SPACE, TIME, AND MECHANICS

Basic Structures of a Physical Theory

Edited by D. Mayr and G. Süssmann

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PREFACE

In connection with the "Philosophy of Science" research program conducted by the Deutsche Forschungsgemeinschaft a colloquium was held in Munich from 18th to 20th May 1979. This covered basic structures of physical theories, the main emphasis being on the interrelation of space, time and mechanics. The present volume contains contributions and the results of the discussions. The papers are given here in the same order of presentation as at the meeting.

The development of these "basic structures of physical theories" involved diverging trends arising from different starting points in philosophy and physics. In order to obtain a clear comparison between these schools of thought, it was appropriate to concentrate discussion on geometry and chronology as the common foundation of classical and quantum mechanics. As a rather simple and well prepared field of study, geochronometry seemed suited to analysing these mutually exclusive positions.

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INTRODUCTION

The distinct positions present at the symposium may be roughly divided into three schools that differ in their philosophical interpretation of physics and their metatheoretical conception of what a physical theory is or should be: (1) The Constructivism of the Erlanger Protophysics (P. Lorenzen, P. Janich), (2) the Structuralism within the Analytical Philosophy of Science (P. Suppes, J.D. Sneed, W. Stegmüller), and (3) the Marburg Physics Theory (G. Ludwig).

(1) As a protagonist of constructivism, the "second Erlanger program" tries to give a normative foundation of the sciences, including physics, which originates from every-day experience (Lorenzen and Schwemmer 1975, Janich 1980). Its aim is to adopt an immediate, theory-independent approach which leads from the prescientific experience of civilized mankind with handicrafts, to a normal language and conceptual system covering basic notions of science. The norms necessary to characterize the basic concepts are required to initiate prescriptions and manual procedures which deliver artifactual realisations of these very norms, which all include standard etalons and gauges. The intention is to establish constructively all the fundamental concepts of lenght, duration, inertial mass and electric charge. The protophysical concepts of action, experience and norm serve as indisputably given basic terms which are assumed to have a theory-independent, prescientific adequacy. This prephysical program aims at a non-circular and purposive foundation of physics in the framework of Protophysics that - surprisingly enough - turned out to be antirelativistic. Consequently, protophysical statements are supposed to be valid beyond all scientific experience. In its own view, Protophysics professes to be a certain kind of formal pretheory for every physical theory, in which basic concepts may be operationally defined, and manufacturing norms should uniquely determine the construction of measuring instruments.

Recently, however, more substantial aspects have arisen

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in the protophysical analysis of real physical theories. Within the very basic concept of a process, Protophysics now seems to distinguish between <u>physical courses</u> and purpose-directed <u>human operations.</u> Physical courses have features that are not influenced by human actions, but are uniquely determined by natural regularities. There are thus situations in which the handicrafts performance of a protophysical norm depends not only on the individual ability of the craftsman but also on the compatibility with some real structure of the environment. This plausible statement is based on the well known physical fact, that it is not usually true that "matter can be compelled to adapt itself to ideal norms" (Lorenzen und Schwemmer p. 236). Consequently, the possibility or validity of norms is restricted by physical experience. At the same time a remarkable mitigation of the antirelativistic position was proposed by its co-originator, P. Janich - at least in the case of special relativity - which now seems compatible with a normative foundation.

(2) The second school, the structuralism of the analytical philosophy of science, may be called an informal conception relative to the formal approach of the so-called Statement-View (Stegmüller 1979, §1). Carnap and the other protagonists of this older philosophy tried to get a rational reconstruction of scientific theories by means of a formal language and an axiomatic set theory. This formalistic approach, however, is burdened with extensive complications which already arise within the formal reconstruction of mathematics. As we have known since Frege, Peano, Russel and Whitehead, and Hilbert and Skolem, even small proofs in the theory of sets are so profuse in formal language, that the foundations of more complex branches are hardly workable in this style. But if the formalized text of the syntax is enlarged by the introduction of new notions and additional rules, the resulting language is more manageable. This is an essential feature of, for example, Bourbaki's presentation, in which formalized language has been condensed to a more ordinary level, the usual language of all mathematical texts in practice (Bourbaki 1968). In this manner Bourbaki constructed a rather rigorous fondation for most of contemporary mathematics. This could serve as an example and as a tool for the philosophy of physics.

