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GEORGE J. HADEMENOS

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SCHAUM'S OUTLINE OF

THEORY AND PROBLEMS

OF

PHYSICS FOR

PRE-MED, BIOLOGY, AND

ALLIED HEALTH STUDENTS

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SCHAUM'S OUTLINE SERIES

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Schaum's Outline of Theory and Problems of
PHYSICS FOR PRE-MED, BIOLOGY, AND ALLIED HEALTH STUDENTS

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5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 VFM VFM 0 9 8 7 6

ISBN 0-07-025474-5

Sponsoring Editor: Barbara Gilson
Production Supervisor: Sherri Souffrance
Editing Supervisor: Maureen B. Walker

Library of Congress Cataloging-in-Publication Data

Hademenos, George J.

Schaum's outline of physics for pre-med, biology, and allied health students / George J. Hademenos.

p. cm.—(Schaum's outline series)

Includes index.

ISBN 0-07-025474-5

1. Biophysics—Outlines, syllabi, etc. 2. Physics—Outlines, syllabi, etc. I. Title II. series.

QH505.H23 1997

612'.014—dc21

97-31139
CIP

McGraw-Hill

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*This book is dedicated to:
Kelly, Alexandra, and Dr. George Kattawar,
my three greatest motivators*

Preface

Schaum's Outline of Physics for Pre-Med, Biology, and Allied Health Students is a text which complements the standard physics textbook in a fundamental physics course. The book is structured in a format similar to that of most physics textbooks used for such courses, except that the presented topics—including kinematics, statics, dynamics, thermal physics, electricity and magnetism, optics, and radioactivity—are supplemented with illustrative examples from the biological and medical sciences. The primary goal of this book is to provide the physics student with unique examples and applications to the life sciences as well as to simplify and enlighten the biology student in these sometimes difficult physical concepts and related principles.

Although a tremendous amount of painstaking effort was devoted to this project, there will, most probably, be errors in this publication that were overlooked throughout the editorial process and I assume full responsibility for these errors. I would greatly appreciate if these errors could be brought to my attention either by direct correspondence with me or with the editorial staff at McGraw-Hill.

GEORGE J. HADEMENOS

Contents

Chapter 1	MATHEMATICS FUNDAMENTALS	1
1.1	Functional Analysis	1
	Illustrative Example 1.1. Poiseuille's Law and Blood Flow	1
1.1.1	Slope and y Intercept	2
	Illustrative Example 1.2. Poiseuille's Law and Blood Flow Revisited	3
1.1.2	Coordinate Systems	3
1.2	Types of Functions	4
1.2.1	Polynomial Functions	4
	Illustrative Example 1.3. Applications of Polynomial Functions	5
1.2.2	Power Functions	7
	Illustrative Example 1.4. Applications of Power Functions	8
1.2.3	Trigonometric Functions	8
	Illustrative Example 1.5. Applications of Trigonometric Functions	10
1.2.4	Exponential Functions	11
	Illustrative Example 1.6. Applications of Exponential Functions	11
1.2.5	Logarithmic Functions	12
	Illustrative Example 1.7. Applications of Logarithmic Functions	13
1.3	Differential Calculus	14
	Illustrative Example 1.8. Particle Motion	15
1.4	Integral Calculus	16
	Illustrative Example 1.9. Particle Motion Revisited	16
1.5	Ordinary Differential Equations	17
	Illustrative Example 1.10. Growth and Decay Biological Processes	18
	Illustrative Example 1.11. Harmonic Oscillator	18
	Supplementary Problems	19
Chapter 2	UNITS AND DIMENSIONS	25
2.1	Significant Figures	25
2.2	Scientific Notation	26
2.3	Units and Dimensional Analysis	27
2.4	Conversion Factors	28
2.5	Systems of Units	30
2.6	Problem-Solving Techniques	30
	Supplementary Problems	31
Chapter 3	VECTORS	33
3.1	Definitions of Vectors	33
3.2	Resolution of Vectors	34
	Illustrative Example 3.1. Vector Representation of DNA	37
3.3	Addition of Vectors	38
3.3.1	Graphical Method of Vector Addition	38
3.3.1.1	Parallelogram Method of Vector Addition	38
3.3.1.2	Polygon Method of Vector Addition	38
3.3.2	Analytic Method of Vector Addition	41

