CALCULUS

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Students:

A *Study Guide* and a *Student Solutions Manual* have been specially created to help you master the concepts presented in this textbook. Information about these can be obtained from your bookstore. Also, a *computer software supplement* specially designed to enhance your understanding of selected topics in this text is available for purchase through your bookstore or through the special order form in the back of this book.

Preface

The image of Stonehenge on the cover represents many of the themes that motivated the writing of this book. Seen from this angle, the arrangement of stones suggests a gateway, which calculus surely is—not only to modern mathematics, but also to science, engineering, and the quantitative sophistication a liberal education seeks to impart. A major goal of the text is to convey to student readers the power of calculus as a tool for understanding the world around them. It is hoped that this will motivate them not merely to pass through this gateway, but to work their way through it actively. They need to emerge from calculus with solid mastery both of its computational procedures and its concepts, as well as confidence in using them to analyze and solve problems in other fields.

To foster those goals, there is a significant effort in the text to relate calculus to topics studied in other lower-division college courses—physics, chemistry, biology, economics, engineering, etc.—as well as to familiar occurrences in everyday life. Such topics are used in some cases to motivate new mathematical ideas and in others to illustrate extra-mathematical applications of calculus. Sometimes, they are used to do both. For instance, limits at infinity are motivated by the question of why new editions of textbooks appear periodically, an occurrence that often prevents students from reselling their texts. One-sided limits are related to change-of-phase in chemistry. And improper integrals are motivated by, and used to compute, the work done in sending a deep space probe out of the solar system. This kind of motivation and application is thoroughly integrated into the text, rather than being confined to isolated sections or subsections.

The current generation of students, whose background is so rich in pictorial experience, seems to learn best when given ample visual aids. The power of Stonehenge to evoke mental images in its viewers suggests the strong visual emphasis throughout the text. In addition to approximately 800 figures in the text itself, tables are frequently used to summarize the results of repeated calculations. And the answer section includes graphs whenever they are asked for in the exercises. Nearly all the art for the text was computer-generated at least twice to ensure accuracy and the most illuminating images possible.

The erection of Stonehenge was a triumph both of insight (in designing it) and persistent hard work (in completing it). Calculus also requires insight to understand its ideas and persistence to really master it, and the text makes no attempt to mislead its readers about that. To be sure, the primary thrust is toward helping the student develop the conceptual understanding and insight that must be the foundation of true mastery of calculus. But there is also emphasis on building the solid computational skills and the confidence in using them that are vital to successful advanced study and work in mathematics, science, and engineering. Many examples show the computational persistence needed to complete some of the more demanding exercises and to fulfill the expectations of later courses. There are discussions of the significance of central concepts, including the necessity as well as the impact of the hypotheses of some important theorems. There are counterexamples as well as examples, and the last few exercises in most sets challenge the students to think in some depth about the material of the section.

Because the most efficient computational implementation of mathematical ideas is usually algorithmic, the text provides suggested algorithms for solving problems in such areas as optimization, related rates, change of variable in integration, integration by parts, and convergence tests for infinite series. It is hoped that working with these can help the students develop the habit of thinking algorithmically and looking at problems as opportunities to apply general methods.

The book provides a wide range of mathematical proofs—from those essential to understanding the concepts and performing the calculations to more subtle ones that yield deeper theoretical insights. These proofs can be omitted without breaking the continuity of the text; instructors who wish to treat the theory more thoroughly, however, may find some of the more conceptual discussions of special interest. The section on differentials, for example, develops the connection between differentiation and linear approximation in some detail, which the instructor can assign according to students' needs and abilities.

There is enough material for a three-semester or four- or five-quarter course meeting four or five hours per week. (See below for information about the *Instructor's Manual,* which contains specific suggestions about outlines and pacing.) The text is divided into chapters that correspond to natural learning units, so that testing may be keyed to the chapter organization. To help students digest and organize the material of each chapter, there is a review section called *Looking Back* for self study. It summarizes the most important material in the chapter, presents a *Chapter Checklist* of key terms from each section, and contains a set of review exercises.

The trigonometric functions appear early, in Chapter 2, and so are available during the first term for applications and illustrations of the full chain rule (rather than just the general power rule). Besides the traditional logarithmic and exponential topics, Chapter 6 includes an optional section on log-log and semi-log graphing, techniques widely used in many fields but whose mathematical foundation most students never see. This material provides an easy but significant application of logarithmic functions and their properties, one that complements the more substantial applications presented earlier in the chapter. In keeping with the theme of the usefulness of calculus, elementary differential equations are also introduced early (Section 3.10) and are used in Chapter 6 in connection with some applications of exponential, logarithmic, and other transcendental functions. Chapter 15, which continues the study of differential equations through second-order linear equations and their applications, can be covered at any point after the completion of Chapter 6. The discussion of infinite series in Chapter 9 emphasizes the idea of finite approximation of irrational quantities. The latter theme, first discussed in the opening section, culminates in the development of Taylor polynomials and Taylor series.

