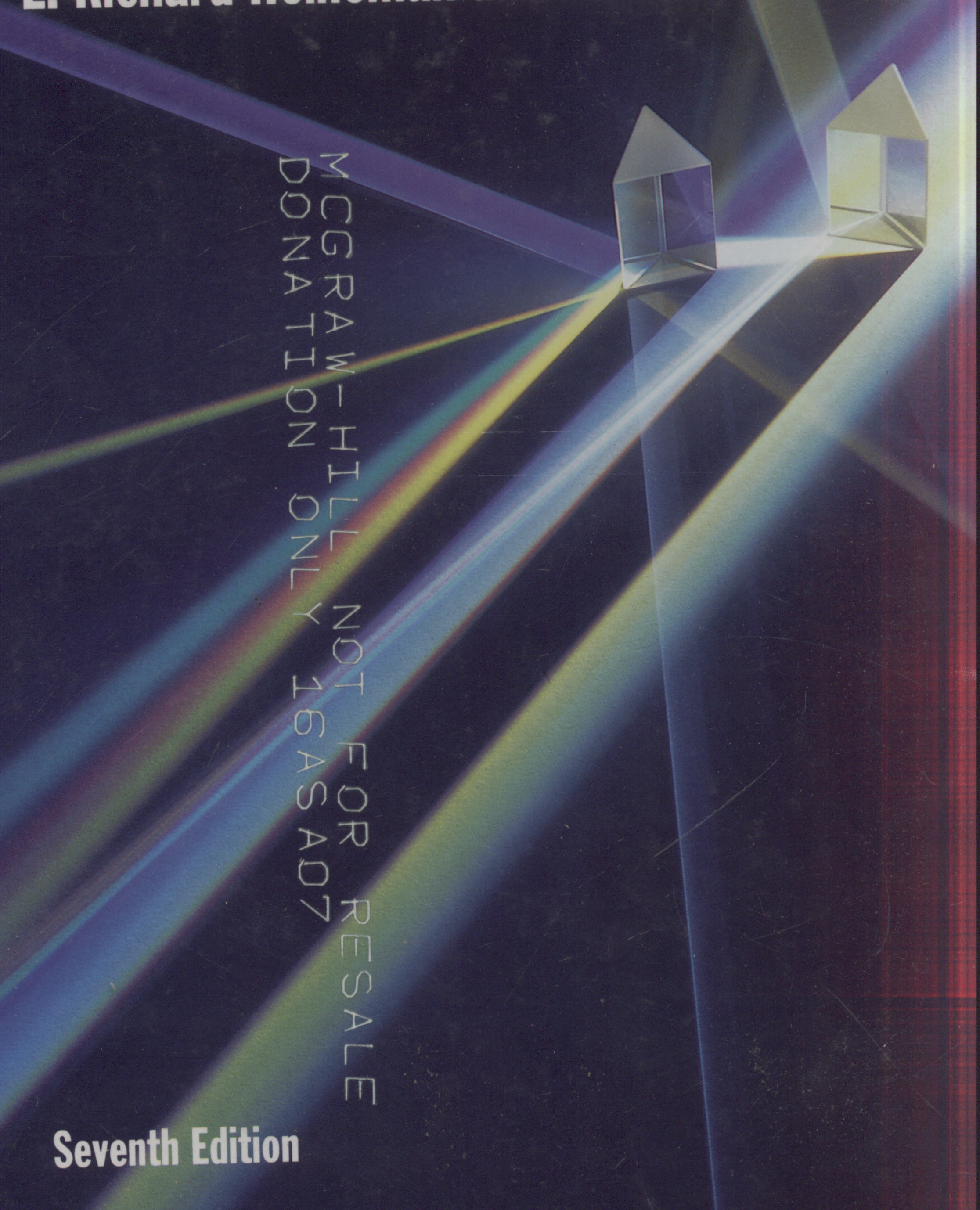
PLANE TRIGONOMETRY

E. Richard Heineman and J. Dalton Tarwater



Seventh Edition

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SEVENTH EDITION

E. Richard Heineman
Late Professor of Mathematics
Texas Tech University

J. Dalton Tarwater
Professor of Mathematics
Texas Tech University

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Dedication

E. Richard Heineman was born and raised in Wisconsin, where he taught high school geometry and chemistry. After earning his degrees from the University of Wisconsin, Madison, he taught at Michigan State University, before moving on to Texas Tech University. After a distinguished career at Texas Tech spanning 45 years, and almost two decades of retirement, he died in 1991.

