-Gaussian processes. As a first step in this direction it was natural to consider the case of stationary processes with finite moments of all order, $E|\xi_t|^k < \infty$, $k \geq 1$, for which one has uniqueness of the moment problem.

Starting from the spectral representation (1) for a process, one can obtain for the moments $m_\xi^{(k)}(t_1, \cdots, t_k) = E\xi_{t_1} \cdots \xi_{t_k}$ and corresponding semiinvariants $S_\xi^{(k)}(t_1, ..., t_k)$ the following spectral representations

$$m_\xi^{(k)}(t_1, \cdots, t_k) = \int e^{i(t,\lambda)} M_\xi^{(k)}(d\lambda) ,$$

$$S_\xi^{(k)}(t_1, \cdots, t_k) = \int e^{i(t,\lambda)} F_\xi^{(k)}(d\lambda) ,$$

where $M_\xi^{(k)}$ and $F_\xi^{(k)}$ are moments and semiinvariant measures of order k, concentrated on hyperspaces

$$L_k = \{(\lambda_1, \cdots, \lambda_k) : \quad \lambda_1 + \cdots + \lambda_k = 0\} .$$

The semiinvariants $S_\xi^{(k)}(t_1, \cdots, t_k)$ play the role of correlation functions of high orders and in some sense their structure is simpler than that of moments. Moreover, higher order moments can be expressed in symmetric form using semiinvariants. For example,

$$\begin{aligned} m_\xi^{(2)}(t_1, t_2) =& S_\xi^{(2)}(t_1, t_2) + S_\xi^{(1)}(t_1) S_\xi^{(1)}(t_2), \\ m_\xi^{(3)}(t_1, t_2, t_3) =& S_\xi^{(3)}(t_1, t_2, t_3) + S_\xi^{(2)}(t_1, t_2) S_\xi^{(1)}(t_3) \\ &+ S_\xi^{(2)}(t_1, t_3) S_\xi^{(1)}(t_2) + S_\xi^{(2)}(t_2, t_3) S_\xi^{(1)}(t_1). \end{aligned}$$

It turns out (V.P. Leonov, Doklady of the Acad. of Sci. USSR, 133,3 (1960), 523-536) that the results of Maruyama for the non-Gaussian case can be generalized as follows

$$(M) \Longleftrightarrow S_\xi^{(k+l)}(t_1, \cdots, t_k, t'_1 + \tau, \cdots, t'_l + \tau) \longrightarrow 0, \quad \tau \to \infty, k, l \geq 1, \quad t_i, t'_j \in \mathbf{R} ;$$

$$(MW) \Longleftrightarrow \frac{1}{t} \int_0^t |S_\xi^{(k+l)}(t_1, \cdots, t_k; t'_1 + \tau, \cdots, t'_l + \tau)|^2 d\tau \longrightarrow 0, \quad t \to \infty, \qquad k, l \geq 1, t_i, t'_j \in \mathbf{R} ;$$

$$(MW) \Longleftrightarrow var|F_\xi^{(k)}(L_k^l)| = 0, \quad 1 \leq l < k, \qquad L_k^l = \{(\lambda_1, \cdots, \lambda_k) : \lambda_1 + \cdots + \lambda_k = 0, \quad \lambda_1 + \cdots + \lambda_l = \nu\}, \quad \nu \in \mathbf{R},$$