The material on three-dimensional analytic geometry and calculus of several variables is approached from the vector point of view. This parallels the notation students see in concurrent science and engineering courses and also puts into sharp focus the analogies between single-variable and multivariable calculus. For example, the gradient is introduced as the multidimensional version of the ordinary derivative of a one-variable function. By looking for such similarities to single-variable calculus, the student is constantly reminded of the latter and thereby helped to deepen his or her mastery of it. The primary emphasis in Chapters 10 through 14 is on computation rather than theory, however. Sophisticated concepts like vector spaces, linear functions and transformations, and Hessian matrices are not introduced.

The computer-generated sky on the cover serves as a backdrop to highlight Stonehenge. It suggests the illuminating potential of computers for bringing many of the key ideas of calculus into sharper focus. The text tries to realize this potential by using computer output to illustrate topics like limits, differentiation, root finding, optimization, integration, Euler's method, calculation of values of the natural logarithm function, Taylor polynomials, and others. Throughout the book, there are optional exercises marked PC. They are designed for programmable calculator or personal computer solution, but are also suitable for a mainframe system or, in many cases, for a nonprogrammable calculator. In addition, there are a number of optional subsections that discuss numerically-oriented topics, and there are comprehensive discussions of numerical integration and polynomial approximation of transcendental functions. These features have been included because of the greater prominence numerical methods have assumed with the spread of affordable programmable calculators and microcomputers. But they are optional enhancements which can be covered in varying degrees of detail or omitted altogether without affecting later work. Indeed, the text has been successfully class-tested in a traditional course that placed little emphasis on numerical matters, in another that integrated programmable calculators, and in one given in conjunction with a computer laboratory section. (Ancillaries designed for use with microcomputers are described below.)

Stonehenge is one of the oldest surviving monuments to humanity's quest to understand our world. The persons who designed and erected its monoliths, and the mathematics they used, are unknown to us. But not so the great mathematicians who have built calculus, and their mathematical ideas. The book contains numerous historical notes to acknowledge their contributions, so that the reader can perceive the continuing evolution of mathematics as an area of human knowledge.

Ancillaries

There are several supplements available to meet special needs and widen the possible uses of the text.

- To assist students in fully mastering the text, a ***Study Guide*** has been prepared by George F. Feissner of the State University of New York College at Cortland. It outlines the content of each section in the form of statements to be filled in by students, to help them organize the material. It also provides additional worked-out examples that isolate the individual skills needed to successfully work the problems in the text. Those worked-out problems are annotated with suggestions on how to analyze, set up, and solve calculus problems. In addition, there are practice problems for the students to test their understanding as they study, and each chapter concludes with a *Self Test* designed to help them gauge their readiness for a chapter test.

- The ***Instructor's Manual*** contains sample outlines and examinations and offers section-by-section suggestions on efficient use of the text. Also included are sample BASIC programs for the numerical methods mentioned in the text. The guidance it provides about choosing material for class discussion is intended especially to assist teaching assistants, part-time instructors, visiting professors, and others new to the calculus classroom.

- A microcomputer diskette that is entitled ***Calculus (Kemeny/Hurley)*** has been specially prepared to accompany the text by John G. Kemeny of Dartmouth College and is available from True BASIC, Inc., 39 South Main Street, Hanover, New Hampshire 03755 (telephone: (800)TR-BASIC or, in New Hampshire, (603) 643-3882). It features several powerful general programs for calculus that provide striking illustrations of such things as tabulating and graphing functions, computing their limits, finding formulas for and plotting their tangents, finding extreme values over intervals, integrating them, using L'Hôpital's rule to evaluate indeterminate limits, and finding formulas for Taylor polynomials and the solution of second-order linear homogeneous differential equations. It also contains custom-designed programs for use in solving the text's PC problems.

- An ***Instructor's Solution Manual*** has been prepared by John T. Hardy, Jr., of the University of Houston and Jeffrey J. Morgan of Texas A & M University in collaboration with Paul R. Fallone, Jr., of the University of Connecticut at Hartford. It contains worked-out solutions of all the exercises and is available on a complimentary basis to those who adopt the text. The ***Student's Solution Manual,*** a shortened version consisting of the solutions to every fourth exercise, is available for sale to students if authorized by a professor who adopts the text.

- The forthcoming paperback ***Computer Laboratory Manual for Calculus*** discusses construction of structured computer programs for calculus and contains some sample BASIC programs for the numerical methods mentioned in the text and exercises. It is organized into chapters corresponding to material that might be presented throughout the year in a computer laboratory section of an introductory calculus course.