3.4	Subtraction of Vectors	41
3.5	Multiplication of Vectors	43
3.5.1	Multiplication of a Vector by a Scalar	43
3.5.2	Dot Product of Vector Multiplication	43
3.5.3	Cross Product of Vector Multiplication	44
3.6	Physical and Biological Applications of Vectors	45
	Illustrative Example 3.2. Particle Motion and Dynamics	45
	Illustrative Example 3.3. Skeletal Mechanics	46
	Illustrative Example 3.4. Vessel Bifurcation	47
	Illustrative Example 3.5. Velocity Profile of Blood Flow	50
	Supplementary Problems	51
Chapter 4	PLANAR MOTION	56
4.1	Definitions of Planar Motion	56
	Illustrative Example 4.1. X-ray Angiography and Blood Flow Measurement	57
4.2	Types of Planar Motion	58
4.2.1	Linear Motion	58
4.2.2	Projectile Motion	64
	Illustrative Example 4.2. Criminal Investigation of a Fall	65
	Supplementary Problems	69
Chapter 5	STATICS	73
5.1	Definitions of Statics	73
5.2	Newton's Laws of Motion	74
5.2.1	Newton's First Law: All Mass Contains Inertia	74
	Illustrative Example 5.1. Blood Flow	74
5.2.2	Newton's Second Law: $\Sigma \mathbf{F} = m\mathbf{a}$	74
	Illustrative Example 5.2. Blood Flow Revisited	74
5.2.3	Newton's Third Law: Law of Action and Reaction	74
	Illustrative Example 5.3. Skeletal Mechanics of Standing	74
	Illustrative Example 5.4. Locomotion of an Octopus	74
5.3	State of Translational Equilibrium	75
5.4	Friction	77
	Illustrative Example 5.5. Osteoarthritis and Friction at Skeletal Joints	77
5.5	Free-Body Diagrams	78
	Illustrative Example 5.6. Skeletal Mechanics: Raising the Arm	79
	Illustrative Example 5.7. Skeletal Mechanics: The Hip	80
	Illustrative Example 5.8. Traction Systems	88
5.6	Center of Gravity	89
5.7	Center of Mass	89
	Illustrative Example 5.9. Center of Mass of a Water Molecule	90
5.8	Newton's Universal Law of Gravitation	90
	Supplementary Problems	91
Chapter 6	PARTICLE DYNAMICS: WORK, ENERGY, POWER	96
6.1	Definitions of Particle Dynamics	96
6.1.1	Work Done by a Constant Force	96
6.1.2	Work Done by Pressure	97
6.1.3	Work Done by a Variable Force	97

