# MSC/NASTRAN

## Static and Normal Modes Analysis

*A study of computerized technology*

*Harry G. Schaeffer*

# MSC/NASTRAN Primer

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# Preface

The field of linear structural analysis has matured to the point where useful technology has been coded and is available as computer programs. This state of affairs could well be called computerized technology.

The computer programs which support structural analysis represent the cutting edge of technology in structural mechanics as well as the fields of computer science and numerical analysis. It is an unfortunate fact of life that the structural engineer must be aware of the technological content of the large analysis programs such as **MSC/NASTRAN** in order to use them efficiently and effectively.

It has been my observation that the typical user of general purpose programs based on the finite element analysis has a marginal background in structural analysis with little or none in numerical analysis or computer science. The task of confronting the user is thus staggering. The user must master the vocabulary of the program sufficiently well to cause it to execute, and meet delivery and cost schedules.

The resulting state of affairs is truly dangerous. The user will be frustrated by poor documentation, a limited background and a series of fatal error messages from the program. When the program finally accepts the data and produces results the user may be mesmerized into believing that just because the program ran, the results are correct. Nothing could be further from the truth. In fact, the engineers' job has just begun since the computer results must be validated.

The purpose of this book is twofold. The first goal is to provide the user with a description of the technological content of a current general purpose finite element program using **MSC/NASTRAN** as a model. The second goal is to describe the **MSC/NASTRAN** vocabulary and capability for static and normal modes analysis.

The material presented in this text is not new or original, but its organization is a departure from finite element text books and the **MSC/NASTRAN** manuals. The material is organized functionally rather than in alphabetic order. Thus all of the input data associated with specifying degrees of freedom or in modifying the stiffness matrix are described in Chapter 7. Similarly, the finite elements are described in Chapter 9, material properties in Chapter 10 and static loads in Chapter 11.

The organization of the **MSC/NASTRAN** program is described in Chapter 1 which includes a discussion of the various data decks in **MSC/NASTRAN**. Chapter 2 provides a review of matrix and index notation and Chapter 3 is included to try to motivate the reader to learn more about the **MSC/NASTRAN** language called DMAP.

The theoretical foundation for the finite element method is presented in Chapters 4 through 6, and the behavioral functions used to approximate element behavior are presented in Chapter 8.

Chapters 12 and 13 describe the use of **MSC/NASTRAN** for static and normal modes analysis, respectively. Chapter 13 includes the formulation of real eigenvalue problems and a discussion of the eigenvalue extraction routines.

The material in this book describes **MSC/NASTRAN** as it appeared in the Spring of 1979. Since the software/hardware environment is continually changing it is inevitable that the program will change over a period of time to the point where the descriptive material contained herein is out of date.

The material presented in this book has been used to support intensive short courses on the finite element method and static and normal modes analysis using **MSC/NASTRAN**. In the university environment the material could be adequately covered in two semesters. Students should be encouraged to use the program during the course since the program provides a source of example problems and is the ultimate learning tool.

This book is similar to the *NASTRAN Primer: Statics and Normal Modes Analysis* which was printed in 1977 and which reflected Level 15.5 COSMIC NASTRAN. In order to reflect the current version of **MSC/NASTRAN** it was necessary to essentially rewrite those portions of the original text that described the program capability. A close comparison of the two books will show that Chapters 2, 4, 5, 6, and 8, which present the technological content of the program, are basically unchanged. The remainder of the book, while organized similarly to the original, has been rewritten.

I would like to acknowledge the cooperation of the MacNeal-Schwendler Corp. in preparing this book. In particular, Dr. Richard MacNeal who encouraged me to undertake the project and who later spent long hours at the Boulder Patch proofreading the copy; and Steve Wall, Jerry Joseph, Mike Gockel, Dean Bellinger, and C. W. McCormick for their technical assistance.

It is also a personal pleasure to acknowledge the day-to-day assistance and support from my close friend, business associate, and wife, Judith Ann. This book is truly a joint venture and would not have been written in any other way.