In closing, it is a happy duty to express my appreciation to my editors, Heather Bennett and Kevin Howat, and their colleagues, Richard Giggey, Richard Greenberg, Hal Humphrey, Marta Kongsle, Lisa Mirski, Catherine Read, Stephen Rutter, Anne Scanlan-Rohrer, Ruth Singer, and Ira Zukerman of Wadsworth Publishing Company for the support, encouragement, and hard work they have contributed to the project that culminated in publication of this text. The uniformly high integrity, professional skill, and personal dedication of the people at Wadsworth—who truly live their professed desire to be partners with their authors in enhancing higher education—have benefitted both me and this book in no small degree.

James F. Hurley

Storrs, Connecticut
July, 1986

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Since 1962, I have taught calculus from texts authored by many mathematicians, and have consulted numerous others while preparing lectures. All of these have helped me shape my view of the material presented here, and I am indebted to them all.

The students at the University of Connecticut and the University of Michigan at Flint who class-tested the text in manuscript form deserve thanks for the valuable feedback they provided to improve the book. The encouragement of my students over the years has played a major role in my decision to start, and persevere with, the writing of my texts.

Charles Paskewitz developed a customized three-dimensional surface plotting program for the IBM Personal Computer and Hewlett-Packard HP-7475A plotter. That was used to generate most of the preliminary illustrations in Chapters 11 and 12 and the corresponding part of the answer section. I am very grateful for his dedicated work, whose high caliber materially improved the visual quality of the manuscript. The final art for the text was produced from a Hewlett-Packard HP-86 computer driving a Hewlett-Packard HP-7550 plotter. The software used was developed by George E. Morris of Scientific Illustrators of Champaign, Illinois. Accuracy of the plots is to within a tolerance of approximately 0.001 in. As will be apparent upon examining the art, Mr. Morris's skills have significantly enhanced the appearance and quality of the text.

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Foreword to the Student

This book is written for you to read and learn from. This *Foreword* offers some suggestions on how to get the most out of the features that have been included to help you as you learn calculus. When you have completed your calculus study, you may find the text useful to keep for reference. Many students find it easiest to review a topic directly from the source where they first learned it.

Examples and Exercises

The *essential* ingredient in mastering calculus is working as many of the exercises as possible. Don't kid yourself. Anybody who has passed a precalculus course can *understand* the concepts of calculus and follow the solutions of the examples in the text. In fact, a careful effort has been made to avoid omitting any computational steps in those solutions. But that is ***not*** the same as learning to do calculus yourself, which can come only through *extensive practice with exercises.* Such practice is needed not only to learn the mechanical processes, but also to become proficient enough in using them so that you can finish examinations in the allotted time.

The examples worked out in the text illustrate how the mathematics presented in each section is applied to solve concrete problems, in calculus and in other fields of study. The examples are designed to prepare you to work the exercises, because most of those resemble the examples or are based on ideas discussed there. To build the skill and confidence needed for quizzes and tests, try to do as many of the exercises as you can without referring back to the worked examples.

Some exercises, especially those toward the end of each list, are more challenging. They are designed to let you dig beneath the surface: to think about the material more deeply and thereby to improve your mastery of it. Often hints are given for those problems. Whether or not they are formally assigned, you should try some of them in order to achieve the best proficiency you can in calculus.

Organization

The text is organized into chapters and sections. The first section of each chapter gives a general introduction to the upcoming topics. Each later section typically begins with one or two paragraphs that set the scene for what follows. They may pose a problem whose solution requires new methods or they may focus on a previous topic that will be explored further in the section.

Next come any new definitions that may be necessary to precisely state the results or procedures to be presented. Those statements specify exactly when the techniques under development can be used. Pay careful attention to them, to avoid the frustration many students experience when they try to apply a technique to a case where it is inapplicable!

Optional material is often separated from the main part of the text in subsections. Your instructor will indicate which of those subsections should be studied in detail.

Proofs Mathematics is a deductive science, whose results are provable from basic axioms and earlier theorems by using the rules of logical inference. Throughout the text, proofs of theorems are interspersed with the definitions, computational rules, and examples. Their primary purpose is to indicate how key results are derivable from previously established or assumed facts. They are written in enough detail for you to follow them step-by-step should your instructor advise you to study them carefully. Mastery of the proofs, however, is not a prerequisite to doing most of the exercises or to learning the subsequent computational methods. While reading them and understanding them can help you build a more solid foundation on which to learn computational calculus, many first courses do not put major emphasis on them.

Answers Answers to all odd-numbered computational problems appear at the end of the text, in Appendix IV. If one of your answers differs from that in the answer section, don't simply assume that you are wrong and need help to proceed. Instead, carefully check your work as you would if there were a discrepancy in answers with a fellow student. Such checking is one of the most effective ways of thoroughly learning calculus.

