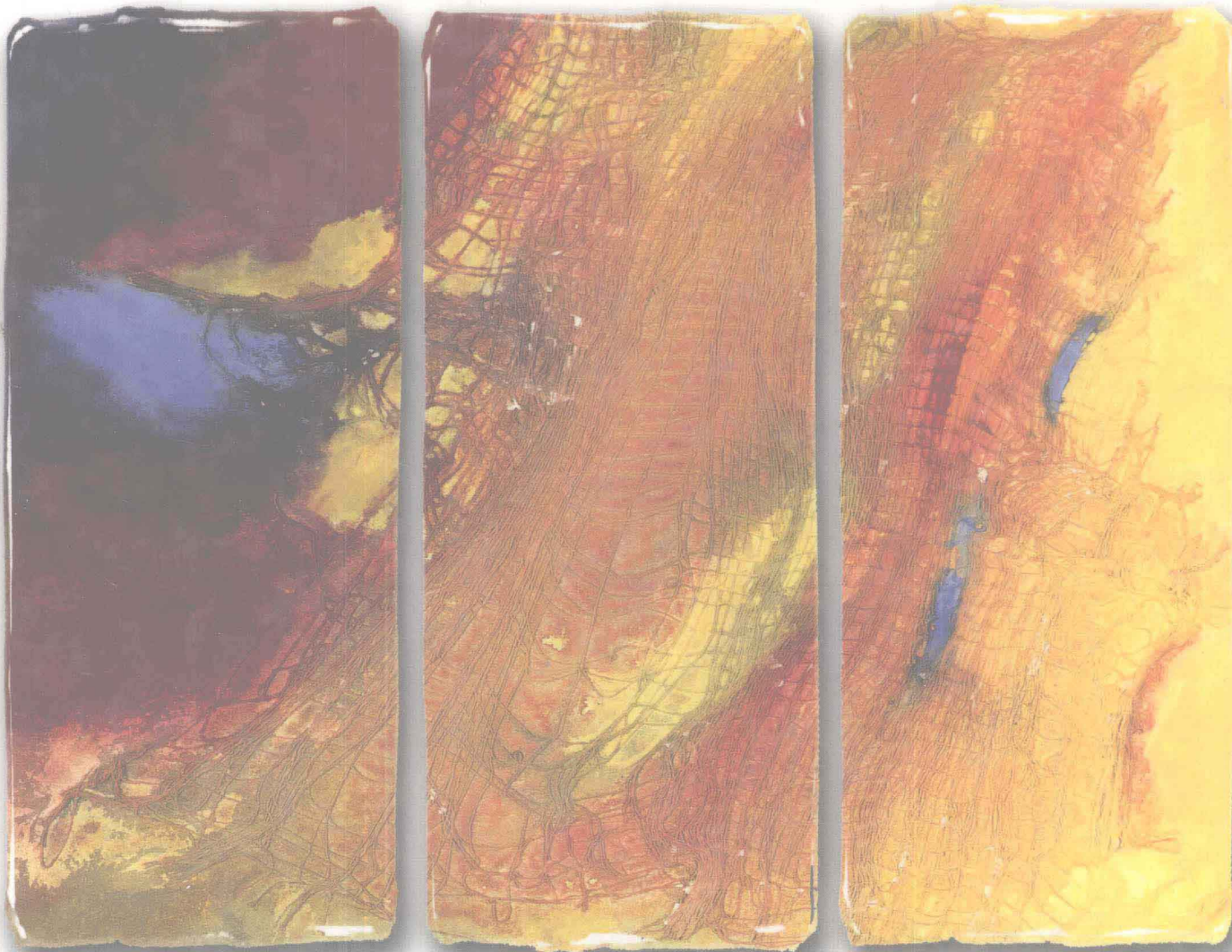


# TAN Applied Calculus

FOR THE MANAGERIAL, LIFE, AND SOCIAL SCIENCES



F I F T H   E D I T I O N

# **Applied Calculus**

for the Managerial, Life, and Social Sciences

FIFTH EDITION

**S. T. Tan**  
Stonehill College

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# P R E F A C E

*Applied Calculus for the Managerial, Life, and Social Sciences, Fifth Edition*, is suitable for use in a two-semester or three-quarter introductory calculus course for students in the managerial, life, and social sciences. As with the previous editions, our objective in *Applied Calculus for the Managerial, Life, and Social Sciences* is twofold: (1) to write a textbook that is readable by students and (2) to make the book a useful teaching tool for instructors. We hope that with the present edition we have come one step closer to realizing our goal. The fifth edition of this text incorporates many suggestions by users of the earlier editions.