Finally, this book is dedicated to you, the reader and the soon-to-be **MSC/NASTRAN** expert. If you really want to understand and master the technique for using the program you have a long journey ahead of you. Bon Voyage!

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March, 1979

# List of Symbols

The following is a list of the principal symbols used in this text. Other symbols are defined where they appear.

Matrices are denoted by a boldface symbol. Subscripts, if they appear, are generally used to denote the **MSC/NASTRAN** displacement set associated with the matrix. The order of the matrix is then indicated by the number of subscripts with the first indicating the row set and the second the column set. For example $\mathbf{M}_{ao}$ is a partition of the mass matrix which has "a" rows and "o" columns. Overbars and tildas (~) are used to denote special quantities. The dot notation is used to represent derivatives with respect to time.

## Matrices and Sets

| | |
|---|---|
| $\mathbf{a}$ | Set of polynomial coefficients (eq. 8.1) or acceleration vector |
| $a_{ij}$ | Direction cosines (Sec. 4.1.5) |
| $\mathbf{B}$ | Boolian transformation matrix (eq. 7.2) |
| $\mathbf{b}$ | Defined by (eq. 6.33) |
| $\mathbf{C}$ | Compliance matrix (eq. 4.72) |
| $\mathbf{c}$ | Forces of constraint |
| $\mathbf{c}_i$ | Number of active columns in $i^{th}$ row of a matrix (Sec. 7.3.5) |
| $\mathbf{D}$ | Matrix of differential operators (eq. 6.29) or rigid body transformation matrix |
| $\mathbf{E}$ | Elasticity matrix (eq. 4.64) |
| $\mathbf{E}_f$ | Bending rigidity matrix (eq. 4.93) |
| $E_{ijkl}$ | Elasticity tensor (Sec. 4.3.1) |
| $\mathbf{e}$ | Physical components of strain (eq. 4.66) |
| $\mathbf{e}_I$ | Initial strain (Sec. 4.3.6) |
| $\mathbf{e}^o$ | Reference surface strains (eq. 4.79) |
| $\mathbf{F}$ | Force |
| $\mathbf{f}$ | Flexibility matrix (Sec. 7.9) |
| $\mathbf{G}$ | Transformation matrix |
| $\mathbf{G}_1, \mathbf{G}_2, \mathbf{G}_3, \mathbf{G}_4$ | Material matrices associated with membrane, bending, shear, and membrane-bending coupling, respectively (eq. 9.41) |
| $\mathbf{I}$ | Unit matrix or inertia tensor |
| $\mathbf{i}_1, \mathbf{i}_2, \mathbf{i}_3$ | Unit vectors in coordinate directions |
| $\mathbf{J}$ | Jacobian of transformation (Sec. 8.6.2) or transformed matrix in the standard form of the eigenvalue problem (eq. 13.30) |
| $\mathbf{k}$ | Stiffness matrix (Sec. 6.2) |
| $\mathbf{L}$ | Lower triangular matrix (Sec. 2.2.2.5) |
| $L_{ij}$ | Lagrangian small strain tensor (eq. 4.14) |
| $\ell_{ij}$ | Lagrangian small strain tensor (eq. 4.14) |
| $\mathbf{M}$ | System mass matrix (eq. 13.4) or moment resultants (eq. 4.82) |
| $\mathbf{m}$ | Element mass matrix (eq. 13.2) |
| $\mathbf{N}$ | Shape functions (Sec. 8.2), force resultants (eq. 4.83), or defining direction of force (eq. 11.2) |
| $\mathbf{n}$ | Unit normal to surface, matrix representation |
| $n_i$ | Unit normal to surface, indicial representation |
| $\mathbf{P}$ | Set of node point forces (Sec. 6.2) |
| $\mathbf{p}(m)$ | Set of polynomial functions up to $m^{th}$ order (eq. 8.1) |
| $\mathbf{Q}$ | Shear resultant vector (eq. 9.42) |
| $\mathbf{R}$ | Position of material point in deformed body (Sec. 4.1), or coefficient matrices in MPC relation (Sec. 7.5.1) |
| $\mathbf{r}$ | Position of material point in undeformed body (Sec. 4.1) |