	Illustrative Example 6.1. Cardiac Stress and Treadmill Exercise	98
	Illustrative Example 6.2. Work Done by Normal and Diseased Hearts	99
	Illustrative Example 6.3. Work Done by Breathing	102
	Illustrative Example 6.4. Hooke's Law and the Work of a Spring	102
	6.1.4 Power	103
	Illustrative Example 6.5. Power Output of the Heart	104
	6.2 Mechanical Efficiency	104
	Illustrative Example 6.6. Mechanical Efficiency of the Heart	104
	6.3 Energy of a Particle: Kinetic and Potential	105
	Illustrative Example 6.7. Dynamics of Bumblebee Flight	105
	Illustrative Example 6.8. Conversion of Potential Energy to Kinetic Energy during a Fall	107
	Supplementary Problems	109
Chapter 7	MOMENTUM AND IMPULSE	113
	7.1 Definitions of Momentum and Impulse	113
	Illustrative Example 7.1. Impulsive Force and Injury due to a Fall	113
	7.2 Conservation of Linear Momentum	116
	7.2.1 Elastic Collisions	117
	7.2.2 Inelastic Collisions	117
	Supplementary Problems	122
Chapter 8	ROTATIONAL MOTION	125
	8.1 Definitions of Rotational Motion	125
	Illustrative Example 8.1. Blood Flow in Curved and Tortuous Vessels	128
	Illustrative Example 8.2. Ultracentrifuge	129
	Supplementary Problems	136
Chapter 9	ROTATIONAL DYNAMICS	139
	9.1 Definitions of Rotational Dynamics	139
	9.2 Parallel-Axis Theorem	143
	9.3 Rotational Equilibrium	143
	9.4 Rotational Dynamics	144
	9.5 Conservation of Angular Momentum	144
	9.6 Newton's Laws of Rotational Motion	145
	Illustrative Example 9.1. Skeletal Mechanics of the Joint: Elbow	151
	Illustrative Example 9.2. Skeletal Mechanics of the Joint: Hip	153
	Illustrative Example 9.3. Skeletal Mechanics of the Joint: Ankle	154
	Supplementary Problems	156
Chapter 10	OSCILLATORY MOTION	159
	10.1 Definitions of Oscillatory Motion	159
	10.2 Physical Characteristics of Simple Harmonic Motion	159
	Illustrative Example 10.1. Blood Flow Waveforms	160
	Illustrative Example 10.2. Electrocardiogram	161
	10.3 Dynamics of a Spring	164
	10.3.1 Potential Energy of a Spring	164
	10.3.2 Kinetic Energy of a Spring	165
	10.3.3 Total Mechanical Energy of a Spring	165

10.4	Pendulum Motion	166
10.4.1	Simple Pendulum	166
10.4.2	Conical Pendulum	167
10.4.3	Physical Pendulum	168
	Illustrative Example 10.3. Physical Pendulum and the Process of Walking	169
10.4.4	Torsional Pendulum	170
	Supplementary Problems	171
Chapter 11	ELASTICITY	174
11.1	Definitions of Elasticity	174
	Illustrative Example 11.1. Elastic Properties of Blood Vessels	179
	Illustrative Example 11.2. Stresses of the Leg during Movement	179
	Illustrative Example 11.3. Bone Fracture from a Fall	180
11.2	Elastic Limit	180
11.3	Laplace's Law	181
	Illustrative Example 11.4. Application of Laplace's Law for an Elastic Cylinder: Forces Acting within a Blood Vessel	181
	Illustrative Example 11.5. Application of Laplace's Law for an Elastic Sphere: Forces Acting within a Brain Aneurysm	182
	Supplementary Problems	183
CHAPTER 12	FLUID STATICS	186
12.1	Definitions of Fluid Statics	186
	Illustrative Example 12.1. Physical Properties of Human Blood	189
	Illustrative Example 12.2. Pressure in the Eye and Glaucoma	189
	Illustrative Example 12.3. Pressure and Infection	189
	Illustrative Example 12.4. Intravenous Delivery	189
	Illustrative Example 12.5. Blood Pressure in Human Circulation	190
	Illustrative Example 12.6. The Sphygmomanometer and Blood Pressure Measurement	191
12.2	Surface Tension	191
12.3	Capillary Action	191
	Illustrative Example 12.7. Lung Surfactant and Respiratory Distress Syndrome	192
12.4	Pascal's Principle	192
12.5	Archimedes' Principle	193
	Supplementary Problems	194
CHAPTER 13	FLUID DYNAMICS	198
13.1	Definitions of Fluid Dynamics	198
	Illustrative Example 13.1. Water Transport in Plants	200
13.2	Equation of Continuity	201
	Illustrative Example 13.2. Blood Flow in a Tapering Blood Vessel	202
	Illustrative Example 13.3. Blood Flow in a Vessel Bifurcation	202
13.3	Bernoulli's Principle	202
	Illustrative Example 13.4. Bernoulli's Principle and Vessel Disease	204
13.4	Torricelli's Theorem	204
	Supplementary Problems	206

