Introduction to Operations Research

A COMPUTER-ORIENTED ALGORITHMIC APPROACH

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INTRODUCTION TO OPERATIONS RESEARCH
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PREFACE

It was not until the advent of computers that operations research (OR) made its impact on our society. Although theoretical results had been worked out for the solution of many types of problems earlier, quite often they could not be applied to "real" problems because of the amount of calculation involved. Today the computer is the core of every operations research group; however, very little has been done to fulfill the need for computer-oriented textbooks in operations research. Thus, the motivation for *this* textbook.

A computer-oriented approach is used throughout this book to acquaint those interested in problem solving with the important methods of operations research in a way that they can start solving realistic problems immediately. This approach is especially appealing because of the role the computer plays in the problem-solving process. The general approach throughout is:

1 Formulation of the problem to be solved
2 Construction of a model of the problem
3 Development of a method to solve the model
4 Presentation of a concise computer-oriented algorithm to solve the model
5 Presentation of a FORTRAN computer program for the algorithm

The development of most of the methods of solution is preceded by one or more illustrative examples, many with computer solutions. Parts 1–3 above help the reader learn about the types of problems that can be solved by a particular method and how to model the problem for solution. Part 4, which is written to cover the method in general, is a detailed step-by-step procedure that can be easily programmed for the computer. Then finally, part 5 gives the reader an opportunity to solve a large variety of problems and to emphasize the analysis of the results, thus enhancing the reader's problem-solving ability.

This book is designed to serve as an introductory OR textbook for undergraduates interested in problem solving, as a textbook for a short course in computer-oriented methods, and as a handbook of optimization methods and computer programs for the practitioner. It is meant as a survey of the important methods of operations research integrating the computer and the methods. It can equally serve the practitioner, the student who plans to study operations research in depth, or the student who only desires an appreciation of the general area of operations research.

The text is divided into two parts. Part I covers deterministic models and methods of solution, while Part II covers probabilistic models and methods of solution.

Each part is independent of the other. The deterministic methods are presented first so that the reader who is unfamiliar with probability can become acquainted with the general computer-oriented algorithmic approach first. The chapter on probability theory is not intended as a comprehensive coverage of the subject. Rather, it presents only the essential eiements of probability that are required throughout the remaining chapters in Part II.

Since computer programming has become an essential tool in most academic programs and is taught on the majority of campuses, a basic knowledge of FORTRAN programming is assumed throughout the book. A multitude of introductory programming texts is available to those who are deficient in this area. The only mathematical background assumed is a first course in calculus.

This book differs from other texts in the area in its overall computer-oriented approach. In addition, many of the latest OR methods from the literature are presented in easy to understand algorithmic form suitable for programming.

Many of the algorithms are also followed by a useful FORTRAN program that has been run on an IBM 370/168 computer. The processing time for each example problem is for the IBM 370/168. A description of how to modify the storage requirements to meet the needs of users with a limited amount of storage is given.

All computer programs are available from the author for a fee to cover reproduction, handling, and mailing.

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Questions, suggestions, and comments from hundreds of students during the past 11 years have been extremely valuable. My appreciation is extended to Professors F. Garnett Walters, Howard Pyron, and C. Y. Ho for using preliminary versions of the book in their classes. A special thanks goes to Professor Thomas B. Baird for reading the inventory chapters and offering valuable suggestions.

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CONTENTS

1 INTRODUCTION

1.1 THE BEGINNING AND PROGRESS OF OPERATIONS RESEARCH

Any problem that requires a positive decision to be made can be classified as an operations research (OR) type problem. However, the approach used in decision making has changed considerably over the years. Although OR problems have existed since the creation of man, it was not until World War II that the name *operations research* was coined and the scientific approach of modern OR came into being. The name probably came from a program undertaken by Great Britain during World War II, "research in military operations," thus operations research. It was during the early part of the war that Great Britain brought together a group of specialists from a number of areas to work on the military defense of their country. The work of this first OR group involved, among other things, studies to determine the best use of air-power and the newly invented radar. Because of the success of OR in military operations, it quickly spread to all phases of industry and government. By 1951, OR had taken its place as a distinct science in the United States. Trefethen [11] gives an excellent history of OR since its beginning.

As big business emerged after World War II, many businesses became so diversified and complex that top management quickly lost control of the business as a whole. This prompted a division of management. Further divisions evolved until finally each department or division was interested only in its own welfare without regard for the other areas of the business. This drastically constrained the potential overall effectiveness. Consequently, OR groups, which consisted of specialists from several areas, were formed to assist management in optimizing the overall effectiveness of the business, while recognizing the importance of separate functional units within the business.

Churchman, Ackoff, and Arnoff [2] point out that

> The systems approach to problems does not mean that the most generally formulated problem must be solved in one research project. However desirable this may be, it is seldom possible to realize it in practice. In practice, parts of the total problem are usually solved in sequence.