**S** Element rigid body transformation matrix (Sec. 7.9) or compliance matrix (Sec. 10.1.4)
**T** Set of surface tractions (Sec. 6.5)
**U** Upper triangular matrix (Sec. 2.2.2.5)
**u** Set of node point displacements (Sec. 6.2)
**v** Displacement vector or orientation vector used to define local coordinate system for finite elements (Sec. 9.5.1.1.1)
$v_i$ Displacement vector, indicial representation
**v*** Prescribed displacement on the boundary (eq. 5.30)
**X** Body force
**Y** Prescribed displacement values

## Special Symbols

$\mathcal{M}$ Modal mass matrix (13.28)
$\mathcal{K}$ Modal stiffness matrix (13.28)

## Scalars

A Area
C Degree of freedom code
D Bending rigidity (eq. 4.94)
E Modulus of elasticity
G Grid or scalar point identification number
I Moment of inertia
$I_{yy}, I_{zz}, I_{zy}$ Area moments of inertia
J Torsional constant
$K_y, K_z$ Shear coefficients
L Length
M Moment or experimentally determined multiply and add time (Sec. 7.3.5)
m Mass
$Q_x, Q_y$ Shear resultants (Sec. 4.4.1)
q Distributed surface pressure (Sec. 4.4.1)
t Time or thickness
U Internal strain energy (eq. 4.19)
$U_o$ Internal strain energy density (eq. 5.17)
U* Complementary internal strain energy (eq. 5.31)
V Potential, or beam forces shear resultant, or volume
(u, v, w) Scalar components of displacement (eq. 4.76)
W Work or wavefront (Sec. 7.3.5)
$W_e$ Work of external forces
$W_i$ Work of internal forces

## Greek Symbols

$\alpha$ Coefficients of thermal expansion (Sec. 4.3.6)
$\delta_{ij}$ Kronecker delta
$\sigma$ Stress vector (eq. 4.37) or components of stress (eq. 4.66)
$\sigma_i$ Stress vector, indicial representation
$\sigma_{ij}$ Components of stress, indicial representation
$\sigma^*$ Prescribed stress vector on boundary (eq. 5.21)
$\lambda$ Lame coefficient (eq. 4.70), eigenvalue, or transformation matrix
$\nu$ Poisson's ratio
$\mu$ Lame coefficient (eq. 4.70)
$\chi^o$ Curvatures of reference surface (eq. 4.81)
$\theta$ Rotation of angular coordinate

| | |
|---|---|
| $\rho$ | Density |
| $\epsilon$ | Error |
| $(\xi, \eta, \zeta)$ | Intrinsic coordinates |
| $\phi$ | Eigenvector |
| $\omega$ | Circular frequency or Angular velocity vector |
| $\omega_i$ | Components of rotation vector (eq. 4.24) |

**Subscripts**

| | |
|---|---|
| a | Analysis set |
| b | Set of physical degrees of freedom retained in generalized dynamic reduction |
| c | Set of generalized degrees of freedom in generalized dynamic reduction |
| e | Denotes $e^{th}$ element |
| f | Set of free displacements |
| g | Set of all degrees of freedom |
| i | Denotes $i^{th}$ node |
| $\ell$ | Set left over |
| m | Set removed by use of multipoint constraint equations |
| n | Set not removed by multipoint constraints |
| o | Set omitted by static condensation |
| r | Set which removes rigid body motion |
| s | Set specified by single point constraint |
| w | Partition of a-set that has null columns |
| x | Partition of a-set that has non zero matrix coefficients |
| (x, y, z) | Denote component in coordinate sense |

# Table of Contents

## Appendices

# 1

# Structural Analysis Using NASTRAN

## 1.1. Introduction

**NASTRAN** (**NA**sa **STR**uctural **AN**alysis) was conceived and developed by the National Aeronautics and Space Administration (NASA) to fill a need for a universally available finite element program. The program was originally to be machine independent to facilitate its dissemination, but differences in word length, overlay structure, and system input/output routines made this goal impractical and led to the development of **NASTRAN** versions for the three most widely used computers.