In the fifties, P. Suppes started with usual or ordinary language of mathematics to reconstruct the mathematical part of a physical theory (Suppes 1957). Without particular elaboration of the formalized language and set theory (cf. Scheibe 1981), he introduced so-called set theoretical predicates, which in any example are defined by a vocabulary of condensed mathematical notions, yielding his informal-syntactical approach. With these predicates, the mathematical part of a physical theory is represented by a class of structures which are finite tuples of sets, relations and functions. Later, Suppes' approach was enlarged by J.D. Sneed with so-called informal-semantical methods (Sneed 1971). In this more "model theoretic" view Sneed separates two levels, corresponding to a distinction between empirical and theoretical domains (called 'non-theoretical' and 'theoretical' in relation to the given theory). These domains are represented by classes of so-called partial possible models (M_{pp}) and possible models (M_p), which again are characterized by set theoretical predicates. In the empirical domain, the members of M_{pp} contain only structures (sets, relations) defined and physically clarified in predecessor theories. The essential feature of the empirical part is a certain subclass of M_{pp} - the intended applications - whose important members may correspond to paradigmatic examples and crucial experiments of the physical theory. In the domain of the possible models new concepts are introduced by "theoretical" relations. Roughly speaking, they describe the new physical aspects of the theory relative to its pretheories, i.e. its genuinely novel features. Those members of M_p which obey the physical laws are called models of the theory. The two domains are connected by a projection from M_p to M_{pp} which simply omits the theoretical relations. Finally, certain theories combine to yield a theory tree connected by the intertheoretical order relation of model theoretic inclusion which represents a kind of superstructure on the class of all theories (net-structure). This informal structuralistic view "allows a constructive criticism and a partial vindication of the philosophies of Kuhn and Lakatos" (Stegmüller 1979, p. 14).

(3) In contrast to the philosophical attempts mentioned, G. Ludwig's Physics Theory has been developed from physical problems - a metatheoretical by-product, as it were, derived from the axiomatization of quantum theory (Ludwig 1970). As is well known, the treatment of foundational questions in physics often produces rough drafts of metatheories. These physical answers to the philosophical question "What is physics?" have a long tradition including such famous exponents as Aristotle, Galileo, Newton, Leibniz, Mach, Duhem,

Poincaré, Einstein and Heisenberg.

An essential part of Physics Theory's mathematical dressing was fashioned by Nicolas Bourbaki, a pseudonym for a group of young French mathematicians who started their famous "Eléments de Mathématique" in the winter term of 1934/35. In the more than 20 volumes which have appeared to date, Bourbaki consistently employed the so-called axiomatic method and created something like the Euclid of the 20th century. This foundation of mathematics sums up the pioneering efforts of many celebrated mathematicians. For example, the works of Riemann, Dedekind, Cantor, Poincaré, Hilbert, Fréchet, Brouwer, Hausdorff and others were necessary to obtain that concept of a topological space which we can find in the lucidly and elegantly codified form of Bourbaki. The idea of the axiomatic foundation and presentation is simple. A mathematical object is not conceived by an explicit construction and not described by an ad hoc procedure peculiar to its specific nature; instead, it is presented as the combined result of a number of more general features or structures, each of which may also be found in other objects. The concrete concept is produced as a synthesis of more abstract notions. For example, the real line is defined as the commutative number field which has a continuous ordering. Explicit constructions are only needed for proofs of existence, being the self-consistency of the implicit definition. Some properties of the real number are thus already treated in the more general domains of algebraic (especially group theoretical) structures, others within the context of topological structures, or among order theoretical structures. The very combination of these parental qualities yields, to be sure, some novel features peculiar to the concept of a real number. But the axiomatic architecture leads to a much deeper understanding of the relations between the various mathematical disciplines, and they are presented in a unified form. According to Bourbaki, a mathematical theory is a species of structures: a relation, typified on the echelon above some principal and auxiliary sets, together with transportable axioms (Bourbaki 1968, IV, § 1.4). On this basis, special insights may easily be transported to other mathematical branches. Mathematics has been given a quite remarkable boost by the systematic use of the axiomatic method as practised on a grand scale by Bourbaki.

Furthermore, what is even more important in our context, Bourbaki helped to clarify what contemporary mathematics, with its remarkable standard of rigor, really is. The philosophy of physics is well advised to accept this formal paradigm. We need not, for example, discard the law of the excluded middle, or the axiom of choice, as most constructivists would have us believe. A mathematical "axiom" or postulate is nothing but part of the nominal definition of an abstract structure which does not need any concrete interpretation. Mathematics is independent of physics and is prior to all laws. The notion of geometry should thus be split into mathematical geometry as a descendant of topology, and physical geometry together with chronometry as the zeroth chapter of mechanics.

In this respect, Ludwig's Physics Theory may be regarded as a program for physics, somewhat analogous to that of Bourbaki's for mathematics. Clearly, unlike in theoretical mathematics, physical concepts are submitted to empirical interpretation, and they need additional characterization describing the correspondence to real objects and experimental results.

