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KINEMATICS ANALYSIS AND SYNTHESIS

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Professor Emeritus
of Mechanical Engineering
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PREFACE

The design of any new machine or the analysis of an existing machine generally starts with the kinematics that are involved. After the existing or proposed motions are analyzed for velocity, acceleration, and static forces, it is possible for the designer to consider strength, mass, inertia forces, and dynamic balancing. Kinematic analysis does not consider strength of the members but only concerns itself with motions involved. Thus the study of kinematics is a vital element of the machinery design process.

Analysis is done either graphically or analytically depending upon the experience and/or desires of the designer. A designer well experienced in kinematic analysis may opt to do analytic analysis, while the novice will gain greater insight using the graphical procedures. Often it is desirable to construct a simple paper or wood model of the mechanism in order to better visualize the motions or to evaluate space requirements.

The ever-increasing demand for automation in industry has caused more emphasis to be placed on kinematic analysis and design of mechanisms. A natural extension of the planar mechanisms considered in this text is that of robotic systems. Many favor the three-dimensional robotic systems, while two or more planar systems with coordinated motion may perform more dependably. The "pick and place" robotic system is essentially two coordinated planar systems.

The text is arranged in a systematic order starting with simple motion analysis and proceeding through velocity and acceleration considerations. The traditional graphical techniques are explained in detail. Following the graphical techniques the analytic techniques are explained. The presentation rather than the mathematical niceties is stressed. An energetic student should be able to follow the analytic procedures and even bring some original thinking to the process.

With the increasing capabilities of personal computers and their adoption in industry, computer programs written in GW BASIC are included in the appendix. The GW BASIC language was selected since it is a popular programming language found with most personal computers. The programs are written in rather simple format without taking advantage of many step-saving procedures provided by the language. In this manner, the student should have little or no trouble adapting the programs to his or her particular computer. It is expected that the student can shorten the programs by utilizing the string functions and data input procedures that may be available with many BASIC programming units.

Since kinematics and the geometry of motions are very closely interwoven, a chapter is devoted to the geometry of motion. It is expected that an understanding of the geometry involved will provide the student with a much firmer foundation for advanced study of kinematics.

The section on cams and cam design can be viewed as another means of providing programmed motion to a system. The constant velocity, constant acceleration, simple harmonic, and cycloidal cam follower motions are discussed in detail. A computer program is provided to aid in analysis of a system using simple harmonic or cycloidal follower motion. Handling the dynamics problem of cam-driven mechanisms should be a natural consequence of the application of the previous chapters on velocity, acceleration, and acceleration forces.

Geometry and notation involved in spur, helical, bevel, and worm gears is included. The text uses standard notation currently recommended by the American Gear Manufacturers Association (AGMA) and also that recommended by the International Standards Organization (ISO) for metric gears. Examples are provided for analysis or design using the AGMA or the ISO units.

Mathematical abilities commonly found in the fourth semester of college study are assumed. Some knowledge of vectors and vector notation is desirable. An awareness of linear algebra is helpful but not essential. Since the equations developed are all linear, a computer program for solution of simultaneous linear equations is included in the appendix. Although matrix manipulations are not necessary, some may prefer that technique, and a short review of matrix manipulations is included in Appendix B along with a computer program for matrix inversion.

For design of mechanisms to satisfy up to four specified positions, the equations are all linear. For design of four-bar mechanisms to satisfy more than four positions, the least-squares technique results in linear equations. In order to design a generalized mechanism with more than four bars, the equations become nonlinear and the Newton-Raphson technique is recommended. The bibliography includes references which present the theory of the Newton-Raphson technique and sources for computer programs. An example is included along with a BASIC program using the Newton-Raphson method.

Other than the section dealing with synthesis using the instant center for acceleration the author does not claim originality in the topics considered. Many very informative references are provided by subject in the bibliography.

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Jack T. Kimbrell

CONTENTS

CHAPTER 1

MECHANISMS ANALYSIS AND SYNTHESIS

1.1 INTRODUCTION

The design engineer plays a significant role in mechanization and automation of modern industry. The designer is called upon to devise a machine or system of machines which is capable more often than not of accepting and following instructions emanating from a computer, performing those required tasks reliably, and notifying the computer when the task is complete. The designer must be familiar with and consider such factors as stress, strength, flexibility, dynamic balance, material properties, manufacturing influences, safety, costs, noise, codes and standards, efficiency, aesthetics, wear, lubrication, maintenance, deflections, accuracy, and reliability as well as control systems and a host of other influencing elements.

In the face of all that must be considered, it is often more effective to set aside many of the influencing factors and reduce the problem to a more readily solved series of smaller tasks. Other influences are then recalled individually and the problem solution modified accordingly. Unfortunately, the final solution inevitably represents a series of necessary compromises.

The study of mechanisms removes all influencing factors except those concerned with position and time. As such it represents a study of motion and geometry aimed at creating a mechanism which will satisfactorily meet the requirements of the problem at hand. After the mechanism is devised, the concepts of mass and inertia forces are introduced to allow thoughts of dynamic balance, shaking forces, and noise. This procedure has the effect of replacing empiricism and practicality with theory.

Although there is much to be said in favor of empirical and practical practice tempered with experience, it is difficult to imagine effective treatment of new and unsolved problems without strong reliance on theory. It is essential that the theoretical basis be thoroughly understood before the empirical ideas are used to temper the results. This is not to belittle the experience factor but simply to provide a sound basis for decision making. It is the engineering method in practice.

1.2 HISTORICAL BACKGROUND

The study of mechanisms began in antiquity under the pressure of necessity. At the outset there was neither a plan nor unity but rather a random growth resulting in an amazing assemblage of mechanisms. The many wars throughout history resulted in development of such items as "throwing machines" for throwing larger rocks greater distances and progressing through rather complex guidance systems for missiles. If any good at all has resulted from wars, it must include advancement of the study of mechanisms.