**NASTRAN** was initially released into the public domain in 1969 through the Computer Software management and information center (COSMIC). The program originally was available on a purchase basis until the COSMIC release of Level 16.0. This and all subsequent COSMIC releases have been available only on a restricted lease basis.

In addition to the NASA-supported version of **NASTRAN** which is commonly called COSMIC/**NASTRAN** there are several proprietary versions of **NASTRAN**. The most widely known of these propriety versions is called **MSC/NASTRAN** which is developed and maintained by the MacNeal-Schwendler Corporation.

**MSC/NASTRAN** and COSMIC/**NASTRAN** have common origins in Level 15.5 **NASTRAN**. Both versions have been developed by separate organizations and although the programs bear a superficial resemblance they are different programs.

The **MSC/NASTRAN** version is taken to be the standard for **NASTRAN** for several reasons including:

1. Its wide usage
2. Advanced features
3. Responsiveness to user needs

**MSC/NASTRAN** is used throughout the world. These installations include many of the world's largest corporations and most of the large commercial data centers in the United States, Europe and Japan.

**MSC/NASTRAN** has been continually maintained and developed. The program includes a consistant set of isoparametric two and three dimensional elements, rigid elements, superelements for substructural analysis, and improved cyclic symmetry as well as numerical analysis and computer science enhancements.

Although **MSC/NASTRAN** is significantly different from COSMIC/**NASTRAN** in terms of relative efficiency and program capability it still retains downward compatibility to Level 15.5. Thus, the data decks for static and normal modes analyses will generally be the same or similar for both versions. Data decks that have been developed using COSMIC/**NASTRAN** can be executed with few if any changes on **MSC/NASTRAN**. Data decks which have been developed on **MSC/NASTRAN** and use unique **MSC/NASTRAN** capability cannot generally be executed using COSMIC/**NASTRAN**.

**MSC/NASTRAN** is also available from various data centers throughout the United States and the world. **MSC/NASTRAN**, thus, is becoming the de facto standard for structural analysis. Because of **MSC/NASTRAN**'s wide-spread use and ready availability, this book will present the theory of matrix structural analysis and the finite element method in terms of the **MSC/NASTRAN** program.

## 1.2. Overview of MSC/NASTRAN

**MSC/NASTRAN** is a system that will create and manipulate a data base to solve problems using matrix structural analysis. The program acronym, is now out of date since **MSC/NASTRAN** has been used to formulate and solve problems in all fields of continuum mechanics.

The system, shown schematically in Fig. 1-1, is composed of a data base, an executive system, and modules that perform modeling, data base manipulation, and program I/O. The data base can be created directly from the input stream by the I/O modules, or it can be created by modeling modules. The data base is then manipulated by the functional modules (addition, subtraction, equation solving, etc.) to obtain a solution data set that is then selectively displayed by the user. The whole process is controlled by the **MSC/NASTRAN** executive, which is, in turn, under user control by means of the **MSC/NASTRAN** language that is called DMAP (**D**irect **M**atrix **A**bstraction **P**rogramming). The **MSC/NASTRAN** executive is always controlled by a sequence of DMAP statements, but sets of precoded DMAP sequences, called rigid formats, have been included in the program. The user can then incorporate the entire set of DMAP instructions associated with a particular rigid format by a single directive in the **MSC/NASTRAN** data deck.