The first step of Ludwig therefore takes the following direction. Each well formulated physical theory PT consists of three parts: a mathematical theory MT, a domain of reality W, and a correspondence (——) between MT and W; in short, we have the identification PT = MT(——)W. Here, MT is a theory in the sense of Bourbaki, a structure which is richer than the theory of sets (an appropriate species of structures). Yet the empirical correspondence is not a mapping; it is more like a many-many relation because, in general, certain elements of MT are related to various physical states. This blurred kind of assignment reflects the typically physical situation where the mathematical results of PT do not tally exactly with the corresponding experimental results.

The second step allows for the fact that some physical concepts cannot be defined independently of PT since the physical content of PT is not completely exhausted by the components MT, (——) and W. For example, the force fields of electrodynamics, the dissipation notions of thermodynamics, and the state concepts of quantum theory are theoretical relations which can only be given physical interpretations if the whole physical theory is used. This foundational problem of physical concepts, the so-called problem of theoretical terms, is answered in Ludwig's view by restriction of the domain of reality W to a subdomain. This subset G of the given facts has to be selected in such a way that the physical concepts of MT(——)G can be defined independently of PT, and that the theoretical content of PT, corresponding to the domain of reality, may already be built up with MT(——)G.

We are not going to present a detailed representation of Ludwig's Physics Theory (cf. Ludwig 1978 or the short account of the L-program in Hartkämper and Schmidt 1981). Let us just mention two aspects which may indicate the physical adequacy of Physics Theory.

Imprecisions are a fundamental fact of life in physics: Results of measurements hold only within a certain range of accuracy, which precludes the empirical correspondence from becoming an exact mapping. Moreover, a mathematical theory trying to picture reality is not a precise representation of the world, and we cannot single out one approximating scheme. In Ludwig's approach the various concepts of imprecision are described by one mathematical tool, the uniform structure imposed on the base sets of MT. By canonical extensions of the uniform structure to power and product sets, the defined relations of PT are endowed with uniformities which may characterize the properties of the imprecisions. In addition, this imprecision structure plays a fundamental role in the comparison of theories, particularly in the case of approximative reduction and embedding, so typical of all theoretical physics.

In the second aspect we return to the problem of the foundation of physical concepts. Is it possible, by the axiomatic method of structure species, to ensure a physical interpretation of theoretical relations? Physics Theory offers a simple criterion for solving this problem. Let us assume that in MT the (principally) undefined base sets and structures are interpreted by known physical concepts; for example, they may coincide with the interpreted sets and structures of a pretheory. Such a form of MT is called the axiomatic base of PT. (It is certainly not a trivial task to find, for an arbitrary PT, a physically equivalent form which satisfies the conditions of an axiomatic base.) Now, a new structure deduced in (the axiomatic base representation of) MT will get a physical interpretation. It is uniquely determined by the interpreted sets and relations which are used for the deduction. If there are certain interpreted relations of MT and corresponding measurements which can be combined to yield an indirect measurement of the deduced structure, it is reasonable to say that the structure represents a set of real physical facts. This may justify calling it an established physical structure or a new physical concept (cf. Ludwig 1978, § 10.5).

REFERENCES

Bourbaki, N.: 1968, 'Theory of Sets', Hermann, Paris.
Hartkämper, A. and H.-J. Schmidt: 1981, 'Structure and Approximation in Physical Theories', Plenum Press, New York.
Janich, P.: 1980, 'Die Protophysik der Zeit', Suhrkamp Verlag, Frankfurt am Main.
Lorenzen, P. und O. Schwemmer: [2]1975, 'Konstruktive Logik, Ethik und Wissenschaftstheorie', BI Mannheim.
Ludwig, G.: 1970, 'Deutung des Begriffs "physikalische Theorie" und axiomatische Grundlegung der Hilbertraumstruktur der Quantenmechanik durch Hauptsätze des Messen', Lecture Notes in Physics, 4, Springer (in addition: 'Foundation of Quantum Mechanics', 2volumes, Translation in Englisch in preparation, to appear by Springer).
Ludwig, G.: 1978, 'Die Grundstrukturen einer physikalischen Theorie', Springer.
Scheibe, E.: 1981, 'A Comparison of Two Recent Views on Theories' in Hartkämper and Schmidt 1981.
Sneed, J.D.: 1971, 'The Logical Structure of Mathematical Physics', Reidel, Dordrecht.
Stegmüller, W.: 1979, 'The Structuralistic View of Theories', Springer.
Suppes, P.: 1957, 'Introduction to Logic', D.v. Nostrand, Princeton.

CONTENTS

Günther Ludwig

IS THE GEOMETRY OF PHYSICAL SPACE A FORM OF PURE SENSIBLE INTUITION? A TECHNICAL CONSTRUCTION? OR A STRUCTURE OF REALITY?