## FEATURES

The following list includes some of the many important features of the book:

■ **Coverage of Topics** The book contains more than enough material for the usual applied calculus course. Optional sections have been marked with an asterisk in the table of contents, thereby allowing the instructor to be flexible in choosing the topics most suitable for his or her course.

■ **Approach** The problem-solving approach is stressed throughout the book. Numerous examples and solved problems are used to amplify each new concept or result in order to facilitate students' comprehension of the material. Figures are used extensively to help students visualize concepts and ideas.

■ **Level of Presentation** Our approach is intuitive, and we state the results informally. However, we have taken special care to ensure that this approach does not compromise the mathematical content and accuracy. Proofs of certain results are given, but they may be omitted if desired.

■ **Applications** The text is application oriented. Many interesting, relevant, and up-to-date applications are drawn from the fields of business, economics, social and behavioral sciences, life sciences, physical sciences, and other fields of general interest. Some of these applications have their source in newspapers, weekly periodicals, and other magazines. Applications are found in the illustrative examples in the main body of the text as well as in the exercise sets. In fact, one goal of the text is to include at least one real-life application in each section (whenever feasible).

■ **Sources** We have included sources for those applications that are based on real-life data.

■ **Exercises** Each section of the text is accompanied by an extensive set of exercises containing an ample set of problems of a routine, computational nature that will help students master new techniques. The routine problems are followed by an extensive set of application-oriented problems that test students' mastery of the topics.

■ **Self-Check Exercises** Every section has self-check exercises, with solutions, to help students monitor their own progress.

■ **Portfolios** These interviews are designed to convey to the student the real-world experiences of professionals who have a background in mathematics and use it in their professions.



■ **Group Discussion Questions** These are optional questions, appearing throughout the main body of the text, that can be discussed in class or assigned as homework. These questions generally require more thought and effort than the usual exercises. Complete solutions to these exercises are given in the *Complete Solutions Manual*.

## TECHNOLOGY



### Exploring with Technology Questions

These optional questions appear throughout the main body of the text and serve to enhance the student's understanding of the concepts and theory presented. Complete solutions to these exercises are given in the *Complete Solutions Manual*.

### Using Technology Subsections



These pages contain optional material and are placed at the end of the sections for which their use is appropriate. The subsections are written in the traditional example–exercise format, with answers given at the back of the book. They may be used in the classroom if desired or as material for self-study by the student.

As many up-to-date and relevant applications have been introduced in these subsections, they provide students with an opportunity to interpret results in a real-life setting.

### Student Resources on the Web



Students and instructors will now have access to these additional materials at the Brooks/Cole World Wide Web site:

<http://www.brookscole.com/product/0534378439>

- Review material and practice chapter quizzes and tests
- Group projects and extended problems for each chapter
- Instructions, including keystrokes, for the procedures referenced in the text for specific calculators (TI-82, TI-83, TI-85, TI-86, and other popular models)
- Modified *Using Technology* sections for CAS systems, including the command statements for Mathematica, Maple, and other popular system

## NEW IN THE FIFTH EDITION

- Exercises that emphasize the understanding of concepts and theory have been added to most sections. These new exercises are usually near the end

of the exercise sets and take the form of true or false questions. More questions asking for the interpretation of graphs have also been added.

- More real-life applications, with sources, have been added; for example, the narrowing gender gap, net-connected computers in Europe, the growth of the Indian gaming industry, accumulation years for baby boomers, the growth of online shopping, online banking usage, the document management business, portable phone usage, growth of international e-mail, the growth of the prison population, marijuana arrests, blood alcohol level after drinking, senior citizen's health care costs, and the spread of HIV. Examples of other new applications are a coast guard patrol search mission, the yield of an apple orchard, blowing soap bubbles, absorption of drugs, loan amortization, and the transmission of disease.
- The section on mathematical modeling has been expanded and strengthened. It now includes a new subsection on constructing mathematical models. A Using Technology section, *Constructing Mathematical Models from Raw Data*, has also been added.
- A new section, Series with Positive Terms, has been added to Chapter 11. The test for divergence, the integral test, the convergence of  $p$ -series, and comparison tests are discussed in this section.
- More Group Discussion questions and Exploring with Technology questions have been added.
- More Using Technology sections and exercises have been added.
- Coverage of the equation of a circle has been added to Chapter 1.
- Instructions for the TI-83 and TI-86 calculators are on the Web site.