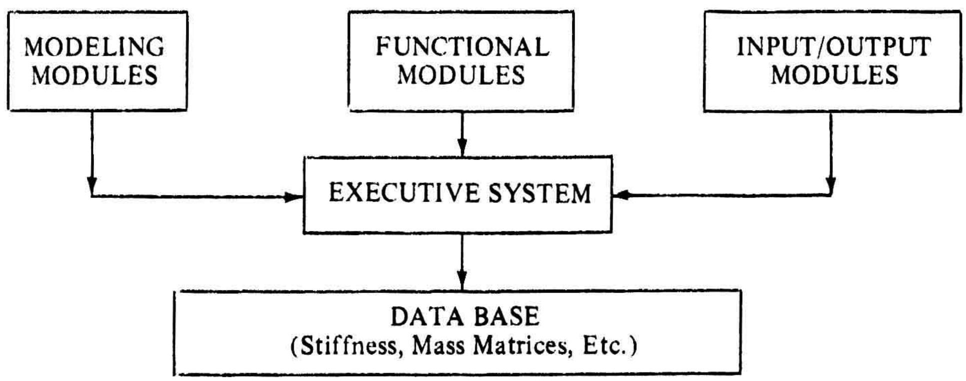


**Figure 1-1. Schematic of NASTRAN Functional Organization**

A specific analysis capability is thus an ordered execution of a set of DMAP instructions. **MSC/NASTRAN** includes rigid formats for performing a large number of analyses types including

- Static Analysis
- Static Analysis with Inertia Relief
- Normal Modes Analysis
- Static Analysis with Differential Stiffness
- Buckling Analysis
- Static Analysis with Nonlinear Elastic Modulus
- Direct Complex Eigenvalue Analysis
- Direct Frequency and Random Response
- Direct Transient Analysis
- Modal Complex Eigenvalue Analysis
- Modal Frequency and Random Response
- Modal Transient Analysis
- Static Analysis with Cyclic Symmetry
- Normal Modes with Cyclic Symmetry

Only those rigid formats that perform static and normal modes analysis will be described in this Primer.

There is a natural aversion to learning a new programming language, and these rigid formats allow one to use certain **MSC/NASTRAN** capability without having to learn the **MSC/NASTRAN** DMAP language. However, the user's ability to utilize DMAP will increase the scope of the problems that can be solved. In addition, the user can bypass unnecessary steps in a rigid format and selectively stop, start, and print intermediate results. Finally, an understanding of DMAP will increase the user's understanding of **MSC/NASTRAN**.

The DMAP language allows the user to specify symbolic names for matrices in much the same sense that any higher level programming language uses a variable name without considering where in the system the data will be stored. The executive, under DMAP control, will write data blocks on a checkpoint tape and corresponding restart instruction, which contains sufficient information to identify the data block on the system punch file. The user is thus provided with a checkpoint/restart capability that is desirable for large problems or when a change of rigid format is required.

There is some rationale for studying **MSC/NASTRAN** without actually implementing it. First, the mnemonics used to describe the various subsets of input data are gaining wide acceptance within the structural mechanics community. It is conceivable that the language will become rather universal, and that independent of one's specialty field, we will understand what is meant by CBAR, CQUAD4, MPC, SPC, etc. Second, the program documentation is rather good and moderately complete. Third, **MSC/NASTRAN** is available in a data center environment on a variety of computers so that the program can be evaluated (and used) without making a sizeable investment. In this book the program is described from the point of view of a general purpose finite element program, i.e., we are interested in the capability represented by the rigid formats for static and normal modes analysis of structures.

## 1.3. NASTRAN Documentation

The **MSC/NASTRAN** documentation is both one of the program's principal assets and one of its principal liabilities. The completeness of the documentation is an asset, while its magnitude is a liability to the new **MSC/NASTRAN** user. The principal **MSC/NASTRAN** documents are described by Table 1-1.

| TITLE | SOURCE |
|---|---|
| MSC/NASTRAN User's Manual | MSC* |
| MSC/NASTRAN Programmer's Manual | MSC |
| MSC/NASTRAN Applications Manual | MSC |
| MSC/NASTRAN Theoretical Manual | MSC |
| MSC/NASTRAN Basic Training Manual | MSC |
| MSC/NASTRAN Demonstration Manual | MSC |
| MSC/NASTRAN MSGMESH Analyst's Guide | MSC |

**Table 1-1. MSC/NASTRAN Source Documentaion**

The theoretical manual describes the sparse-matrix and finite element technology that has been incorporated in the program, the user's manual describes the **MSC/NASTRAN** input data, and the programmer's manual describes the **MSC/NASTRAN** program. In order to use and understand the program it has been necessary in the past for the user to have all three manuals.