In this paper we shall not be able to present a complete or definitive answer to the above question. We only attempt to examine this question without prejudice and we shall not make the claim that only one of the above three possibilities is correct. In fact, we find that each of these viewpoints has a certain justification. We shall attempt to clarify this problem by examining the relationships between these three viewpoints.

We wish to confine the problem to the case in which Euclidean geometry can be used to describe the geometry of the real space - that is - we shall not consider problems concerning "cosmology" or "black holes".

§ 1 Three extreme viewpoints

In my opinion each of following three extreme viewpoints I to III is incorrect.

I) Euclidean geometry is nothing other than a form of pure sensible intuition which is a "necessary basic requirement" for all physics. We leave open the question how human beings may have gained this form of pure sensible intuition, whether this form was gained during evolution by natural selection or was gained in the first years of our life, or is a form impressed in our mind. As a "necessary basic requirement" we mean that Euclidean geometry will be a necessary structure for the formulation of experience.

The following two arguments can be raised in opposition to this opinion:

1) In the history of physics we have found that it is possible to use non-Euclidean geometries in physics. On the other hand it is not possible to see how we are able by pure sensible intuition to state whether a physical realization of a plane - obtained for example by means of a "grinding procedure of three plates" - corresponds to an intuitive plane. In other words: How are we be able to determine whether a geo-

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metric figure obtained by technical procedures corresponds to our intuitive notions.

2) The discussion of problems in the foundations of mathematics has weakened our faith in intuition. We no longer believe that it is possible to verify the axioms of Euclidean geometry by intuition - that is - we are not convinced by intuition that Euclidean geometry is internally consistent. For example we are not convinced that there can be only one line passing through a given point which is parallel to a given line. By intuition we cannot exclude the possibility that there exists a finite range of angles (see fig. 1) for straight

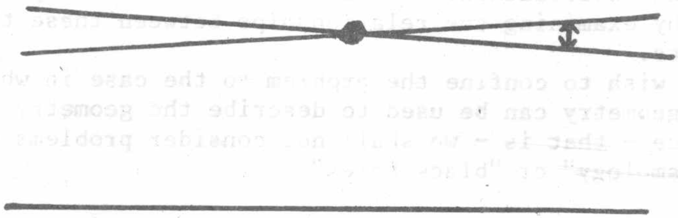

Fig. 1

lines passing through a given point which does not intersect a given straight line. This angle may depend upon the distance between the given line and the given point.

If we examine our understanding of geometrical figures it is not difficult to see that we have developed a certain intuition which is not sufficiently precise that one may deduce precise mathematical relationships. For example we have no intuition for the behaviour of straight lines, planes, etc. at infinity. Our intuition corresponds to the case in which only a bounded portion of geometry can be seen, and the remaining portion cannot be observed.

II. Another extreme opinion is that Euclidean geometry is a technical constructed structure which is used to obtain a reference system for the description of physical phenomena and that such constructions are arbitrary as - for instance - the well known Euclidean maps of our earth. Such maps are very useful. Nevertheless the Euclidean geometry of the map is not identical with the geometry of the surface of the earth. In physics we are not interested in arbitrary maps of the space but in the actual ("physical") geometry of the space regardless of how the actual geometry is determined - from experience or by other methods.

III. The third extreme opinion is as follows: The theorems of Euclidean geometry may be found in nature, the theorems can be "read" from nature despite the fact that we know that the mathematical form of the theorems includes idealizations about nature. In this sense one believes Euclidean geometry to be a structure which is present in nature.

It is evidently no simple matter to read the Euclidean geometry from nature. Realizations of geometrical figures such as straight lines or planes cannot be found in nature, they have to be "made". But how do we make such geometrical figures?

If we reflect on these three extreme viewpoints it seems that it is possible to raise objections to each of these viewpoints by means of the others. Therefore we have to conclude that each of these viewpoints is right in some respect and we are wrong if we claim that one of these is the complete solution of the space problem.

§ 2 Methods and interests

We may obtain a better insight into the problem if we admit that various persons have various interests concerning the space problem and that these various interests will determine the methods used to describe and solve the problem, or more precisely, to solve the parts of the problem of interest.

For example the method depends upon whether one is asking what has to be anticipated before every experience with objects existing side by side; or whether one is asking, what are the necessary postulates on manipulation of solid bodies for the purpose of measuring distances between spots (finite physical realizations of points) in the real space; or whether one is asking, what are the "real structures" of space which make it "possible" to construct measuring apparatuses in the usual way.

Since our primary interest is in this last question, we shall not consider questions concerning the historical development of physics. We only seek the formulation of a physical theory by which it is possible to answer the question: what aspects of geometry describe a real structure?