CHAPTER 14	THERMAL PHYSICS	209
	14.1 Definitions of Thermal Physics	209
	Illustrative Example 14.1. Hypothermia: Human Response to Cold	
	Temperatures	213
	14.2 Heat	213
	14.3 Mechanisms of Heat Transfer	215
	Illustrative Example 14.2. Heat Stroke	216
	14.4 Thermodynamics	216
	Supplementary Problems	217
CHAPTER 15	WAVES AND SOUND	221
	15.1 Definitions of Waves and Sound	221
	15.2 Standing Waves	226
	15.3 Resonance	226
	15.4 Principle of Superposition	228
	15.5 Sound	229
	15.6 Doppler Effect	230
	Illustrative Example 15.1. Dolphins and Echolocation	232
	Illustrative Example 15.2. Ultrasound and Assessment of Stroke Risk	232
	Supplementary Problems	233
CHAPTER 16	ELECTRICITY	237
	16.1 Definitions of Electricity	237
	Illustrative Example 16.1. Electric Signal Transmission through Nerves	245
	Illustrative Example 16.2. Electrical Origin of the Heartbeat	247
	Illustrative Example 16.3. Electrical Potential of Cellular Membranes	247
	Supplementary Problems	247
CHAPTER 17	DIRECT-CURRENT CIRCUITS	250
	17.1 Definitions of DC Circuits	250
	Illustrative Example 17.1. Electrical Analogs and Blood Flow	258
	Illustrative Example 17.2. Electric Shock and the Human Body	259
	17.2 Kirchhoff's Laws of Circuit Analysis	260
	Illustrative Example 17.3. Arteriovenous Fistula: Short Circuit of the Human	
	Circulation	262
	Supplementary Problems	263
CHAPTER 18	MAGNETISM	267
	18.1 Definitions of Magnetism	267
	Illustrative Example 18.1. Mass Spectrometry and Quantitation of Ionic	
	Mass	270
	Illustrative Example 18.2. Magnetic Resonance Imaging	271
	18.2 Magnetic Induction	272
	18.3 Induced EMF	275
	18.4 Lenz's Law	276
	Illustrative Example 18.3. Electromagnetic Flowmeters and Blood Flow	
	Measurements	276

	18.5 Self-induced EMF	277
	Supplementary Problems	278
CHAPTER 19	ALTERNATING-CURRENT CIRCUITS	282
	19.1 Definitions of AC Circuits	282
	19.2 Ohm's Law	283
	19.3 Power Dissipated in an AC Circuit	285
	19.4 Resonance in an AC Circuit	285
	19.5 Transformer	288
	Illustrative Example 19.1. Electrical Analogs and Blood Flow Revisited	289
	Supplementary Problems	290
CHAPTER 20	LIGHT	292
	20.1 Definitions of Light	292
	Illustrative Example 20.1. Marine Organisms and Bioluminescence	293
	20.2 Reflection and Refraction of Light	293
	20.3 Total Internal Reflection	296
	Illustrative Example 20.2. Endoscopy: Imaging inside the Body	297
	Supplementary Problems	298
CHAPTER 21	GEOMETRIC OPTICS	301
	21.1 Definitions of Geometric Optics	301
	21.2 Mirrors	302
	21.2 Thin Lenses	306
	Illustrative Example 21.1. The Physics of Vision and Common Visual Defects: Myopia and Hyperopia	310
	Illustrative Example 21.2. Radial Keratotomy	311
	Illustrative Example 21.3. Compound Microscope	312
	Supplementary Problems	314
CHAPTER 22	NUCLEAR PHYSICS AND RADIOACTIVITY	316
	22.1 Definitions of Nuclear Physics and Radioactivity	316
	22.2 Isotopes	317
	22.3 Quantum Theory of Radiation	318
	22.4 Radioactivity	318
	22.5 Types of Radioactive Decay	321
	Illustrative Example 22.1. Nuclear Medicine and Radioactive Tracers	321
	Illustrative Example 22.2. Radiotherapy of Tumors	322
	Supplementary Problems	322
INDEX		325