A principal motivation for the present text was that of incorporating all the information required to use **MSC/NASTRAN** to solve static and normal modes problems in a single volume. The **MSC/NASTRAN** Primer provides the user with a complete description of the **MSC/NASTRAN** technology for this subset of **MSC/NASTRAN** capability. The material presented in this text is thus adequate for those who intend to utilize the program without making modifications. The ambitious **MSC/NASTRAN** user will find use for all the documentation described by Table 1-1.

*Available from the MacNeal-Schwendler Corporation, 7442 North Figueroa Street, Los Angeles, CA 90041

## 1.4. The MSC/NASTRAN Data Deck

The **MSC/NASTRAN** program is completely controlled by user-specified input data. The functions of the **MSC/NASTRAN** input can be logically described as

1. Defining the physical problems or the system of equations to be solved.
2. Providing user control over **MSC/NASTRAN** I/O
3. Providing user control over the **MSC/NASTRAN** executive functions.

The **MSC/NASTRAN** data deck is thus logically composed of three subdecks each of which performs one of the functions described above. The complete data deck is shown schematically in Fig. 1-2 and consists of

1. The EXECUTIVE CONTROL DECK performs function 3 above and is physically the first subdeck. (Section 1.9)
2. The CASE CONTROL DECK performs function 2 above and is physically the second subdeck. (Section 1.8)
3. The BULK DATA DECK performs function 1 above and is physically the third subdeck. (Section 1.6)

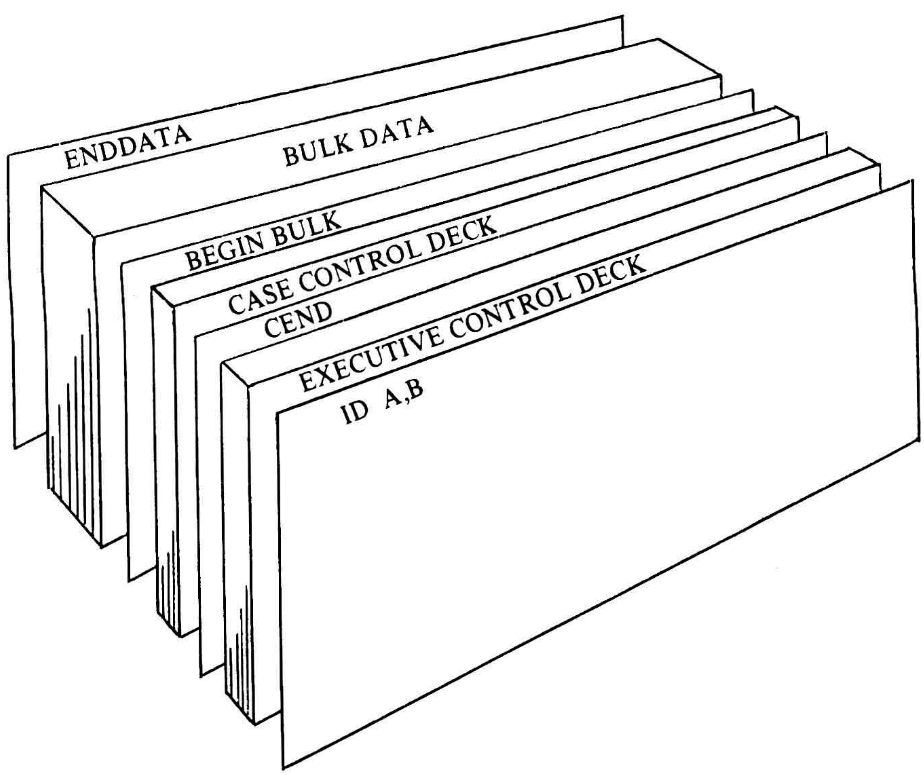


**Figure 1-2. NASTRAN Data Deck**