Since its first edition, Professor Heineman's *Plane Trigonometry* has helped well over a million students learn trigonometry. It remains the standard for all other trigonometry texts.

This book is dedicated to his memory.

PREFACE

The seventh edition of *Plane Trigonometry* is a significant revision of this classic text. Several chapters have been left basically intact, a few have been merged or revised slightly, while others have been completely rewritten. Two new chapters have been added. Every effort has been made to retain the brief expository style of the previous edition while updating the content to meet current standards.

Major changes include:

1. The introduction of radian measure in Chapter 4 and its extensive use afterward
2. An optional Section 8.6 on graphing calculators
3. A new Chapter 11 on polar coordinates and a new Chapter 13 on topics from analytic geometry
4. The assumption that the student uses a scientific calculator; however, the Appendix has a section on the use of tables for students without a calculator
5. The addition of a list of Key Terms and a set of Review Exercises at the end of each chapter
6. More emphasis on graphing throughout the text and the addition of graphs in the Answers section

Major features retained from the previous editions include:

1. Miscellaneous points include (*a*) a note to the student, (*b*) problems that are encountered in calculus, (*c*) a careful explanation of the concept of infinity, (*d*) memory schemes, (*e*) the uses of the sine and cosine curves, and (*f*) interesting applied problems.
2. The problems in each exercise are so arranged that by assigning numbers 1, 5, 9, etc., or similar sets beginning with 2, 3, or 4, the instructor can obtain balanced coverage of all points involved without undue emphasis on some principles at the expense of others.
3. Answers to three-fourths of the problems are given at the back of the book.

4. Many of the exercises contain true-false questions to test the student's ability to avoid pitfalls and to detect camouflaged truths. The duty of the instructor is not only to teach correct methods but also to convince the student of the error in the false methods.
5. All problem sets are carefully graded and contain an abundance of simple problems that involve nothing more than the principles being discussed. There is also an ample supply of problems of medium difficulty and some "head-scratchers."

A number of *supplements* have been provided for both the student and instructor. They include:

The Instructor's Resource Manual, prepared by Joan Van Glabek of Edison Community College, contains teaching suggestions, chapter exams and final exams, and solutions to every fourth problem from the text.

The Student's Solutions Manual, also prepared by Joan Van Glabek, contains the solutions to three-fourths of the problems from the text.

The Videotape Series, prepared by John Jobe of Oklahoma State University, consists of 11 tapes that present essential trigonometry topics for review.

The Professor's Assistant is a computerized test generator that allows the instructor to create tests using algorithmically generated test questions and those from a standard testbank. This testing system enables the instructor to choose questions either manually or randomly by section, question type, difficulty level, and other criteria. This system is available for IBM, IBM compatible, and Macintosh computers.

The Print Test Bank is a printed and bound copy of the questions found in the standard test bank.

For further information about these supplements, please contact your local College Division sales representative.

For over half of a century many people have contributed to the success of this text. We are grateful to the many students and instructors who have made suggestions and have offered criticisms. For this edition we are particularly indebted to Patrick Tarwater of Texas Tech University for the graphs in the Answers Section, and to Beth Martin, also of Texas Tech, for her excellent typing of the manuscript. Several reviewers have made substantial improvements in the presentation. Thanks to Sandra Beken, Horry-Georgetown Technical College; Steven Blasberg, West Valley College; Forrest G. Lowe, Longview Community College; Glenn T. Smith II, Santa Fe Community College; and Carol M. Walker, Hinds Community College.

Special thanks to Karen M. Minette and Michael Johnson of McGraw-Hill for their undying patience.