Chapter 1

Mathematics Fundamentals

Physical principles and interactions encountered in physics are expressed typically in terms of qualitative reasoning and observations and rely on mathematics to translate these laws into quantitative or substantive results. Mathematics allows the student to intuitively and intellectually grasp physical concepts and to understand the effects of various factors on the particular physics problem. This chapter provides a brief description of the mathematics integral in the description and exemplification of physics as it relates to biological and medical applications.

1.1 FUNCTIONAL ANALYSIS

A function is defined as a mathematical one-to-one relationship between elements of a given set $X = \{x\}$, referred to as the *domain*, and another set $Y = \{y\}$, referred to as the *range*. A function represents a mathematical relationship between a dependent variable y and an independent variable x , which can represent any number within a specified range or interval. A function, denoted typically as $y = f(x)$, allows one to mathematically characterize a physical system in terms of variables that directly influence the system's behavior and stability.

ILLUSTRATIVE EXAMPLE 1.1. Poiseuille's Law and Blood Flow

Blood flow through a blood vessel in the human circulation can be approximated by the elementary case of fluid flow through a rigid tube. Suppose, for example, we want to determine the influence of tube radius R on the rate of fluid flow Q through the tube. Although there exist variables such as fluid density and viscosity which are embedded in the system function as constants, theory and experimental data reveal a relationship between the variable R and the system variable Q according to

$$Q = \frac{\pi \Delta P R^4}{8L\eta}$$

where Q is the volumetric flow rate, ΔP is the pressure gradient, L is tube length, and η is the fluid viscosity. The relationship is known as *Poiseuille's law*. The rate of fluid flow Q is a dependent variable since its value depends upon the value of R , which is an independent variable and can be expressed as a function

$$Q = f(R)$$

When displayed graphically on a rectangular coordinate system with the two axes representing the two variables Q and R , the function defined by $Q = f(R) = \text{constant} \times R^4$ is described by a continuous curve in two dimensions, as shown in Figure 1-1. This example illustrates the strong "power" dependence of flow on the tube radius.

Solved Problem 1.1. Identify the following mathematical statements as either functions or relations:

- (a) $A = \pi R^2$
- (b) $x + 5y = 15$
- (c) $F = \{\text{human fingerprints}\}$, $P = \{\text{humans}\}$
- (d) $x^2 + y^2 = 16$
- (e) $f(x) = 7(-\infty < x < \infty)$
- (f) $b(r) = 3r^2$
- (g) $f(x) = \sqrt{x}$
- (h) $x = \frac{1}{2}at^2$

Poiseuille's Law and Blood Flow

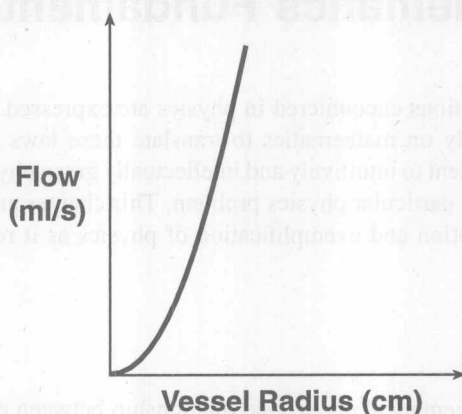


Fig. 1-1

Solution

- (a) Function, $A = A(r)$ where π is a fixed constant.
- (b) Function, $y = 3 - x/5$.
- (c) Function, there is a one-to-one relationship between humans and their fingerprints.
- (d) Relation, $y^2 = 25 - x^2$ is not a function since for a single value of x , y can be either positive or negative.
- (e) Relation, not a function since different x values are one-to-one related to the same element, 7, of the range.
- (f) Function, since one value of r corresponds to a single value of $b(r)$.
- (g) Relation, depending on the domain. This is a function only for positive real x , and the square root means the positive square root.
- (h) Function, for reasons similar to (f) in that one value of t corresponds to a unique value of x .