Ancient rulers and their whims were effective in promoting the study of mechanisms. A classic example is the Marly machine devised for the sole purpose of creating beautiful fountains in the garden of the palace at Versailles. The Marly machine utilized fourteen 40-foot(ft)-diameter undershot water wheels at the river to develop 1200 horsepower(hp). The water wheels drove 64 pumps at the river, 79 pumps at a 160-ft elevation, and 82 pumps at a 343-ft elevation. Water was delivered to an aqueduct at 533 ft above the river. All the pumps were driven with parallelograph linkages which required 64,000 ft of iron bars.

Commercial necessity also prompted mechanical innovation. The need to move merchandise resulted in some fascinating mechanisms such as the paddle wheel of Buchannan (Fig. 1.1) arranged to keep the paddles perpendicular to the water stream and Watts' steam engine (Fig. 1.2).

Perhaps the earliest writings on the subject were those of Pollio Marcus Vitruvius, a military engineer who wrote *De Architectura* in about 28 B.C. Vitruvius was principally concerned with methods for moving heavy objects. It was Hero of Alexandria in about the first century A.D., essentially a mathematician but enamored with applications, who named the components from which all complex mechanisms of that day were

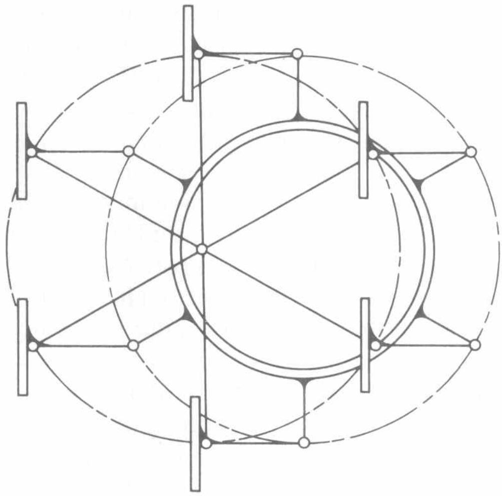

FIGURE 1.1
Buchannan's paddle wheel.

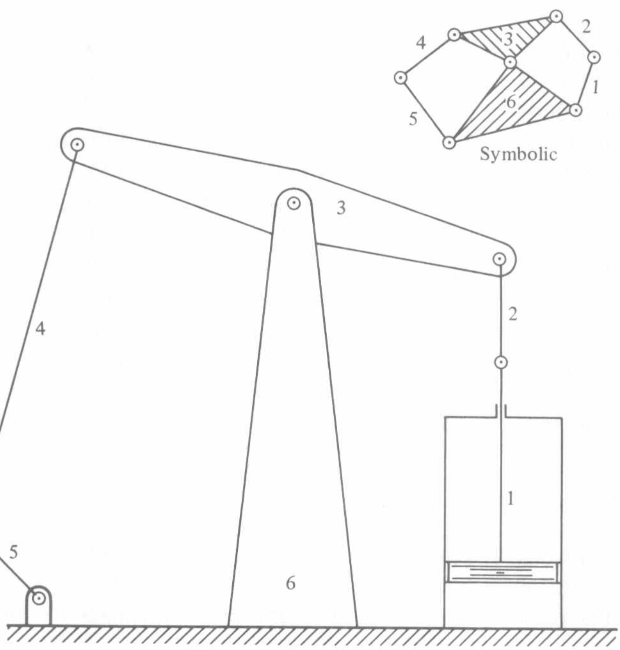

FIGURE 1.2
Watts' steam engine.

assembled. He recognized the wedge, lever, screw, windlass, and pulley. In these early times machines were considered as whole objects, not as groups of parts known today as mechanisms. In 1588, Ramelli in *Arteficiose Machine* describes each machine separately and completely without any recognition of similarity of the machine components. Jacob Leupold appears to have been the first to recognize mechanisms as components of machines and to suggest the idea of modifying motion. His nine-volume series of books published between 1724 and 1739 was directed toward craftspeople. The books contained almost 500 copper engravings depicting the then current practice in machines. The illustrations were so complete that any competent craftsperson could reproduce the machines and mechanisms.

It was the work of André Marie Ampère (1775–1836) that stimulated the science of kinematics as a separate study. During his critical examination of mechanisms at Ecole Polytechnique, Ampère published his *Essai sur la Philosophie de Science*. He gave the name kinematics (*Cinématique*) to the study and encouraged separate treatment of the subject. Following the lead of Ampère, the science of kinematics was taken up as a separate study and continues today supported by a wealth of literature.

Prior to about 1940, significant literature on the subject of kinematics originated in Europe and Australia. The literature followed either a discussion of graphical techniques (applied geometry) or highly analytic methods. The bulk of the writings was devoted to plane mechanisms. In 1948 the Mechanisms Conference organized at Purdue University sparked an interest in the subject in the United States, and significant literature has resulted. With the advent of computers, the literature expanded from analysis of plane mechanisms to analysis and synthesis of plane and spatial mechanisms. The applications to robots are obvious, and computer-controlled automated systems using plane and spatial mechanisms are a very natural development. Unfortunately, very little has been accomplished in recent years in extending or developing new theories of kinematics. Major efforts in the United States and abroad have been in application of existing theories. It is doubtful though that all theories possible have been developed.

1.3 SCHEMATIC NOTATION

In considering time-dependent motion of machine parts, it is unnecessary, and in fact complicating, to be concerned with the actual shape and mass of the parts. For this reason, several simplifying conventions have been adopted.

A mechanism consists of several component parts which move relative to one another. Each component part is called a *link*. When two