Finally, we acknowledge that the quotation attributed to Thomas Jefferson is taken from E. R. Hogan, The Beginnings of Mathematics in a Howling Wilderness, *Historia Mathematica,* v. 1, 1974, p. 156. Also, some exercises, examples, and formulas in Chapters 11 and 13 were borrowed from G. Fuller/D. Tarwater, *Analytic Geometry,* 7th ed., © 1992, Addison-Wesley Publishing Company, Inc. They are reprinted with permission of the publisher.

J. Dalton Tarwater

NOTE TO THE STUDENT

A mastery of the subject of trigonometry requires (1) a certain amount of memory work, (2) a great deal of practice and drill in order to acquire experience and skill in the application of the memory work, and (3) an insight and understanding of "what it is all about." Your instructor is a "troubleshooter" who attempts to prevent you from going astray, supplies missing links in your mathematical background, and tries to indicate the "common sense" approach to the problem.

The memory work in any course is one thing that students can and should perform by themselves. The least you can do for your instructor and yourself is to *commit to memory each definition and theorem as soon as you contact it.* This can be accomplished most rapidly not by reading, but by writing the definition or theorem until you can reproduce it without the aid of the text.

In working the problems, do not continually refer back to the illustrative examples. Study the examples so thoroughly (by writing them) that you can reproduce them with your text closed. Only after the examples are entirely clear and have been completely mastered should you attempt the unsolved problems. These problems should be worked *without referring to the text.*

Bear in mind, too, that *memory* and technical *skill* are aided by *understanding;* therefore, as the course develops you should review the definitions and theorems from time to time, always seeking a deeper insight into them.

Plane Trigonometry

CONTENTS

1

The Trigonometric Functions

1.1 HISTORICAL OVERVIEW

Mathematics, like music and poetry, is a creation of the human mind. Just as we can't remember who wrote the first song or the first verse, the origins of mathematics are lost in antiquity. This is especially true of the branch of mathematics known as trigonometry, which means "triangle measurement." We have evidence of some aspects of trigonometry in Babylonia as early as 1600 B.C. Later, around 200 B.C., the Greek astronomers of Alexandria were well aware of the uses of trigonometry.

Since then, there have been many theoretical advances in trigonometry. There have been countless applications of trigonometry to astronomy, cartography, electronics, engineering, music theory, navigation, physics, surveying, and other endeavors. Indeed, in the early days of America, trigonometry was learned for its central role in surveying, navigation on the high seas, and astronomy. It was essential for a young country which was trying to explore, settle, and defend a continent. Thomas Jefferson, in 1799, wrote that

> trigonometry . . . is most valuable to every man. There is scarcely a day in which he will not resort to it for some of the purposes of common life . . .

For two centuries Jefferson's view of the utility of trigonometry has proved sound, if not in ways he meant or even in ways he could have imagined.

In the last half century, since this text was first published, the appearance of computers, hand calculators with trigonometric functions, and graphing calculators has removed the tedium of many trigonometric calculations, increased the accuracy of such calculations, and provided visual reiteration of the principles of the subject. While we embrace these technological advances, we caution the student that these devices do not lessen your need to know, or even to commit to memory, basic principles, definitions, and theorems of trigonometry.

We begin our study of trigonometry with a brief review of fundamental ideas of coordinate geometry.

1.2 THE RECTANGULAR COORDINATE SYSTEM

A directed line is a line on which one direction is considered positive and the other negative. Thus, in Figure 1.1, the arrowhead indicates that all segments measured from left to right are positive. Hence if $OA = 1$ unit of length, then $OB = 3$ and $BC = -5$. Observe that since the line is directed, CB is not equal to BC. However, $BC = -CB$; or $CB = -BC$. If A, B, and C are any three points on the line, then $AB + BC = AC$ and $AB + BC + CA = 0$.

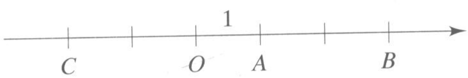

Figure 1.1

A rectangular (or cartesian) coordinate system consists of two perpendicular *directed* lines. It is conventional to draw and direct these lines as in Figure 1.2. The ***x* axis** and the ***y* axis** are called the **coordinate axes;** their intersection O is called the **origin.** The position of any point in the plane is fixed by its distances from the axes.