1.1.1 Slope and y Intercept

Consider the function describing a line:

$$y = mx + b$$

Two important features of a function presented by the above equation are the y intercept b and the slope of the function or curve m . These features are illustrated in Figure 1-2. Ideal functions originate at the origin $(0, 0)$ of the coordinate system. Most functions, however, either originate from or pass through a point along the y axis other than the origin $(0, 0)$. The value along the y axis, known as the y intercept, is given by the parameter b . The *slope* defines the rate of change of the curve in the y axis with respect to the corresponding change in the x direction and is given mathematically as

$$\text{Slope} = m = \frac{\Delta y}{\Delta x}$$

The above relation is applicable only to linear functions.

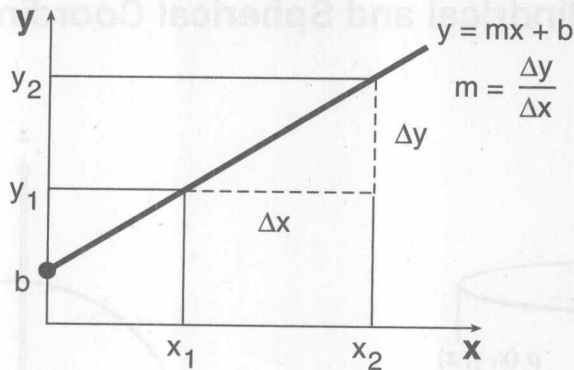


Fig. 1-2

ILLUSTRATIVE EXAMPLE 1.2. Poiseuille's Law and Blood Flow Revisited

As mentioned previously, Poiseuille's law for fluid flow is a power function described by

$$y = cx^n \Rightarrow Q = \frac{\pi \Delta P}{8\eta L} R^4$$

where c is a constant and n is any real number. The power function presented by Poiseuille's law serves as a unique variation of the above discussion. As can be seen from Figure 1-1, the curve describing fluid flow is not linear, and thus the above expression for slope cannot be applied directly. It is possible, however, to transform the power function to a linear one by taking the logarithm of both sides:

$$\log y = n \log x + \log c$$

$$y = mx + b$$

By comparison, the slope for a power function is given by

$$n = \frac{\Delta(\log y)}{\Delta(\log x)}$$

Application of Poiseuille's law for fluid flow yields

$$\log Q = 4 \log R + \log \frac{\pi \Delta P}{8\eta L}$$

Thus, the slope of Poiseuille's law is 4, implying a 4-fold increase in flow with respect to a change in tube radius. To put this in perspective, when applied to the human circulation, doubling the radius of a blood vessel increases blood flow by a factor of 16 whereas halving the radius reduces blood flow by a factor of $\frac{1}{16}$.

1.1.2 Coordinate Systems

Almost as important as the function itself are the geometric boundaries within which the function is defined. This can be understood by the following illustrative example. Consider a point in space $p(x, y, z)$ bounded by a rectangular coordinate system given by $x = x$, $y = y$, and $z = z$. However, if that same point is now defined within a different geometry such as a cylinder or sphere, it is a cumbersome, although possible, task to define the point in rectangular coordinates. It thus becomes a matter of convenience to denote any point within a sphere or cylinder in terms of the variables that more precisely define its geometry in space through coordinate transformations between different geometries.