Review Sections Each chapter ends with a review section called *Looking Back*, a short summary of the chapter that puts its contents into perspective. Included are a *Chapter Checklist*, which gives the principal terms from the chapter, and a set of *Review Exercises* to help you judge how well you have learned the major problem-solving techniques of the chapter.

Inside Covers The inside front cover contains lists of key formulas from algebra, geometry, and trigonometry that are needed in calculus. Refer to it whenever an example uses an algebraic technique or geometric or trigonometric formula that isn't familiar. There is also a list of common algebraic errors, titled *Algebraic Pitfalls*. In checking your work for possible mistakes, it may be helpful to refer to this list to make sure that you have not fallen victim to one of those pitfalls, which are often very difficult for students to spot in their work. The inside back cover contains an index of symbols and notation used throughout the book, arranged by first page of occurrence of the symbols.

Format Definitions, lemmas, theorems, and corollaries are numbered sequentially within each section, with a double number that gives the section and item number of each numbered statement. For example, Definition 4.3 is the third numbered item in Section 4. The most important statements and formulas are boxed or printed in color. When new terms are introduced, they are printed in ***bold print.*** Examples are numbered separately, starting from Example 1 in each section. When two or more steps are combined on a single line of an example's solution, the steps are linked by a colored arrow →. A colored box ■ marks the end of the solution to each example. Proofs end with the symbol QED. That is the abbreviation of the Latin phrase *quod erat demonstrandum*, meaning *which was to be proved*. Until the 1800s, most mathematics texts were written in Latin, and that phrase is the Latin translation of the Greek phrase ὅπερ ἔδει δεῖξαι, with which Euclid ended his proofs more than 2000 years ago.

Mathematics is a very old subject rich in impressive discoveries, brilliant breakthroughs, and illustrious people. The cover of the text shows a digitized image of Stonehenge, where the ancient monoliths arranged with precise attention to key cyclical events have long intrigued those who have gazed upon this scene. The design of that arrangement constitutes an ancient application of mathematics to better understand the world around us. The digitized image suggests the contemporary interaction between calculus and the modern world that you will learn of in this text as you read and study the applications of calculus and the algorithms for performing some of its calculations on electronic devices. The *Historical Notes* you will find throughout the text present information about some of the men and women who contributed to the development of calculus and related areas, a process that, as we suggested, continues to this day.

Study Aids To provide additional help in learning from the text, supplementary books and software are available.

- A ***Study Guide***, prepared by Professor George F. Feissner of the State University of New York College at Cortland, spotlights each section's major concepts and also provides strategic analysis of the steps needed to work the exercises for that section. Its suggestions about the exercises are followed by practice problems that focus on the specific skills needed to become proficient at solving calculus problems. Each chapter ends with a *Self Test*, a multiple-choice examination to help you gauge your readiness to take an examination on the material of that chapter.
- A ***Student Solutions Manual*** has been prepared by Professors John T. Hardy, Jr., and Jeffrey J. Morgan of the University of Houston. It contains solutions to every fourth problem and can be stocked by your college bookstore on your instructor's request.
- A disk entitled ***Calculus (Kemeny/Hurley)*** has been prepared by Professor John Kemeny of Dartmouth College, one of the inventors of the BASIC programming language. It contains a number of powerful general programs that illustrate major concepts of calculus quickly and clearly on a microcomputer screen, as well as specially designed programs for use in working the PC problems in the text. Those are exercises intended for solution on personal computers or programmable calculators, which extend the scope of calculus beyond hand calculation. Many problems in science and industry that are solved using calculus are too complex for hand solution, and the PC problems give you a chance to use modern electronic devices in the way they are actually used to apply calculus in contemporary work. Versions of this software are available for the IBM PC

and computers that are compatible with it (most of those that use the MS-DOS operating system), the Apple Macintosh, and the Commodore Amiga.

Finally, a word about how to read a mathematics text: *actively!* Read with a pencil in your hand and paper at your side. Whenever the text does a calculation, work through it on your own, checking that you understand and can reproduce the steps shown in the book. Don't expect to see everything immediately without any thought. If calculus were *that* easy, then calculus courses would be short and calculus books thin. However, this book is written so that by performing routine algebraic calculations you can follow derivations presented or solutions given in the text. Such active reading is the best way to understand calculus and build confidence in using it. It also sharpens your algebraic skills, which are indispensable in mastering this subject.

Many students find it helpful to read through each section before it is discussed in lecture. That helps them get an idea of what the lecture will cover and often reveals points they may want to ask questions about during class as their instructor discusses the material. Other students prefer to let their instructor introduce them to the material, and then study the text and their notes after class. They find that combination often helps them understand points they may not have fully grasped the first time they saw them. Both approaches involve at least one careful reading of the text. Try each one to find which works better for you.

Do you have a willing mind, a sharp pencil, and a blank pad of paper ready? Then let's get started!

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* Optional section