The methods presented herein are applicable to the well-defined subproblems that are solved in sequence. The emphasis throughout this book is on learning computer-oriented OR methods that are applicable to a large number of problems, rather than on learning the overall systems approach to solving very large problems.

Churchman, Ackoff, and Arnoff [2], as well as Hillier and Lieberman [5], discuss in detail the six standard phases of an OR project, namely:

1 Formulating the problem
2 Constructing a mathematical model to represent the system under study
3 Deriving a solution from the model
4 Testing the model and the solution derived from it
5 Establishing controls over the solution
6 Putting the solution to work: implementation

However, the nature of this text dictates that we concentrate primarily on phases 1, 2, and 3. This in no way is meant to minimize the importance of the other necessary phases of every OR project, but our emphasis is on the methods used to derive an optimal solution for a given mathematical model of a problem.

Of course, the concurrent development of the digital computer is credited with the rapid progress of OR in this country. For example, the simplex method of linear programming was developed in 1947 by George B. Dantzig, but it lay dormant with respect to realistic problems until the mid- to late 1950s when the computer, with its high speed and large storage capacity, became commonplace in many universities and businesses, as well as in government agencies. Since that time, the computer has assisted the development and/or implementation of most of the OR methods in use today. It is clear to most that OR is vitally dependent upon the computer, for without it OR would be reduced to a theoretical science rather than the ever-expanding field that it is. It is because of the computer's essential and vital role in OR that we have merged the two in this textbook.

1.2 CLASSIFICATION OF PROBLEMS IN OPERATIONS RESEARCH

Although there is no single classification of problems that are candidates for solution by methods of operations research, most problems fall into one of the following categories:

1 Sequencing
2 Allocation
3 Routing
4 Replacement
5 Inventory
6 Queueing
7 Competitive
8 Search

Mathematical models have been constructed for each of these categories, and methods for solving the models are available in many cases.

Sequencing problems involve placing items in a certain sequence or order for service. For example, in a job shop, N jobs requiring different amounts of time on different machines must each be processed on M machines in the same order with no passing between machines. How should the jobs be ordered for processing to minimize the total time to process all of the jobs on all of the machines? The solution is quite simple for the two-machine problem and for a special case of the three-machine problem, but is several magnitudes more difficult for the general M-machine problem.

Allocation problems involve the allocation of resources to activities in such a way that some measure of effectiveness is optimized. For example, if the measure of effectiveness can be represented as a linear function of several variables subject to a number of linear constraints involving the variables, then the allocation problem is classified as a *linear programming problem*. Likewise, if the resource is people who can each perform any one of several jobs, possibly in different amounts of time, and the measure of effectiveness is the total time to perform all of the jobs when one and only one person is "allocated" to each job, then the problem is classified as an *assignment problem*. Suppose person A takes 2 min to perform job 1 and 4 min to perform job 2. Likewise, suppose person B takes 3 and 2 min to perform jobs 1 and 2, respectively. Which person should be assigned to each job to minimize the total time to perform both jobs? Obviously, by inspection or enumeration, person A should perform job 1 and person B should perform job 2 for a total effectiveness of 3. Suppose further that three people and three jobs are involved with the corresponding times given by

	Job		
Person	1	2	3
A	2	6	3
B	8	4	9
C	5	7	8

Which person should perform each job? If we enumerate all possible assignments, we have

	Assignments					
	A:1	A:1	A:2	A:2	A:3	A:3
	B:2	B:3	B:1	B:3	B:1	B:2
Total	C:3	C:2	C:3	C:1	C:2	C:1
effectiveness:	14	18	22	20	18	12

Thus, the minimum total effectiveness (time) is 12 units and is obtained by assigning person A to job 3, B to job 2, and C to job 1. These are trivial problems, but suppose 20 people are available to perform 20 jobs. What is the minimum time to perform all jobs? How long do you suppose it would take our fastest computer to find the solution by enumeration? If you guessed thousands of years, you guessed correctly, for there are

$$20! = 20 \cdot 19 \cdot 18 \cdot \ldots \cdot 1 \approx 2.433 \times 10^{18}$$

different assignments to be checked. Obviously, some other means of solution must be used.

Routing problems involve finding the optimal route from an origin to a destination when a number of possible routes are available. The classical traveling salesman problem is an example. A salesman wishes to visit each of N cities once and only once before returning to his home office. In what order should he visit the cities to minimize the overall distance traveled? This problem arises as a subproblem of the *vehicle dispatch or delivery problem.* Once a set of distinct locations have been assigned to a certain truck route in the delivery problem, in what order should the locations be visited to minimize the total distance traveled?