Cylindrical and Spherical Coordinates

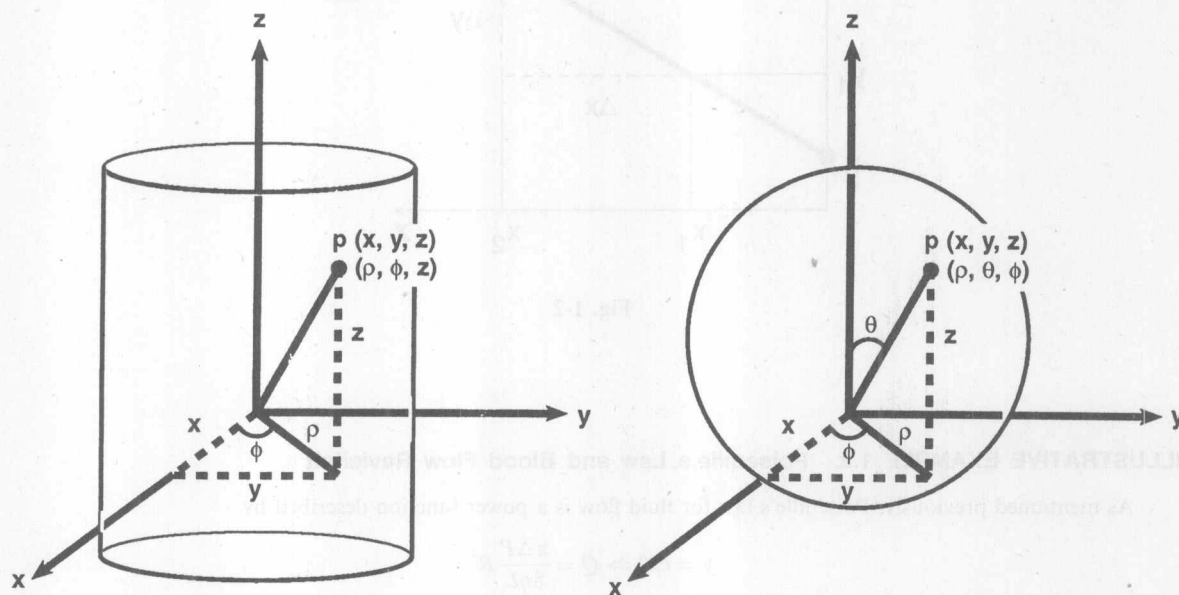


Fig. 1-3

Given the geometry of the cylinder and sphere depicted in Figure 1-3, the three-dimensional coordinates of a point in space which were defined originally as x , y , and z , in the rectangular coordinate system, are now redefined for the cylinder in cylindrical coordinates (ρ, ϕ, z) as

$$x = \rho \cos \phi$$

$$y = \rho \sin \phi$$

$$z = z$$

where $\rho \geq 0$, $0 \leq \phi < 2\pi$, $-\infty < z < \infty$; and for the sphere in spherical coordinates (ρ, θ, ϕ) as

$$x = \rho \sin \theta \cos \phi$$

$$y = \rho \sin \theta \sin \phi$$

$$z = \rho \cos \theta$$

where $\rho \geq 0$, $0 \leq \theta \leq \pi$, $0 \leq \phi < 2\pi$.

1.2 TYPES OF FUNCTIONS

Although numerous mathematical statements meet the criteria of a function, there are five general classifications of functions, described below.

1.2.1 Polynomial Functions

Polynomial functions are the most general of the five types of functions and of the form