## SUPPLEMENTS

- *Student's Solutions Manual*, available at extra cost to students, includes the solutions to odd-numbered exercises. ISBN 0-534-38788-8
- *Instructor's Complete Solutions Manual*, available only to instructors, includes solutions to all exercises. ISBN 0-534-38787-X.
- *BCA Testing* is a browser-based test and quiz generator with the capacity to post quizzes on the Web with automatic grading. ISBN 0-534-38793-4. A printed copy of the test items is available by request through your Brooks Cole/Thomson Learning sales representative.
- *Graphing Calculator Supplement*, by Ryan & Hester, both of Texas A&M University, is available to both students and instructors. The manual develops selected examples and exercises and also includes additional problems for reinforcement. It is specifically written for use with the TI line of programmable graphics calculators. ISBN 0-534-37403-4
- *Applied Calculus with Microsoft Excel*, by Chester Piascik, Bryant College, illustrates key topics in applied calculus through the use of Microsoft Excel. Explanations of Excel instructions and formulas reinforce underlying mathematical concepts. The author encourages students to be active learners, asking them to verbalize and verify the mathematical concepts behind the spreadsheet results. ISBN 0-534-37058-6

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S. T. Tan  
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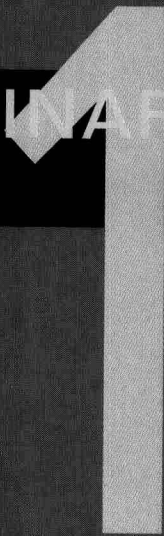
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**Applied Calculus**  
for the Managerial, Life, and Social Sciences  
FIFTH EDITION

# PRELIMINARIES



- 1.1** Precalculus Review I
- 1.2** Precalculus Review II
- 1.3** The Cartesian Coordinate System
- 1.4** Straight Lines

The first two sections of this chapter contain a brief review of algebra. We then introduce the Cartesian coordinate system, which allows us to represent points in the plane in terms of ordered pairs of real numbers. This in turn enables us to compute the distance between two points algebraically. This chapter also covers straight lines. The slope of a straight line plays an important role in the study of calculus.

What sales figure can be predicted for next year? In Example 10, page 46, you will see how the manager of a local sporting goods store used sales figures from the previous years to predict the sales level for next year.



## 1.1 Precalculus Review I

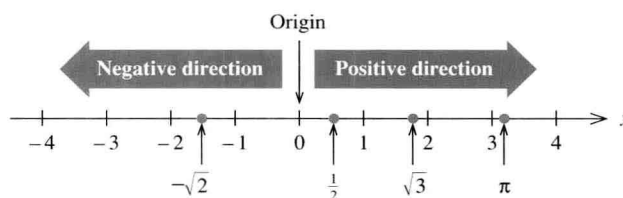
Sections 1.1 and 1.2 review some of the basic concepts and techniques of algebra that are essential in the study of calculus. The material in this review will help you work through the examples and exercises in this book. You can read through this material now and do the exercises in areas where you feel a little “rusty,” or you can review the material on an as-needed basis as you study the text. We begin our review with a discussion of real numbers.

### THE REAL NUMBER LINE

The real number system is made up of the set of real numbers together with the usual operations of addition, subtraction, multiplication, and division.

Real numbers may be represented geometrically by points on a line. Such a line is called the **real number**, or **coordinate**, **line** and can be constructed as follows. Arbitrarily select a point on a straight line to represent the number zero. This point is called the **origin**. If the line is horizontal, then a point at a convenient distance to the right of the origin is chosen to represent the number 1. This determines the scale for the number line. Each positive real number lies at an appropriate distance to the right of the origin, and each negative real number lies at an appropriate distance to the left of the origin (Figure 1.1).

**FIGURE 1.1**  
The real number line



A *one-to-one correspondence* is set up between the set of all real numbers and the set of points on the number line; that is, exactly one point on the line is associated with each real number. Conversely, exactly one real number is associated with each point on the line. The real number that is associated with a point on the real number line is called the **coordinate** of that point.