The *x* coordinate of point *P* is the length of the directed segment *NP* (or *OM*) measured from the *y* axis to point *P*. The *y* coordinate of point *P* is the length of the directed segment *MP*, measured from the *x* axis to point *P*.

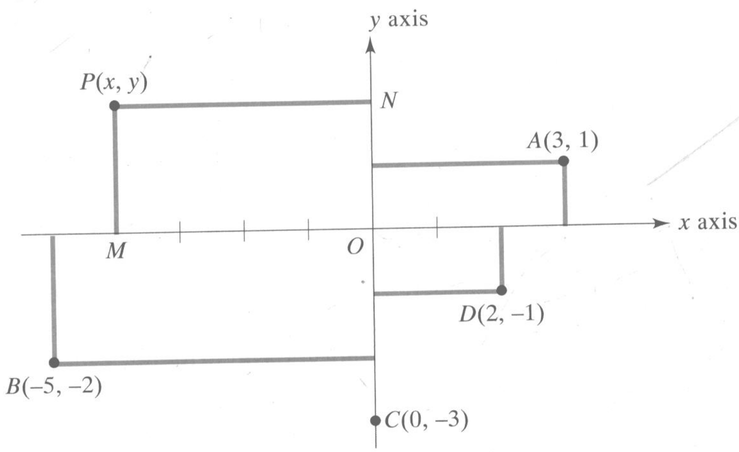

Figure 1.2

It is necessary to remember that each coordinate is measured *from axis to point*. Thus the x coordinate of P is NP (not PN); the y coordinate of P is MP (not PM). The point P, with x coordinate x and y coordinate y, is denoted by $P(x, y)$. It follows that the x coordinate of any point to the right of the y axis is positive; to the left it is negative. Also the y coordinate of any point above the x axis is positive; below, it is negative.

To **plot** a point means to locate and indicate its position on a coordinate system. Several points are plotted in Figure 1.2.

The distance r from the origin O to point P is called the **radius vector** of P. This distance r is not directed and *is always positive* by agreement. Hence with each point of the plane we can associate three numbers: x, y, and r. The radius vector r can be found by using the Pythagorean relation $x^2 + y^2 = r^2$ (see Figure 1.3).

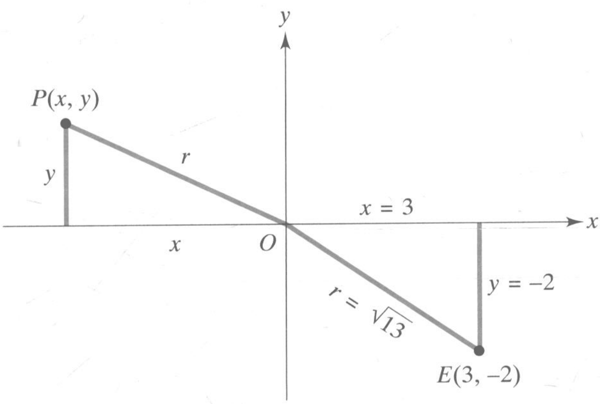

Figure 1.3

The coordinate axes divide the plane into four parts called **quadrants** as indicated in Figure 1.4. We shall sometimes denote these as Q I, Q II, Q III, and Q IV, respectively. We have abbreviated "$x < 0$" by "$x = -$" as an aid to memory.

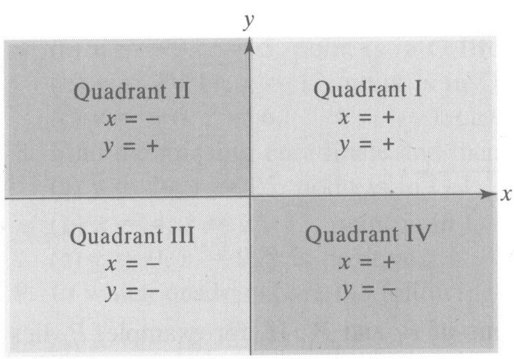

Figure 1.4