$$y = f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots + a_nx^n$$

Polynomial Functions

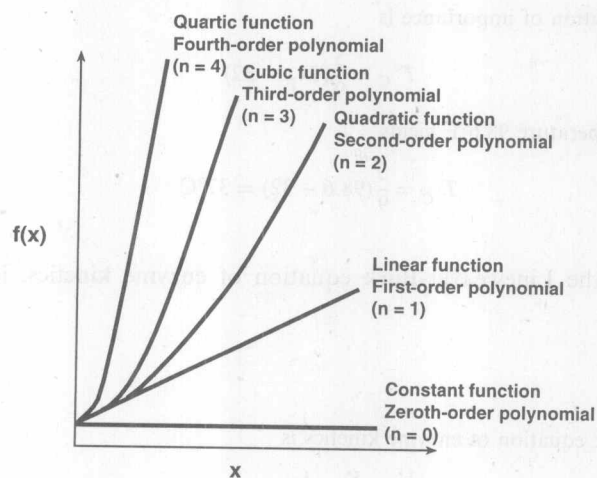


Fig. 1-4

where a_0, a_1, a_2, a_n are coefficients (constants) and n is an integer which describes the degree or order of the polynomial. The most general polynomial function is the first-order polynomial function given by

$$y = f(x) = a_0 + a_1x \quad a_1 \neq 0$$

which is a linear function (compare with Section 1.1.1). In the linear function, a_1 corresponds to the slope m . Figure 1-4 displays graphically the more common polynomial functions.

ILLUSTRATIVE EXAMPLE 1.3. Applications of Polynomial Functions

Polynomial functions represent a wide range of mathematical functions and can be found in numerous physical applications as they pertain to biological sciences, including:

- (1) *Ohm's law of electricity.* The voltage V applied across a wire is related to the current flowing through the wire I , multiplied by the resistance R of the wire, or

$$V = IR$$

- (2) *Temperature conversion.* The Celsius temperature scale ($^{\circ}\text{C}$) and the Fahrenheit scale ($^{\circ}\text{F}$) are linearly related by

$$T_{\text{C}} = \frac{5}{9}(T_{\text{F}} - 32)$$

- (3) *Lineweaver-Burk plot of enzyme kinetics.* The influence of an enzyme on the speed of a chemical reaction can be observed through the relationship between the substrate concentration $[S]$ of the enzyme and the velocity of the reaction v by

$$\frac{1}{v} = \frac{K_m}{V_{\text{max}}} \frac{1}{[S]} + \frac{1}{V_{\text{max}}}$$

where V_{max} is the maximum velocity of the chemical reaction and K_m is a constant.

Solved Problem 1.2. Normal human body temperature is approximately 98.6°F . Calculate this temperature in the Celsius scale.

Solution

The mathematical relation of importance is

$$T^{\circ}\text{C} = \frac{5}{9}(T^{\circ}\text{F} - 32)$$

Substituting the body temperature 98.6°F yields

$$T^{\circ}\text{C} = \frac{5}{9}(98.6 - 32) = 37^{\circ}\text{C}$$

Solved Problem 1.3. For the Lineweaver-Burk equation of enzyme kinetics, identify the slope and y intercept.

Solution

The Lineweaver-Burk equation of enzyme kinetics is

$$\frac{1}{v} = \frac{K_m}{V_{\max}} \frac{1}{[S]} + \frac{1}{V_{\max}}$$

Since it is a linear function, the above equation follows the general form of

$$y = mx + b$$

By an elementary comparison, the slope m is

$$\text{Slope} = \frac{K_m}{V_{\max}}$$

and the y intercept b is

$$y \text{ intercept} = \frac{1}{V_{\max}}$$

Solved Problem 1.4. The absorption of ethyl alcohol into the bloodstream in the human body is one of the few known kinetic processes that is zeroth-order. Assume that the process may be represented as



Derive the integrated rate law for the biochemical process, and show that this is a linear function.

Solution

Assuming that ethyl alcohol can be represented by $[B]$, the reaction for the process can be mathematically characterized by

$$\begin{aligned} -\frac{dB}{dt} &= k \\ -dB &= k dt \end{aligned}$$

Integrating both sides of the equation yields

$$\begin{aligned} \int_{B_0}^B dB &= -k \int_0^t dt \\ B - B_0 &= -kt \\ B &= B_0 - kt \end{aligned}$$

Integration is covered in greater detail in Section 1.4. By comparison, it can be seen that this rate law is a linear function, where $-k$ is the slope and B_0 is the y intercept.