### INTERVALS

Throughout this book, we will often restrict our attention to certain subsets of the set of real numbers. For example, if  $x$  denotes the number of cars rolling off a plant assembly line each day, then  $x$  must be nonnegative—that is,  $x \geq 0$ . Further, suppose management decides that the daily production must not exceed 200 cars. Then,  $x$  must satisfy the inequality  $0 \leq x \leq 200$ .

More generally, we will be interested in the following subsets of real numbers: open intervals, closed intervals, and half-open intervals. The set of

all real numbers that lie *strictly* between two fixed numbers  $a$  and  $b$  is called an **open interval**  $(a, b)$ . It consists of all real numbers  $x$  that satisfy the inequalities  $a < x < b$ , and it is called “open” because neither of its end points is included in the interval. A **closed interval** contains *both* of its end points. Thus, the set of all real numbers  $x$  that satisfy the inequalities  $a \leq x \leq b$  is the closed interval  $[a, b]$ . Notice that square brackets are used to indicate that the end points are included in this interval. **Half-open intervals** contain only *one* of their end points. Thus, the interval  $[a, b)$  is the set of all real numbers  $x$  that satisfy  $a \leq x < b$ , whereas the interval  $(a, b]$  is described by the inequalities  $a < x \leq b$ . Examples of these **finite intervals** are illustrated in Table 1.1.

Table 1.1 Finite Intervals		
Interval	Graph	Example
Open $(a, b)$		$(-2, 1)$
Closed $[a, b]$		$[-1, 2]$
Half-open $(a, b]$		$(\frac{1}{2}, 3]$
Half-open $[a, b)$		$[-\frac{1}{2}, 3)$

In addition to finite intervals, we will encounter **infinite intervals**. Examples of infinite intervals are the half lines  $(a, \infty)$ ,  $[a, \infty)$ ,  $(-\infty, a)$ , and  $(-\infty, a]$  defined by the set of all real numbers that satisfy  $x > a$ ,  $x \geq a$ ,  $x < a$ , and  $x \leq a$ , respectively. The symbol  $\infty$ , called *infinity*, is not a real number. It is used here only for notational purposes in conjunction with the definition of infinite intervals. The notation  $(-\infty, \infty)$  is used for the set of all real numbers  $x$  since, by definition, the inequalities  $-\infty < x < \infty$  hold for any real number  $x$ . Infinite intervals are illustrated in Table 1.2.

Table 1.2 Infinite Intervals		
Interval	Graph	Example
$(a, \infty)$		$(2, \infty)$
$[a, \infty)$		$[-1, \infty)$
$(-\infty, a)$		$(-\infty, 1)$
$(-\infty, a]$		$(-\infty, -\frac{1}{2}]$



## PROPERTIES OF INEQUALITIES

In practical applications, intervals are often found by solving one or more inequalities involving a variable. In such situations, the following properties may be used to advantage.

### Properties of Inequalities

If  $a$ ,  $b$ , and  $c$  are any real numbers, then

		Example
<b>Property 1</b>	If $a < b$ and $b < c$ , then $a < c$ .	$2 < 3$ and $3 < 8$ , so $2 < 8$
<b>Property 2</b>	If $a < b$ , then $a + c < b + c$ .	$-5 < -3$ , so $-5 + 2 < -3 + 2$ ; that is, $-3 < -1$
<b>Property 3</b>	If $a < b$ and $c > 0$ , then $ac < bc$ .	$-5 < -3$ , and since $2 > 0$ , we have $(-5)(2) < (-3)(2)$ ; that is, $-10 < -6$
<b>Property 4</b>	If $a < b$ and $c < 0$ , then $ac > bc$ .	$-2 < 4$ , and since $-3 < 0$ , we have $(-2)(-3) > (4)(-3)$ ; that is, $6 > -12$

Similar properties hold if each inequality sign,  $<$ , between  $a$  and  $b$  is replaced by  $\geq$ ,  $>$ , or  $\leq$ .

A real number is a *solution of an inequality* involving a variable if a true statement is obtained when the variable is replaced by that number. The set of all real numbers satisfying the inequality is called the *solution set*.

#### EXAMPLE 1

Find the set of real numbers that satisfy  $-1 \leq 2x - 5 < 7$ .

#### SOLUTION ✓

Add 5 to each member of the given double inequality, obtaining

$$4 \leq 2x < 12$$

Next, multiply each member of the resulting double inequality by  $1/2$ , yielding

$$2 \leq x < 6$$

Thus, the solution is the set of all values of  $x$  lying in the interval  $[2, 6)$ .