Replacement problems occur when one must decide the optimal time to replace equipment that deteriorates or fails immediately. When, if ever, should an automobile be replaced with a new one? This is a problem faced by most of us today. Of course, we each have our own measure of effectiveness, so there would not be a single optimal answer for everyone even if each automobile gave exactly the same service. Much depends on the purpose of the car, the role prestige plays in our lives, how fast we drive, etc. Another type of replacement problem involves equipment that works perfectly until it fails, such as a light bulb or an intricate computer component. What is the optimal replacement policy for this type of equipment?

The problem of deciding how much of a certain product to hold in inventory is one of real concern. If a customer requests a certain quantity of the product but it is not available, this could mean a lost sale. On the other hand, if an excess of the product is carried in inventory, the many costs associated with inventory may be unacceptable. Hence, the *inventory problem* is to determine the level of inventory that will optimize some measure of effectiveness.

Queueing problems plague us from the time we rise in the morning until we retire at night. Wait for the bathroom, wait for breakfast, wait at stoplights, wait for the computer, wait, wait, wait; that's the story of our lives. Any problem that involves waiting for service is classified as a *queueing or waiting-line problem*. The OR literature is bulging with solutions for many types of queueing models; however, most realistic queueing problems are so complex and the components so interrelated that simulation is a vital technique in this general area.

Competitive problems arise when two or more people are competing for a precious resource. The resource may range from an opponent's king in chess to a larger share of the market in business. Quite often a competitive problem involves bidding for a contract to perform a service or to obtain some type of privilege. A number of different types of bidding procedures are used, but in each case competition is involved. Formal models of realistic problems in this area are scarce; however, the underlying concepts of the decision-making process are worthy of some study.

Search problems differ from the other types of problems we have discussed in that they all involve searching for information that is necessary to make a decision. Some examples are:

- Searching the ocean for enemy ships
- Auditing books for errors
- Exploring for valuable natural resources, such as oil, copper, or coal
- Retrieving information from computer storage
- Shopping for a new suit

In each case, the objective is to minimize the costs associated with collecting and analyzing data to reduce decision errors and to minimize the costs associated with the decision errors themselves. We will see later that statistical decision theory provides a basis for solving many search problems.

1.3 MATHEMATICAL MODELING IN OPERATIONS RESEARCH

Suppose we want an optimal solution of a given problem. To get at the solution, it is usually more meaningful and convenient to write out the problem in mathematical terms. This mathematical description or representation of the problem is called a *mathematical model* of the problem. Generally, it is easier to get a "handle" on the model and solve it rather than the problem in its original nonmathematical form. If a mathematical model of a given problem can be solved either analytically or numerically, the solution can then be applied to the original problem. If the mathematical model is a good representation of the problem, the solution of the model will be a good solution of the problem. On the other hand, even an exact solution of a poor model will not be a good solution of the problem.

Many problems can be represented by a number of different models, but one model is usually more appropriate than others. To this end, a number of unique models with appropriate methods of solution have become well known during the past 25 years. For example, linear programming models, dynamic programming models, inventory models, and queueing models have solutions readily available. Thus, if a given problem can be modeled as (put into the mathematical form of) a certain type of linear programming model, the method of solution is immediately available. The object of any OR project is to determine the most appropriate mathematical model for the problem at hand, and either use available methods to solve the model or develop new methods of solution.

It may be that a mathematical model of a given problem cannot be constructed. On the other hand, it may be possible to construct a mathematical model, but exact methods for solving the model may not be readily available or may not be amenable for computer solution because of the large amount of computer storage or time required. Consequently, an alternative in this situation is to use an intuitive or heuristic approach to the solution. This approach has been used successfully to solve problems directly without formulating a mathematical model. It has also been used to provide approximate and/or exact solutions of many mathematical models of problems. Quite often heuristic methods provide exact, or at least adequate, solutions of problems much faster than numerical methods that are used to solve an appropriate model of the given problem. For example, one type of problem involving the allocation of resources to activities may be formulated as an integer linear programming model; however, a heuristic method called the *Hungarian method* provides the solution of the problem much faster in most cases.

Finally, if a problem is so complex that it cannot be modeled adequately for solution by one of the available methods or if it cannot be solved adequately with a heuristic method, then the problem solver usually resorts to simulation. Of course, simulation is not the answer to all problems; nevertheless, it does have a great deal of merit in studying large, complex systems where the components are highly interrelated. We will discuss the positive and negative aspects of simulation in Chapter 14.

It is our purpose to present computer-oriented algorithms for most of the methods used to solve the well-known mathematical models, as well as algorithms for important heuristic methods. In most cases, a computer program for the algorithm is presented so the reader can be exposed to the solution and analysis of a large variety of problems to enhance his problem-solving ability. A number of up-to-date methods from the literature are also presented in easy-to-grasp algorithmic form.

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